CLASS – IX
HIGHER MATHEMATICS
CHAPTER – 2
RATIO AND PROPORTION

SOLUTIONS

EXERCISE 2.1

1. If $a:b=2:3$, find the value of $3a+5b:7a+2b$.

Solution: We have, $a:b=2:3$

Then, $\frac{3a+5b}{7a+2b}=\frac{3(a/b)+5}{7(a/b)+2}$

$=\frac{3(2/3)+5}{7(2/3)+2}$

$=\frac{7\times 3}{14+6}=\frac{21}{20}$

$=21:20$

2. If $2a+5b:3a+2b=26:27$, find $a:b$.

Solution: Here,

$2a+5b:3a+2b=26:27$

$\Rightarrow 27(2a+5b)=26(3a+2b)$ [product of extremes= product of means]

$\Rightarrow 54a+135b=78a+52b$

$\Rightarrow 78a-54a=135b-52b$

$\Rightarrow 24a=83b$

$\Rightarrow \frac{a}{b}=\frac{83}{24}$

$\Rightarrow a:b=83:24$

3. Two numbers are in the ratio 8:9 and their sum is 204. Find the numbers.

Solution: Let the two numbers be a and 204 - a.

Then,

$$\Rightarrow \frac{a}{204-a}=\frac{8}{9}$$
$$\Rightarrow 9a=8(204-a)$$
$$\Rightarrow 9a+8a=8\times 204$$
$$\Rightarrow 17a=1632$$
$$\Rightarrow a=\frac{1632}{17}$$
$$\Rightarrow a=96.$$

Hence, the two numbers are 96 and 204 – 96=108.

4. Two numbers are in the ratio 7:5 and their difference is 60. Find the numbers.

Solution: Let the two numbers be $7x$ and $5x$.

Then,

$$7x-5x=60$$
$$\Rightarrow 2x=60$$
$$\Rightarrow x=30$$

Hence, the two numbers are 7×30 and 5×30 i.e. 210 and 150.

5. Two numbers are in the ratio 3:5 and if 2 be added to each, the sums are in the ratio 5 :8. Find the numbers.

Solution: Let the two numbers be $3x$ and $5x$.

By question we have,

$$\frac{3x+2}{5x+2}=\frac{5}{8}$$

$$\Rightarrow 8(3x+2)=5(5x+2)$$

$$\Rightarrow 24x+16=25x+10$$

$$\Rightarrow x=6.$$

Required numbers are 3×6 and 5×6 i.e. 18 and 30.

6. Find the value of x for which the ratio $17-x:13-x$ is equal to 3.

Solution: We have,

$$\frac{17-x}{13-x}=3$$
$$\Rightarrow 17-x=3(13-x)$$
$$\Rightarrow 17-x=39-3x$$
$$\Rightarrow 2x=22$$
$$\Rightarrow x=11.$$

7. What number must be added to each term of the ratio 23:27 so that it may become equal to 10: 11.

Solution: Let x be the number to be added to each term.

Then,

$$\frac{23+x}{27+x}=\frac{10}{11}$$
$$\Rightarrow 11(23+x)=10(27+x)$$
$$\Rightarrow 11x-10x=270-253$$
$$\Rightarrow x=17.$$

8. Find the third proportional to :
(i) 4, 6 (ii) 16, 12 (iii) 24, 36 (iv) 125, 75.

Solution: (i) Let x be the third proportional to 4 and 6.

