



জাগোৱাফলৈ আৰু নাজুৱাফলৈ (আম)

DEPARTMENT OF EDUCATION (S)

Government of Manipur

CLASS – IX
MATHEMATICS
CHAPTER – 3
COORDINATE GEOMETRY

Cartesian Co-ordinates

Rene Descartes, the great French mathematician and philosopher propounded a system of describing the position of a point in a plane. In honour of Descartes, this system used for describing the position of a point in a plane is known as the Cartesian System of Co-ordinates.

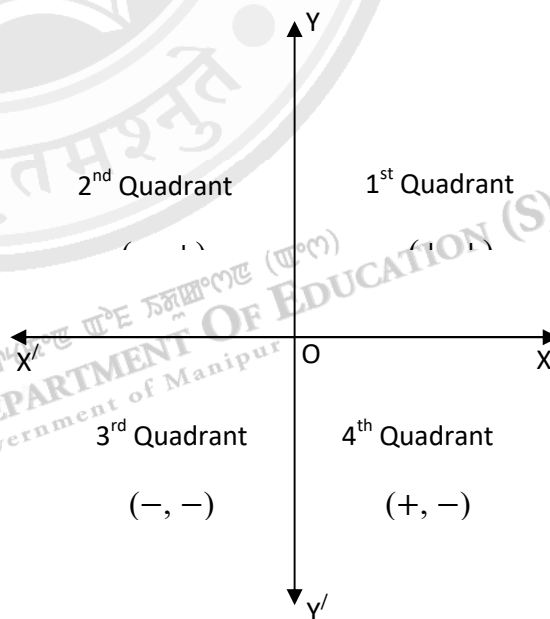
Rectangular Cartesian Co-ordinate System

To fix the position of a point P in a plane, we take two fixed perpendicular lines conventionally one horizontal and other vertical on the plane intersecting at a point. The horizontal line is called X-axis and the vertical line is called Y-axis. The plane with these two co-ordinate axes is known as the Cartesian plane. The point of intersection of the co-ordinate axes is called origin.

Quadrants

The co-ordinate axes divide the plane into four regions. Each region is called a Quadrant.

- If a point lies in the 1st quadrant, the signs of its co-ordinates are of the form $(+, +)$.
- If a point lies in the 2nd quadrant, then the signs of its co-ordinates are of the form $(-, +)$.
- If a point lies in the 3rd quadrant, then the signs of its co-ordinates are of the form $(-, -)$.
- If a point lies in the 4th quadrant, then the signs of its co-ordinates are of the form $(+, -)$.





Note: (i) For a point $P(a, b)$ on the Cartesian plane, a is called the x -coordinate or abscissa and b is called the y -coordinate or ordinate of the point P .

(ii) If a point lies on the X -axis, its ordinate is zero and if a point lies on the Y -axis, its abscissa is zero.

Plotting of point on a plane:

The steps of locating a point with a given co-ordinates on a plane

Steps 1: We take the co-ordinate axes on the plane so that the origin is at a suitable position preferably at the middle of the plane

Steps 2: We choose the scale on the axes so that the point corresponding to the given co-ordinates may be shown in the plane

Steps 3: We check the sign of the abscissa. If it is positive, we take the required units starting from the origin O along the positive direction of the X -axis. If it is negative, we take required units starting from O along the negative direction of X -axis. If it is zero, it remains at O .

Steps 4: We name the point obtained in step 3, A (say).

Steps 5: We check the sign of the ordinate. If it is positive, we take the required units starting from A along the positive direction of the Y -axis. If it is negative, we take the required units starting from A along the negative direction of the Y -axis. If it is zero, it remains at A .

Steps 6: We name the point obtained in step 5, P (say).

Then P is the required point on the plane with the given co-ordinates.

SOLUTIONS

EXERCISE 3.1

1. In which quadrants, do the following points lie?

(a) $(-2, 7)$

Ans: Second quadrant

(b) $(-2, -3)$

Ans: Third quadrant.

