CLASS – X
MATHEMATICS
CHAPTER – 5
QUADRATIC EQUATIONS

NOTES

- **Quadratic Equation**

An equation of the form $ax^2 + bx + c = 0$ where $a, b, c \in R$ and $a \neq 0$ is called a quadratic equation with real coefficients in the variable x.

- **Roots of a quadratic equation**

A real number α is called a root of the quadratic equation $ax^2 + bx + c = 0, a \neq 0$ if $a\alpha^2 + b\alpha + c = 0$.

Note: (i) A root of the quadratic equation $ax^2 + bx + c = 0$ is a zero of the polynomial $ax^2 + bx + c$ and vice-versa.

(ii) For every quadratic equation, there are two roots.

- **Algebraic methods of solving a quadratic equation**
  - Method of Factorisation
  - Method of Completing Perfect Square.

- **Solve $ax^2 + bx + c = 0, a \neq 0$ by Method of Completing Perfect Square**.

Ans: We have,

$$ax^2 + bx + c = 0, \ a \neq 0$$

$$\Rightarrow 4a^2x^2 + 4abx + 4ac = 0 \text{ (Multiplying both sides by } 4a)$$

$$\Rightarrow (2ax)^2 + 2 \times 2ax \times b + b^2 + 4ac = b^2$$

$$\Rightarrow (2ax + b)^2 = b^2 - 4ac$$

$$\Rightarrow 2ax + b = \pm\sqrt{b^2 - 4ac}$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Thus, the roots of the quadratic equation $ax^2 + bx + c = 0, \ a \neq 0$ are $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$.

- **Quadratic Formula:** Roots of a quadratic equation $ax^2 + bx + c = 0, \ a \neq 0$ can be found directly by using the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. This formula is known as quadratic formula.

Discriminant

The quantity of $b^2 - 4ac$ is called the discriminant of the quadratic equation $ax^2 + bx + c = 0$.

❖ **Nature of the Roots**

In the quadratic equation $ax^2 + bx + c = 0, a \neq 0$,

- if $b^2 - 4ac > 0$, the roots are real & unequal.
  (i) if $a, b, c \in Q$ and $b^2 - 4ac =$ a square number, the roots are rational & unequal.
  (ii) if $a, b, c \in Q$ and $b^2 - 4ac \neq$ a square number, the roots are irrational & unequal.
- if $b^2 - 4ac = 0$, the roots are real & equal.
- if $b^2 - 4ac < 0$, the roots are not real & unequal.

Note: The roots of a quadratic equation $ax^2 + bx + c = 0$ are real if $b^2 - 4ac \geq 0$.

❖ **Relation between Roots and Coefficients**

The two roots of the equation $ax^2 + bx + c = 0$ are

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

(i) $\alpha + \beta =$ sum of the roots

$$= \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-b+\sqrt{b^2-4ac}-b-\sqrt{b^2-bac}}{2a}$$

$$= \frac{-2b}{2a}$$

$$= \frac{-b}{a}$$

$$= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}.$$

(ii) $\alpha . \beta =$ Product of the roots

$$= \frac{-b+\sqrt{b^2-4ac}}{2a} \times \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$= \frac{(-b)^2-(\sqrt{b^2-4ac})^2}{4a^2}$$

$$= \frac{b^2-b^2+4ac}{4a^2}$$

$$= \frac{c}{a}$$

$$= \frac{\text{constant term}}{\text{coefficient of } x^2}$$

❖ **Important Deductions**

In the Quadratic Equation $ax^2 + bx + c = 0, a \neq 0$,

- if $b = 0$, the roots equal in magnitude but opposite in signs.
- if $c = 0$, the roots are 0 and $\frac{-b}{a}$.
- if $a = c$, the roots are reciprocal of each other.
- if $a + b + c = 0$, the roots are 1 and $\frac{c}{a}$.

❖ **Formation of Quadratic Equation when the roots are given**

Let $ax^2 + bx + c = 0$ be the required equation whose roots are α and β.

Now, $ax^2 + bx + c = 0$

$$\Rightarrow x^2 - \left(-\frac{b}{a}\right)x + \frac{c}{a} = 0$$

$$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\text{i.e. } x^2 - (\text{sum of the roots})x + \text{product of the roots} = 0$$

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