Then, 4, 6, x are in continued proportion.

$$\therefore \frac{4}{6}=\frac{6}{x}$$

$$\Rightarrow x=\frac{36}{4}$$
$$\Rightarrow x=9.$$

(ii) Let x be the third proportional to 16 and 12 .
Then, 16, 12, x are in continued proportion.

$$\therefore \frac{16}{12}=\frac{12}{x}$$
$$\Rightarrow x=\frac{144}{16}$$
$$\Rightarrow x=9.$$

(iii) Let x be the third proportional to 24 and 36 .
Then, 24, 36, x are in continued proportion.

$$\therefore \frac{24}{36}=\frac{36}{x}$$
$$\Rightarrow x=\frac{36\times 36}{24}$$
$$\Rightarrow x=54.$$

(iv) Let x be the third proportional to 125 and 75.
Then, 125, 75, x are in continued proportion.

$$\therefore \frac{125}{75}=\frac{75}{x}$$
$$\Rightarrow x=\frac{75\times 75}{125}$$
$$\Rightarrow x=45.$$

9. Find the mean proportional between:

(i) 5, 125 (ii) 3, 27 (iii) $\frac{1}{12}$, 48

Solution: (i) Let x be the mean proportional between 5 and 125.

Then, 5, x , 125 are in continued proportion.

$$\therefore \frac{5}{x}=\frac{x}{125}$$
$$\Rightarrow x^2=5\times 125$$
$$\Rightarrow x=\sqrt{5\times 125}.$$
$$\Rightarrow x=25$$

(ii) Let x be the mean proportional between 3 and 27.

Then, 3, x , 27 are in continued proportion.

$$\therefore \frac{3}{x}=\frac{x}{27}$$
$$\Rightarrow x^2=3\times 27$$
$$\Rightarrow x=\sqrt{3\times 27}$$
$$\Rightarrow x=9.$$

(iii) Let x be the mean proportional between $\frac{1}{12}$ and 48.

Then, , $\frac{1}{12}$ x , 48 are in continued proportion.

$$\therefore \frac{1/12}{x} = \frac{x}{48}$$

$$\Rightarrow x^2 = \frac{1}{12} \times 48$$
$$\Rightarrow x = \sqrt{\frac{1}{12} \times 48}$$
$$\Rightarrow x = 2.$$

10. Find the fourth proportional to :

(i) 3, 4, 6 (ii) 14, 24, 35 (iii) $\frac{1}{\sqrt{3}}, \sqrt{3}, \sqrt{3}.$

Solution: (i) Let x be the fourth proportional to 3, 4, 6.

Then, 3, 4, 6, x are in proportion.

$$\therefore \frac{3}{4} = \frac{6}{x}$$

$$\Rightarrow 3 \times x = 6 \times 4$$
$$\Rightarrow x = \frac{6 \times 4}{3}$$
$$\Rightarrow x = 8.$$

(ii) Let x be the fourth proportional to 14, 24, 35

Then, 14, 24 , 35, x are in proportion.

$$\therefore \frac{14}{24} = \frac{35}{x}$$

$$\Rightarrow 14 \times x = 35 \times 24$$

$$\Rightarrow x = \frac{35 \times 24}{14}$$
$$\Rightarrow x = 60.$$

(iii) Let x be the fourth proportional to $\frac{1}{\sqrt{3}},\ \sqrt{3},\ 13.$

Then, $\frac{1}{\sqrt{3}},\ \sqrt{3},\ 13,\ x$ are in proportion.

$$\therefore\ \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{13}{x}$$

$$\Rightarrow \frac{1}{\sqrt{3}} \times x = 13 \times \sqrt{3}$$

$$\Rightarrow x = 13 \times 3$$

$$\Rightarrow x = 39.$$

11. If $a:b = c:d$, prove the following:

(i) $a:b = a+c:b+d = a-c:b-d$

Solution: We have,

$$\frac{a}{b} = \frac{c}{d} \qquad \ldots\ldots.. \ (1)$$

$$\Rightarrow \frac{a}{c} = \frac{b}{d}$$

$$\Rightarrow \frac{a+c}{c} = \frac{b+d}{d} \qquad \text{[Componendo]}$$

$$\Rightarrow \frac{a+c}{b+d} = \frac{c}{d} \qquad \ldots\ldots\ldots. \ (2)$$

Again, $$\frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow \frac{a}{c} = \frac{b}{d}$$

$$\Rightarrow \frac{a-c}{c} = \frac{b-d}{d} \qquad \text{[Dividendo]}$$

$$\Rightarrow \frac{a-c}{b-d} = \frac{c}{d} \qquad \ldots\ldots.. \ (3)$$

From (1), (2) and (3) we have,

$$\frac{a}{b} = \frac{a+c}{b+d} = \frac{a-c}{b-d}.$$

Hence proved.