(c) $(1, 6)$

Ans: First quadrant

(d) $(5, -3)$

Ans: Fourth quadrant

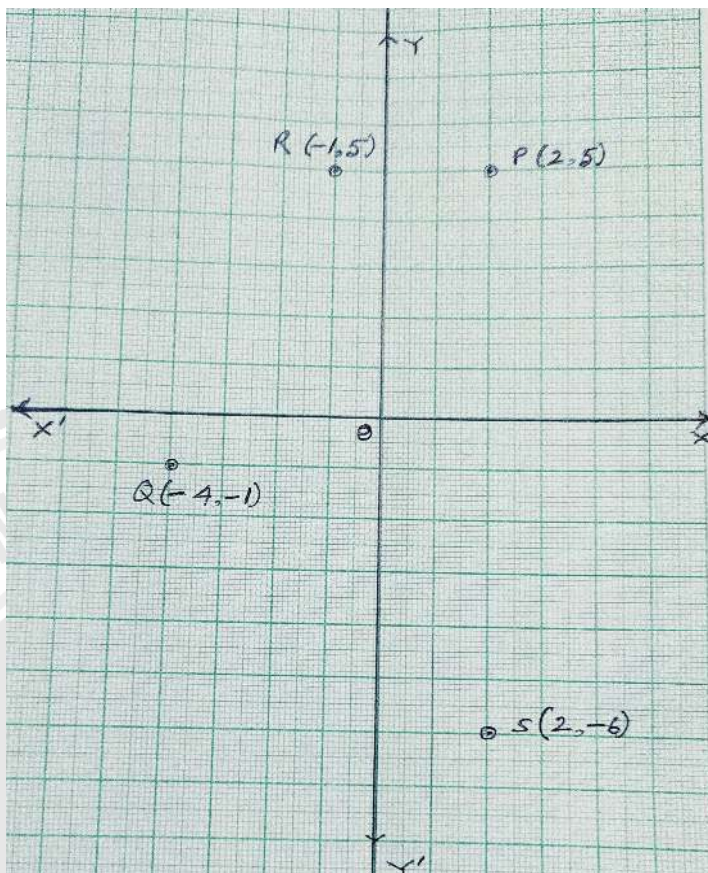




2. Locate the points given below on the Cartesian plane and also state the quadrants in which they lie.

- (a) $(2, 5)$
- (b) $(-4, -1)$
- (c) $(-1, 5)$
- (d) $(2, -6)$

Solution:



- (a) On the Cartesian plane, P represents $(2, 5)$. It lies in the first quadrant.
 - (b) On the Cartesian plane, Q represents $(-4, -1)$. It lies in the third quadrant.
 - (c) On the Cartesian plane, R represents $(-1, 5)$. It lies in the Second quadrant.
 - (d) On the Cartesian plane, S represents $(2, -6)$. It lies in the fourth quadrant.
3. On which axis do the following points lie:

- a) $(1, 0)$

Ans: It lies on the X-axis.

- b) $(0, 5)$

Ans: It lies on the Y-axis.

- c) $(-3, 0)$

Ans: It lies on the X-axis.

- d) $(0, -2)$

Ans: It lies on the Y-axis.

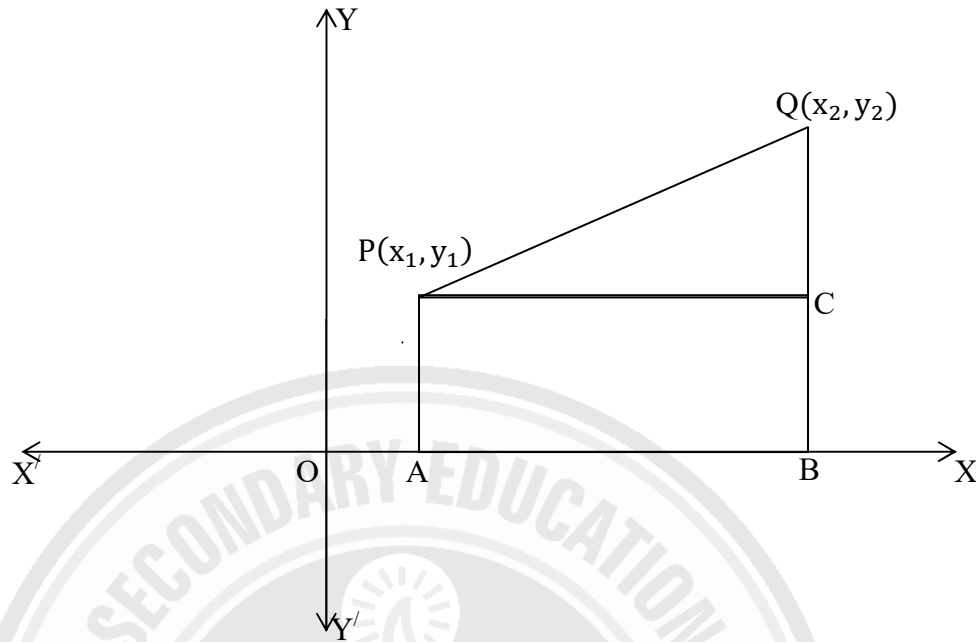
- e) $(0, 0)$

Ans: It lies on both the X and Y axes.





