CLASS – IX
MATHEMATICS
CHAPTER – 2
POLYNOMIALS

NOTES

- **Polynomial:** An algebraic expression involving only non-negative integral powers of a variable is called a polynomial in the variable. Example: $4y^2 - 3y + 2$.
- **Monomial:** A polynomial having only one term is called a monomial.
  Example:$2x^2, 5x, 3$ etc.
- **Binomial:** A polynomial having only two terms is called a binomial.
  Example: $x + 2, x^2 - x, y^2 - 9$ etc.
- **Trinomial:** A polynomial having only three terms is called a trinomial.
  Example: $x^2 - 5x + 6, x^4 + x^2 + 1$ etc.
- **Degree of a polynomial:** The exponent of the variable in a term of a polynomial represents the degree of that term and the highest of the degrees of the term is called the degree of the polynomial.
  Example: In the polynomial,$5x^6 - 3x^4 + 4x^2 + x$, degree = 6
- **General form of a polynomial**
  The most general form of a polynomial of degree n in a single variable x is
  $$a_0 + a_1x + a_2x^2 + \dots\dots\dots\dots\dots + a_nx^n$$
  $$\text{or } a_nx^n + a_{n-1}x^{n-1} + \dots\dots\dots\dots + a_1x + a_0,$$
  where $a_0, a_1, a_2, \dots\dots\dots\dots\dots\dots, a_n$ are constants and $a_n \neq 0$.
  Zero Polynomial: A polynomial in which all the coefficients are zero is called a Zero Polynomial and is denoted by 0.

  Notes:
  (i) Degree of a zero polynomial is not defined.
  (ii) A non-zero constant is a polynomial of degree zero.
- **Standard form of a polynomial**: A polynomial is said to be in the standard form when its terms are arranged in ascending order descending powers of the variable.
- **Monic polynomial**: A polynomial in which the coefficient of the highest degree term is 1 is called a Monic polynomial. Example: $x^3 + x^2 + 1, x^4 - 3x^3 + x - 2$ etc.
  Note: Monic polynomial of degree zero is 1.

- **Some special names of polynomials**
  - **Linear polynomial**: A polynomial of degree one is called a linear polynomial. Example: $x,\ x + 3, 4 - 3x$ etc.
  - **Quadratic polynomial**: A polynomial of degree two is called quadratic polynomial. Example: $2x^2 + 5, x^2 + 5x - 2$ etc.
  - **Cubic polynomial**: A polynomial of degree three is called cubic polynomial. Example: $5x^3 - 1, y^3 - 5y + 2$
  - **Biquadratic or quartic polynomial**: A polynomial of degree four is called quadratic polynomial. Example: $5x^4 + 4x^3 + 4, 2x^4 - 3x^3 + 4x + 10$ etc.

- **Zero of a polynomial:** A real number 'c' is called a zero a polynomial $p(x)$ if $p(c) = 0$.
- Zero of a polynomial $p(x)$ is obtained by equating it to 0 and solving the resulting equation.
- If c is a zero of the polynomial $p(x)$, then c is called a root of the equation $p(x) = 0$.
- A non-zero constant polynomial has no zero.
- Every linear polynomial in one variable has a unique zero.
- 0 (zero) may be a zero of a polynomial.
- A polynomial can have more than one zeros.

- **Factorisation**: The process of expressing a given polynomial as the product of its prime factors is called factorisation.
- **Factorisation of $x^2 + bx + c$, where $a \neq 0$, by splitting the middle term:**

  Let $px + q$ and $rx + s$ be the factors of $ax^2 + bx + c$.

  Then $ax^2 + bx + c = (px + q)(rx + s)$

  $= prx^2 + (ps + qr)x + qs$

  Comparing the coefficients of like terms, we get

  $a = pr, b = ps + qr$ and $c = qs$

  We see that b is the sum of two numbers ps and qrwhose product is

  $(ps)(qr) = (pr)(qs) = ac$

  In particular, to factorise $ax^2 + bx + c$ where a, b, c are integers and $a \neq 0$, we have to write b as the sum of two numbers whose product is ac.

- **Some algebraic identities**:

i) $(\boldsymbol{a}+\boldsymbol{b})^2 = a^2+2ab+b^2$

ii) $(\boldsymbol{a}-\boldsymbol{b})^2 = a^2-2ab+b^2$

iii) $(\boldsymbol{a}+\boldsymbol{b})(\boldsymbol{a}-\boldsymbol{b}) = a^2-b^2$

iv) $(\boldsymbol{a}+\boldsymbol{b})^3 = a^3+3a^2b+3ab^2+b^3$

$= a^3+b^3+3ab\,(a+b)$

v) $(\boldsymbol{a}-\boldsymbol{b})^3 = a^3-3a^2b+3ab^2-b^3$

$= a^3-b^3-3ab\,(a-b)$

vi) $\boldsymbol{a}^3+\boldsymbol{a}^3 = (a+b)(\,a^2-ab+\,a^2)$

vii) $\boldsymbol{a}^3-\boldsymbol{a}^3 = (a-b)(a^2+ab+a^2)$

- **H.C.F. of polynomials:** The H.C.F. of two or more polynomials is defined as the polynomial of highest degree with the greatest leading coefficient, which is a factor of each of the given polynomials.

- **L.C.M. of polynomials:** The L.C.M. of two or more polynomials is defined as the polynomial of lowest degree and smallest leading coefficient which is exactly divisible by each of the given polynomials.

- **H.C.F. of polynomials by factorisation**

To find the H.C.F. of two or more polynomials by the method of factorisation, we may proceed as follows:

i) Resolve each of the given polynomials into irreducible or prime factors.

ii) Find the H.C.F. of the leading coefficients of the polynomials.

iii) Find the product of the H.C.F of the leading coefficients and factors with their highest powers common to all polynomials.

- **L.C.M. of the polynomials by Factorisation**

To find the L.C.M. of two or more polynomials by the method of factorisation, we may proceed as follows:

i) Resolve each of the given polynomials into its prime factors.

ii) Find the L.C.M. of the leading coefficients of the polynomials.

iii) Find the product of the L.C.M. of the leading coefficients and the factors with their highest powers involved in either of the polynomials.

- **Common Zero(s) of polynomials**

A number which is a zero of each of two or more polynomials is said to be a common zero of the polynomials.

Notes: 1. The common zeros of the polynomials are given by the zeros of the H.C.F. of the polynomials.

2. If the H.C.F. of the polynomials is a constant polynomial, they have no common zero as a constant polynomial has no zero.

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