(ii) $a^2 : b^2 = a^2 + c^2 : b^2 + d^2$

Solution: We have,

$$\frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow \left(\frac{a}{b}\right)^2 = \left(\frac{c}{d}\right)^2 \qquad \text{[squaring on both sides]}$$

$$\Rightarrow \frac{a^2}{b^2} = \frac{c^2}{d^2} \qquad \ldots\ldots\ldots (1)$$

$$\Rightarrow \frac{a^2}{c^2} = \frac{b^2}{d^2} \qquad \text{[Alternendo]}$$

$$\Rightarrow \frac{a^2+c^2}{c^2} = \frac{b^2+d^2}{d^2} \qquad \text{[componendo]}$$

$$\Rightarrow \frac{a^2+c^2}{b^2+d^2} = \frac{c^2}{d^2} = \frac{a^2}{b^2} \qquad \text{[from (1)]}$$

$\therefore\ a^2 : b^2 = a^2 + c^2 : b^2 + d^2$

Hence proved.

(iii) $a^2 + c^2 : b^2 + d^2 = ac : db$

Solution: We have, $\frac{a}{b} = \frac{c}{d} = k(say)$

$$\Rightarrow \quad a = bk,\ c = dk.$$

Now, $$\frac{a^2+c^2}{b^2+d^2} = \frac{(bk)^2+(dk)^2}{b^2+d^2}$$

$$= \frac{(b^2+d^2)k^2}{b^2+d^2} = k^2$$

And, $$\frac{ac}{db} = \frac{bdk^2}{db} = k^2$$

Hence, $a^2 + c^2 : b^2 + d^2 = ac : db$.

(iv) $a+b:c+d=\sqrt{a^2+b^2}:\sqrt{c^2+d^2}=\sqrt{3a^2+5b^2}:\sqrt{3c^2+5d^2}$

Solution: We have, $\frac{a}{b}=\frac{c}{d}=k(say)$

$\Rightarrow \quad a=bk,\ c=dk.$

Now,

$$\frac{a+b}{c+d}=\frac{bk+b}{dk+d}=\frac{b(k+1)}{d(k+1)}=\frac{b}{d}$$

$$\frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}=\frac{\sqrt{b^2k^2+b^2}}{\sqrt{d^2k^2+d^2}}=\frac{b\sqrt{k^2+1}}{d\sqrt{k^2+1}}=\frac{b}{d}$$

$$\therefore \quad a+b:c+d=\sqrt{a^2+b^2}:\sqrt{c^2+d^2}=\sqrt{3a^2+5b^2}:\sqrt{3c^2+5d^2}$$

(v) $ma+nc:mb+nd=(a^2+c^2)^{1/2}:(b^2+d^2)^{1/2}$

Solution: We have, $\frac{a}{b}=\frac{c}{d}=k(say)$

$\Rightarrow \quad a=bk,\ c=dk.$

Now,

$$\frac{ma+nc}{mb+nd}=\frac{mbk+ndk}{mb+nd}=\frac{k(mb+nd)}{mb+nd}=k$$

And, $$\frac{(a^2+c^2)^{1/2}}{(b^2+d^2)^{1/2}}=\frac{\sqrt{a^2+c^2}}{\sqrt{b^2+d^2}}=\frac{\sqrt{b^2k^2+d^2k^2}}{\sqrt{b^2+d^2}}=\frac{k\sqrt{b^2+d^2}}{\sqrt{b^2+d^2}}=k$$

$$\therefore \quad ma+nc:mb+nd=(a^2+c^2)^{1/2}:(b^2+d^2)^{1/2}$$

Hence proved.