Distance between two points:



Let XOX' and YOY' be the two co-ordinate axes. $P(x_1, y_1)$ and $Q(x_2, y_2)$ be any two points in the Cartesian plane. PA and QB are drawn perpendicular to the X -axis. PC is also drawn perpendicular to QB meeting QB at C .

We have $BC = AP = y_1$

$$PC = AB = OB - OA = x_2 - x_1$$

$$\text{and } QC = QB - BC = QB - PA = y_2 - y_1$$

In $\triangle PCQ$, $\angle PCQ = 90^\circ$,

\therefore by Pythagoras theorem, we have

$$PQ^2 = PC^2 + QC^2$$

$$\Rightarrow PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\Rightarrow PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

\therefore The distance between any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Note:

1. The distance between any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ can also be taken as

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

2. The distance of any point (x, y) from the origin O is $\sqrt{x^2 + y^2}$.



SOLUTIONS

EXERCISE 3.2

1. Find the distance of the following points from the origin:

(a) (2, 5)

Solution: The distance of the point (2, 5) from the origin $= \sqrt{2^2 + 5^2}$
 $= \sqrt{4 + 25}$
 $= \sqrt{29}$ units

(b) (6, -10)

Solution: The distance of the point (6, -10) from the origin $= \sqrt{6^2 + (-10)^2}$
 $= \sqrt{36 + 100}$
 $= \sqrt{136} = \sqrt{2^2 \times 2 \times 17}$
 $= 2\sqrt{34}$ units

(c) (-6, 12)

Solution: The distance of the point (-6, 12) from the origin $= \sqrt{(-6)^2 + 12^2}$
 $= \sqrt{36 + 144}$
 $= \sqrt{180}$
 $= \sqrt{2^2 \times 3^2 \times 5}$
 $= 2 \times 3\sqrt{5}$
 $= 6\sqrt{5}$ units

2. Find the distance between the pairs of points

(a) (1, 3), (7, 2)

Solution: The distance between (1, 3) and (7, 2) $= \sqrt{(1 - 7)^2 + (3 - 2)^2}$
 $= \sqrt{(-6)^2 + 1^2}$
 $= \sqrt{36 + 1}$
 $= \sqrt{37}$ units

(b) (7, -2), (3, -1)

Solution: The distance between (7, -2) and (3, -1) $= \sqrt{(7 - 3)^2 + \{-2 - (-1)\}^2}$
 $= \sqrt{4^2 + (-2 + 1)^2}$
 $= \sqrt{16 + (-1)^2}$
 $= \sqrt{16 + 1}$
 $= \sqrt{17}$ units



(c) (10, 4), (-1, -2)

Solution: The distance between (10, 4) and (-1, -2) $= \sqrt{\{10 - (-1)\}^2 + \{4 - (-2)\}^2}$
 $= \sqrt{(10 + 1)^2 + (4 + 2)^2}$
 $= \sqrt{11^2 + 6^2}$
 $= \sqrt{121 + 36}$
 $= \sqrt{157}$ units

(d) (-1, 3), (4, -2)

Solution: The distance between (-1, 3) and (4, -2) $= \sqrt{(-1 - 4)^2 + \{3 - (-2)\}^2}$
 $= \sqrt{(-5)^2 + (3 + 2)^2}$
 $= \sqrt{25 + 5^2}$
 $= \sqrt{25 + 25}$
 $= \sqrt{2 \times 25}$
 $= \sqrt{2 \times 5^2}$
 $= 5\sqrt{2}$ units