12. If $a:b=b:c$, show that

(i) $a^2+ab+b^2:b^2+bc+c^2=a:c$

Solution: Let $\frac{a}{b}=\frac{b}{c}=k\ (say)$ Then,

$$a=bk,\ b=ck$$

$$\Rightarrow a=ck^2,\ b=ck$$

Now,

$$\frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{(ck^2)^2+(ck^2)(ck)+(ck)^2}{(ck)^2+ck.c+c^2}=\frac{c^2k^4+c^2k^3+c^2k^2}{c^2k^2+c^2k^2+c^2}=\frac{c^2k^2(k^2+k+1)}{c^2(k^2+k+1)}=k^2$$

$$\frac{a}{c}=\frac{ck^2}{c}=k^2$$

$$\therefore\ a^2+ab+b^2:b^2+bc+c^2=a:c$$

Hence shown.

(ii) $a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right)=a^3+b^3+c^3$

Solution: Suppose $\frac{a}{b}=\frac{b}{c}=k\ (say)$

Then, $a=bk,\ b=ck$

$\Rightarrow a=ck^2,\ b=ck$.

Now, $a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right)=(ck^2)^2(ck)^2c^2\left[\frac{1}{(ck^2)^3}+\frac{1}{(ck)^3}+\frac{1}{(c)^3}\right]$

$$=c^2k^4c^2k^2c^2\left[\frac{1}{c^3k^6}+\frac{1}{c^3k^3}+\frac{1}{c^3}\right]$$

$$=c^6k^6\left[\frac{1}{c^3k^6}+\frac{1}{c^3k^3}+\frac{1}{c^3}\right]$$

$$=c^3+k^3c^3+c^3k^6$$

And, $a^3+b^3+c^3=(ck^2)^3+(ck)^3+(c)^3=c^3+k^3c^3+c^3k^6$.

$$\therefore\ a^2b^2c^2\left(\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}\right)=a^3+b^3+c^3$$

Hence shown.

(iii) $a^3+b^3:b^3+c^3=a^3:b^3$

Solution: Suppose $\frac{a}{b}=\frac{b}{c}=k\ (say)$

Then, $a=bk,\ b=ck$

$\Rightarrow a=ck^2,\ b=ck$.

Now, $\frac{a^3+b^3}{b^3+c^3}=\frac{(ck^2)^3+(ck)^3}{(ck)^3+c^3}=\frac{c^3k^6+c^3k^3}{c^3k^3+c^3}=\frac{k^3(k^3+1)}{k^3+1}=k^3$

$\frac{a^3}{b^3}=\frac{(ck^2)^3}{(ck)^3}=\frac{c^3k^6}{c^3k^3}=k^3$

$\therefore\ a^3+b^3:b^3+c^3=a^3:b^3$

Hence shown.

13. If a,b,c,d,e are in continued proportion, prove that

(i) $a^2+ab+b^2:b^2+bc+c^2=a:c$

(ii) $a:c=a^4:b^4$

Solution: We have, $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=\frac{d}{e}=k\ (say)$

$\Rightarrow a=bk,\ b=ck, c=dk, d=ek$

$\Rightarrow a=ek^4,\ b=ek^3, c=ek^2, d=ek$

(i) Now, $\frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{(ek^4)^2+ek^4.ek^3+(ek^3)^2}{(ek^3)^2+ek^3.ek^2+(ek^2)^2}$

$=\frac{e^2k^8+e^2k^7+e^2k^6}{e^2k^6+e^2k^5+e^2k^4}=\frac{k^6(k^2+k+1)}{k^4(k^2+k+1)}=k^2$

And, $\frac{a}{c}=\frac{ek^4}{ek^2}=k^2$

$\therefore\ \frac{a^2+ab+b^2}{b^2+bc+c^2}=\frac{a}{c}$

(ii) $$\frac{a^4}{b^4} = \frac{(ek^4)^4}{(ek^3)^4} = \frac{e^4k^{16}}{e^4k^{12}} = k^4$$

$$\frac{a}{e} = \frac{ek^4}{e} = k^4$$

$$\therefore \frac{a^4}{b^4} = \frac{a}{e}$$

Hence proved.