3. The Co-ordinates of A are (-4, 8) and those of B are (x, 3). Find x of AB=13.

Solution: We have,

$$AB = 13$$

$$\Rightarrow \sqrt{(-4 - x)^2 + (8 - 3)^2} = 13$$

$$\Rightarrow \{-(x + 4)\}^2 + (8 - 3)^2 = 13^2$$

$$\Rightarrow (x + 4)^2 + 5^2 = 169$$

$$\Rightarrow x^2 + 8x + 16 + 25 = 169$$

$$\Rightarrow x^2 + 8x + 41 = 169$$

$$\Rightarrow x^2 + 8x - 128 = 0$$

$$\Rightarrow x^2 + (16 - 8)x - 128 = 0$$

$$\Rightarrow x^2 + 16x - 8x - 128 = 0$$

$$\Rightarrow x(x + 16) - 8(x + 16) = 0$$

$$\Rightarrow (x + 16)(x - 8) = 0$$

$$\Rightarrow \text{Either } x + 16 = 0 \text{ or } x - 8 = 0$$

$$\Rightarrow x = -16 \text{ or } x = 8$$

$$\therefore x = -16 \text{ or } 8$$



4. Show that the point A (2,7) , B (3,0), C (-4,-1) are vertices of an isosceles triangle and find the length of the base.

Solution: Here,

$$\begin{aligned} AB &= \sqrt{(2-3)^2 + (7-0)^2} \\ &= \sqrt{(-1)^2 + 7^2} \\ &= \sqrt{1+49} \\ &= \sqrt{50} \\ &= \sqrt{2 \times 5^2} \\ &= 5\sqrt{2} \\ BC &= \sqrt{\{3-(-4)\}^2 + \{0-(-1)\}^2} \\ &= \sqrt{(3+4)^2 + (0+1)^2} \\ &= \sqrt{7^2 + 1^2} \\ &= \sqrt{49+1} \\ &= \sqrt{50} \\ &= \sqrt{2 \times 5^2} \\ &= 5\sqrt{2} \\ AC &= \sqrt{\{2-(-4)\}^2 + \{7-(-1)\}^2} \\ &= \sqrt{(2+4)^2 + (7+1)^2} \\ &= \sqrt{6^2 + 8^2} \\ &= \sqrt{36+64} \\ &= \sqrt{100} \\ &= 10 \end{aligned}$$

As, $AB = BC \neq AC$, the points are vertices of an isosceles triangle and the length of the base AC is 10 units.

5. Show that the points A (4,3) , B(1,2) , C(1,0), D (4,1) are vertices of a parallelogram and find the length of its diagonals.

Solution: Here, AB

$$\begin{aligned} &= \sqrt{(4-1)^2 + (3-2)^2} \\ &= \sqrt{3^2 + 1^2} \\ &= \sqrt{9+1} \\ &= \sqrt{10} \end{aligned}$$



$$\begin{aligned} BC &= \sqrt{(1-1)^2 + (2-0)^2} \\ &= \sqrt{2^2} \\ &= 2 \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(1-4)^2 + (0-1)^2} \\ &= \sqrt{(-3)^2 + (-1)^2} \\ &= \sqrt{9+1} \\ &= \sqrt{10} \end{aligned}$$

$$\begin{aligned} \text{and } DA &= \sqrt{(4-4)^2 + (1-3)^2} \\ &= \sqrt{0^2 + (-2)^2} \\ &= \sqrt{2^2} \\ &= 2 \end{aligned}$$

Thus $AB = CD$ and $BC = DA$.

So, the opposite sides of the quadrilateral ABCD are equal in length.

Hence, the points are the vertices of parallelogram.

$$\text{Here, } AC = \sqrt{(4-1)^2 + (3-0)^2} = \sqrt{3^2 + 3^2} = \sqrt{2 \times 3^2} = 3\sqrt{2}$$

$$BD = \sqrt{(1-4)^2 + (2-1)^2} = \sqrt{(-3)^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

\therefore the lengths of the diagonals are $3\sqrt{2}$ units and $\sqrt{10}$ units.

6. Show that (2,1) is the centre of the circum- circle of the triangle whose vertices are (-3,-9), (13, -1) and (-9,3).

$$\begin{aligned} \text{Solution: The distance of (2,1) from } (-3,-9) &= \sqrt{(-3-2)^2 + (-9-1)^2} \\ &= \sqrt{(-5)^2 + (-10)^2} \\ &= \sqrt{25+100} \\ &= \sqrt{125} \\ &= 5\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{The distance of (2, 1) from } (13,-1) &= \sqrt{(13-2)^2 + (-1-1)^2} \\ &= \sqrt{11^2 + (-2)^2} \\ &= \sqrt{121+4} \\ &= \sqrt{125} \\ &= 5\sqrt{5} \end{aligned}$$



$$\begin{aligned}\text{The distance of } (2,1) \text{ from } (-9, 3) &= \sqrt{(-9-2)^2 + (3-1)^2} \\ &= \sqrt{(-11)^2 + 2^2} \\ &= \sqrt{121+4} \\ &= \sqrt{125} \\ &= 5\sqrt{5}\end{aligned}$$

Now, we have (2,1) is equidistant from (-3,-9), (13,-1) and (-9,3).

Hence, (2,1) is the centre of the circum-circle of the triangle whose vertices are (-3,-9), (13,-1) and (-9,3).

7. Show that the points (4, 4), (5,-1), (-6, 2) are the vertices of a right triangle.

Solution: Let A(4,4), B(5,-1), C(-6,2) be the three points.

$$\begin{aligned}\text{Then, AB} &= \sqrt{(4-5)^2 + \{4-(-1)\}^2} \\ &= \sqrt{(-1)^2 + (4+1)^2} \\ &= \sqrt{1+5^2} \\ &= \sqrt{1+25} \\ &= \sqrt{26} \\ \text{BC} &= \sqrt{\{5-(-6)\}^2 + (-1-2)^2} \\ &= \sqrt{(5+6)^2 + (-3)^2} \\ &= \sqrt{121^2 + 9} \\ &= \sqrt{121+9} \\ &= \sqrt{130}\end{aligned}$$

$$\begin{aligned}\text{and AC} &= \sqrt{\{4-(-6)\}^2 + (4-2)^2} \\ &= \sqrt{(4+6)^2 + 2^2} \\ &= \sqrt{10^2 + 4} \\ &= \sqrt{100+4} \\ &= \sqrt{104}\end{aligned}$$

$$\text{Now, } BC^2 = (\sqrt{130})^2 = 130$$

$$\text{and } AB^2 + AC^2 = (\sqrt{26})^2 + (\sqrt{104})^2 = 26 + 104 = 130$$

As $BC^2 = AB^2 + AC^2$, by converse of Pythagoras Theorem, ABC is a right triangle right angled at A.

Hence, the points (4, 4), (5,-1), (-6, 2) are the vertices of a right triangle.



8. Show that the points (p, p) , $(-p, -p)$, $(p\sqrt{3}, -p\sqrt{3})$ are the vertices of an equilateral triangle.

Solution: Let $A(p, p)$, $B(-p, -p)$, $C(p\sqrt{3}, -p\sqrt{3})$ be the three points.

$$\begin{aligned}\text{Then, } AB &= \sqrt{\{p - (-p)\}^2 + \{p - (-p)\}^2} \\ &= \sqrt{(p + p)^2 + (p + p)^2} \\ &= \sqrt{(2p)^2 + (2p)^2} \\ &= \sqrt{2 \times (2p)^2} \\ &= 2\sqrt{2}p\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(-p - p\sqrt{3})^2 + \{-p - (-p\sqrt{3})\}^2} \\ &= \sqrt{\{-(p + p\sqrt{3})\}^2 + (-p + p\sqrt{3})^2} \\ &= \sqrt{(p + p\sqrt{3})^2 + (p\sqrt{3} - p)^2} \\ &= \sqrt{\{p^2 + 2p \cdot p\sqrt{3} + (p\sqrt{3})^2\} + \{(p\sqrt{3})^2 - 2p\sqrt{3} \cdot p + p^2\}} \\ &= \sqrt{p^2 + 2\sqrt{3}p^2 + 3p^2 + 3p^2 - 2\sqrt{3}p^2 + p^2} \\ &= \sqrt{p^2 + 3p^2 + 3p^2 + p^2} \\ &= \sqrt{8p^2} \\ &= 2\sqrt{2}p\end{aligned}$$

$$\begin{aligned}\text{And } AC &= \sqrt{(p - p\sqrt{3})^2 + \{p - (-p\sqrt{3})\}^2} \\ &= \sqrt{(p - p\sqrt{3})^2 + (p + p\sqrt{3})^2} \\ &= \sqrt{p^2 - 2p \cdot p\sqrt{3} + (p\sqrt{3})^2 + p^2 + 2p \cdot p\sqrt{3} + (p\sqrt{3})^2} \\ &= \sqrt{p^2 + 3p^2 + p^2 + 3p^2} \\ &= \sqrt{8p^2} \\ &= 2\sqrt{2}p\end{aligned}$$

Here, $AB = BC = AC$

Hence, the points $A(p, p)$, $B(-p, -p)$ and $C(p\sqrt{3}, -p\sqrt{3})$ are the vertices of an equilateral triangle.