14. If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, prove that (i) $\frac{x^2 - yz}{a^2 - bc} = \frac{y^2 - zx}{b^2 - ca} = \frac{z^2 - xy}{c^2 - ab}$

(ii) $\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{3xyz}{3abc}$

Solution: Suppose $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k \ (say)$.

Then, $\Rightarrow x = ak,\ y = bk, z = ck.$

(i) Now, $$\frac{x^2 - yz}{a^2 - bc} = \frac{a^2k^2 - bck^2}{a^2 - bc} = \frac{(a^2 - bc)k^2}{a^2 - bc} = k^2$$

$$\frac{y^2 - zx}{b^2 - ca} = \frac{b^2k^2 - ack^2}{b^2 - ca} = \frac{(b^2 - ac)k^2}{b^2 - ca} = k^2$$

$$\frac{z^2 - xy}{c^2 - ab} = \frac{c^2k^2 - abk^2}{c^2 - ab} = \frac{(c^2 - ab)k^2}{c^2 - ab} = k^2$$

$$\therefore \frac{x^2 - yz}{a^2 - bc} = \frac{y^2 - zx}{b^2 - ca} = \frac{z^2 - xy}{c^2 - ab}.$$

(ii) $$\frac{a^3 + b^3 + c^3}{x^3 + y^3 + z^3} = \frac{a^3 + b^3 + c^3}{a^3k^3 + b^3k^3 + c^3k^3} = \frac{a^3 + b^3 + c^3}{k^3(a^3 + b^3 + c^3)} = \frac{1}{k^3}$$

$$\frac{abc}{xyz} = \frac{abc}{abck^3} = \frac{1}{k^3}$$

$$\therefore \frac{a^3 + b^3 + c^3}{x^3 + y^3 + z^3} = \frac{abc}{xyz}$$

(iii) $$\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{a^3k^3}{a^3} + \frac{b^3k^3}{b^3} + \frac{c^3k^3}{c^3} = 3k^3$$

$$\frac{3xyz}{abc} = \frac{3abck^3}{abc} = 3k^3$$

$$\Rightarrow \frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{3xyz}{abc}$$

Hence proved.

15. If $x=\dfrac{2ab}{a+b}$**, find the value of** $\dfrac{x+a}{x-a}+\dfrac{x+b}{x-b}$**.**

Solution: We have, $x=\dfrac{2ab}{a+b}$

$$\Rightarrow \frac{x}{a}=\frac{2b}{a+b} \text{ and } \frac{x}{b}=\frac{2a}{a+b}.$$

Now, $\dfrac{x}{a}=\dfrac{2b}{a+b}$

$$\Rightarrow \frac{x+a}{x-a}=\frac{2b+a+b}{2b-a-b} \quad \text{[Componendo-dividendo]}$$

$$\Rightarrow \frac{x+a}{x-a}=\frac{a+3b}{b-a} \quad \text{------ (1)}$$

Similarly, $\dfrac{x+b}{x-b}=\dfrac{3a+b}{a-b}$ ------ (2)

Then, $\dfrac{x+a}{x-a}+\dfrac{x+b}{x-b}=\dfrac{a+3b}{b-a}+\dfrac{3a+b}{a-b}$

$$=\frac{a+3b}{b-a}-\frac{3a+b}{b-a}$$

$$=\frac{a+3b-3a-b}{b-a}$$

$$=\frac{2b-2a}{b-a}$$

$$=2.$$

16. If $x=\dfrac{a+b}{a-b}$ **and** $y=\dfrac{a-b}{a+b}$**, find the value of** $\dfrac{x-y}{x+y}$**.**

Solution: We have, $x=\dfrac{a+b}{a-b}$, $y=\dfrac{a-b}{a+b}$.

Now, $$\frac{x-y}{x+y}=\frac{\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}}{\dfrac{a+b}{a-b}+\dfrac{a-b}{a+b}}$$

$$= \frac{(a+b)^2-(a-b)^2}{(a+b)^2+(a-b)^2}$$

$$= \frac{4ab}{2(a^2+b^2)} = \frac{2ab}{a^2+b^2}.$$

17. If $x = \frac{2\sqrt{10}}{\sqrt{2}+\sqrt{5}}$, find the value of $\frac{x+\sqrt{2}}{x-\sqrt{2}} + \frac{x+\sqrt{5}}{x-\sqrt{5}}$.

Solution: We have, $x = \frac{2\sqrt{10}}{\sqrt{2}+\sqrt{5}} = \frac{2\sqrt{2}\sqrt{5}}{\sqrt{2}+\sqrt{5}}$

$$\Rightarrow \frac{x}{\sqrt{2}} = \frac{2\sqrt{5}}{\sqrt{2}+\sqrt{5}} \text{ and } \frac{x}{\sqrt{5}} = \frac{2\sqrt{2}}{\sqrt{2}+\sqrt{5}}.$$

Now, $\frac{x}{\sqrt{2}} = \frac{2\sqrt{5}}{\sqrt{2}+\sqrt{5}}$

$$\Rightarrow \frac{x+\sqrt{2}}{x-\sqrt{2}} = \frac{2\sqrt{5}+\sqrt{2}+\sqrt{5}}{2\sqrt{5}-\sqrt{2}-\sqrt{5}}$$ [by componendo and dividend]

$$\Rightarrow \frac{x+\sqrt{2}}{x-\sqrt{2}} = \frac{3\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$ ---------- (1)

Similarly, $\frac{x+\sqrt{5}}{x-\sqrt{5}} = \frac{3\sqrt{2}+\sqrt{5}}{\sqrt{2}-\sqrt{5}}$

$$\Rightarrow \frac{x+\sqrt{5}}{x-\sqrt{5}} = -\frac{3\sqrt{2}+\sqrt{5}}{\sqrt{5}-\sqrt{2}}$$ ---------- (2)

Adding eqn. (1) and eqn.(2) we get,

$$\frac{x+\sqrt{2}}{x-\sqrt{2}} + \frac{x+\sqrt{5}}{x-\sqrt{5}} = \frac{3\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} - \frac{3\sqrt{2}+\sqrt{5}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{3\sqrt{5}+\sqrt{2}-3\sqrt{2}-\sqrt{5}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{2(\sqrt{5}-\sqrt{2})}{\sqrt{5}-\sqrt{2}}$$

$$= 2.$$

18. **Prove that** $3bx^2-4ax+3b=0$ **if** $x=\dfrac{\sqrt{2a+3b}+\sqrt{2a-3b}}{\sqrt{2a+3b}-\sqrt{2a-3b}}$

Solution: We have, $x=\dfrac{\sqrt{2a+3b}+\sqrt{2a-3b}}{\sqrt{2a+3b}-\sqrt{2a-3b}}$

$$\Rightarrow \frac{x+1}{x-1}=\frac{\sqrt{2a+3b}+\sqrt{2a-3b}+\sqrt{2a+3b}-\sqrt{2a-3b}}{\sqrt{2a+3b}+\sqrt{2a-3b}-\sqrt{2a+3b}+\sqrt{2a-3b}} \text{ [Componendo-dividendo]}$$

$$\Rightarrow \frac{x+1}{x-1}=\frac{\sqrt{2a+3b}}{\sqrt{2a-3b}}$$

$$\Rightarrow \left(\frac{x+1}{x-1}\right)^2=\frac{2a+3b}{2a-3b}$$

$$\Rightarrow \frac{(x+1)^2}{(x-1)^2}=\frac{2a+3b}{2a-3b}$$

$$\Rightarrow \frac{(x+1)^2+(x-1)^2}{(x+1)^2-(x-1)^2}=\frac{2a+3b+2a-3b}{2a+3b-2a+3b} \text{ [Componendo-dividendo]}$$

$$\Rightarrow \frac{x^2+2x+1+x^2-2x+1}{x^2+2x+1-x^2+2x-1}=\frac{4a}{6b}$$

$$\Rightarrow \frac{2(x^2+1)}{4x}=\frac{2a}{3b}$$

$$\Rightarrow \frac{x^2+1}{4x}=\frac{a}{3b}$$

$$\Rightarrow 3bx^2+3b=4ax$$

$$\Rightarrow 3bx^2-4ax+3b=0.$$

Hence proved.

19. If $\dfrac{a}{b}=\dfrac{b}{c}=\dfrac{c}{d}$**, show that (i)** $\dfrac{a^3+b^3+c^3}{b^3+c^3+d^3}=\dfrac{a}{d}$

(ii) $\dfrac{1}{b}+\dfrac{1}{c}+\dfrac{1}{d}:\dfrac{1}{a}+\dfrac{1}{b}+\dfrac{1}{c}=a:b$

(iii) $(b+c)(b+d)=(c+a)(c+d)$

(iv) $(\boldsymbol{a}+\boldsymbol{d})(\boldsymbol{b}+\boldsymbol{c})-(\boldsymbol{a}+\boldsymbol{c})(\boldsymbol{b}+\boldsymbol{d})=(\boldsymbol{b}-\boldsymbol{c})^2$

Solution: We have, $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}=k$ (say)

$$\Rightarrow a=bk, b=ck, c=dk$$
$$\Rightarrow a=dk^3, b=dk^2, c=dk$$

(i) $$\frac{a^3+b^3+c^3}{b^3+c^3+d^3}=\frac{d^3k^9+d^3k^6+d^3k^3}{d^3k^6+d^3k^3+d^3}$$
$$=\frac{k^3(k^6+k^3+1)}{k^6+k^3+1}$$
$$=k^3.$$

$$\frac{a}{d}=\frac{dk^3}{d}=k^3.$$

$$\therefore \frac{a^3+b^3+c^3}{b^3+c^3+d^3}=\frac{a}{d}$$

(ii) $$\frac{1}{b}+\frac{1}{c}+\frac{1}{d}:\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{dk^2}+\frac{1}{dk}+\frac{1}{d}:\frac{1}{dk^3}+\frac{1}{dk^2}+\frac{1}{dk}$$
$$=\frac{1}{d}\left(\frac{1}{k^2}+\frac{1}{k}+1\right):\frac{1}{dk}\left(\frac{1}{k^2}+\frac{1}{k}+1\right)$$
$$=k.$$

And, $\frac{a}{b}=\frac{dk^3}{dk^2}=k.$

$$\therefore \frac{1}{b}+\frac{1}{c}+\frac{1}{d}:\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=a:b$$

(iii) $$(b+c)(b+d)=(dk^2+dk)(dk^2+d)$$
$$=d^2k(k+1)(k^2+1)$$

And, $(c+a)(c+d)=(dk+dk^3)(dk+d)$
$$=d^2k(k+1)(k^2+1)$$

$$\therefore (b+c)(b+d)=(c+a)(c+d).$$

(iv) $$(a+d)(b+c)-(a+c)(b+d)$$
$$=(dk^3+d)(dk^2+dk)-(dk^3+dk)(dk^2+d)$$
$$=d^2k(k^3+1)(k+1)-d^2k(k^2+1)^2$$
$$=d^2k[(k^3+1)(k+1)-(k^2+1)^2]$$
$$=d^2k[k^4+k^3+k+1-k^4-2k^2-1]$$
$$=d^2k[k^3-2k^2+k]$$
$$=d^2k^2[k^2-2k+1]=d^2k^2(k-1)^2.$$

And, $(b-c)^2=(dk^2-dk)^2$
$$=d^2k^2(k-1)^2.$$

$$\therefore (a+d)(b+c)-(a+c)(b+d)=(b-c)^2.$$

Hence Shown.

20. If $\dfrac{x}{b+c-a}=\dfrac{y}{c+a-b}=\dfrac{z}{a+b-c}$**, show that** $(b-c)x+(c-a)y+(a-b)z=0$**.**

Solution: Let $\dfrac{x}{b+c-a}=\dfrac{y}{c+a-b}=\dfrac{z}{a+b-c}=k$ (say)

Then, $x=k(b+c-a),\ y=k(c+a-b),\ z=k(a+b-c).$

Now, $(b-c)x+(c-a)y+(a-b)z$

$$=(b-c)k(b+c-a)+(c-a)k(c+a-b)+(a-b)k(a+b-c)$$
$$=k[(b-c)(b+c-a)+(c-a)(c+a-b)+(a-b)(a+b-c)]$$
$$=k[b^2-c^2-ab+ac+c^2-a^2-bc+ab+a^2-b^2-ac+bc]$$
$$=k\times 0$$
$$=0.$$

21. If $\dfrac{a+x+\sqrt{a^2-x^2}}{a+x-\sqrt{a^2-x^2}}=\dfrac{b}{x}$**, show that** $x^2=2ab-b^2$**.**

Solution : We have, $\dfrac{a+x+\sqrt{a^2-x^2}}{a+x-\sqrt{a^2-x^2}}=\dfrac{b}{x}$

$$\Rightarrow \frac{a+x+\sqrt{a^2-x^2}+a+x-\sqrt{a^2-x^2}}{a+x+\sqrt{a^2-x^2}-a-x+\sqrt{a^2-x^2}}=\frac{b+x}{b-x}$$ [Componendo-dividendo]

$$\Rightarrow \frac{2(a+x)}{2\sqrt{a^2-x^2}}=\frac{b+x}{b-x}$$

$$\Rightarrow \frac{(a+x)}{\sqrt{(a-x)(a+x)}}=\frac{b+x}{b-x}$$

$$\Rightarrow \sqrt{\frac{a+x}{a-x}}=\frac{b+x}{b-x}$$

$$\Rightarrow \frac{a+x}{a-x}=\frac{(b+x)^2}{(b-x)^2}$$ [squaring on both sides]

$$\Rightarrow \frac{a+x+a-x}{a+x-a+x}=\frac{(b+x)^2+(b-x)^2}{(b+x)^2-(b-x)^2}$$

$$\Rightarrow \frac{2a}{2x}=\frac{2(b^2+x^2)}{4bx}$$

$\Rightarrow a = \dfrac{b^2 + x^2}{2b}$

$\Rightarrow x^2 = 2ab - b^2.$

Hence shown.

22. If $\dfrac{a}{b} = \dfrac{c}{d}$**, prove that** $\dfrac{(a^2+c^2)^2}{(b^2+d^2)^2} = \dfrac{a^4+c^4}{b^4+d^4}$.

Solution: Let $\dfrac{a}{b} = \dfrac{c}{d} = k$ (say) .

Then, $\Rightarrow a = bk, c = dk.$

Now, $\dfrac{(a^2+c^2)^2}{(b^2+d^2)^2} = \dfrac{(b^2k^2+d^2k^2)^2}{(b^2+d^2)^2}$

$= \dfrac{k^4(b^2+d^2)^2}{(b^2+d^2)^2}$

$= k^4.$

And, $\dfrac{a^4+c^4}{b^4+d^4} = \dfrac{b^4k^4+d^4k^4}{b^4+d^4}$

$= \dfrac{(b^4+d^4)k^4}{b^4+d^4}$

$= k^4.$

$\therefore \dfrac{(a^2+c^2)^2}{(b^2+d^2)^2} = \dfrac{a^4+c^4}{b^4+d^4}.$

Hence proved.

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