CHAPTER – 1
NUMBER SYSTEM

NOTES

➢ Euclid's Division Lemma (or Euclid's Division Algorithm)

Let a and b be any two integers and b>0. Then there exists unique integers q and r such that a = bq + r and $0 \leq r < b$.

➢ Euclid's Algorithm for finding HCF of two given positive integers

1. Find the quotient and remainder of the division of the greater number by the smaller.
2. If the remainder is zero, then the divisor is the HCF.
3. Else, taking the previous remainder as the new divisor and the previous divisor as the new dividend, find the quotient and remainder.
4. Continue the process till the remainder is zero.

   The last divisor is the required HCF.

➢ Fundamental Theorem of Arithmetic or Unique Factorisation Theorem

Every composite number can be expressed as a product of primes uniquely except for the order of the factors.

OR

Every integer n > 1 can be expressed uniquely in the form

$$n = {p_1}^{a_1}{p_2}^{a_2}{p_3}^{a_3}, \ldots \ldots \ldots \ldots \ldots . {p_k}^{a_k}$$

where $p_1, p_2, p_3, \ldots \ldots \ldots \ldots, p_k$ are primes such that $p_1 < p_2 < p_3 < \ldots \ldots \ldots \ldots < p_k$ and $a_1, a_2, a_3, \ldots \ldots \ldots \ldots, a_k$are all positive integers.

➢ Eleven field properties of real numbers

1. Closure under addition: The sum of two real numbers is a real number i.e. $x + y \in R$ whenever $x, y \in R$.
2. Associativity of addition: For every $x, y, z \in R, (x + y) + z = x + (y + z)$
3. Commutativity of addition: $x + y = y + x$ for every $x, y \in R$.
4. Existence of additive identity: There exists a real number 0 (zero) called the additive identity such that $x + 0 = x$ for every $x \in$ R.
5. Existence of additive inverse: For each $x \in R$, there exists $-x \in R$ called the additive inverse or negative of x such that $x + (-x) = 0$(additive identity).
6. Closure under multiplication: The product of two real numbers is a real number i.e. $xy \in R$ whenever $x, y \in R$.

7. Associativity of multiplication: For every $x, y, z \in R$, $(xy)z = x(yz)$
8. Commutativity of multiplication: $xy = yx$ for every $x, y \in R$.
9. Existence of multiplicative identity: There exists a real number 1, called the multiplicative identity such that $x \times 1 = x$ for any $x \in R$.
10. Existence of multiplicative inverse: For each non-zero real number x, there exists $\frac{1}{x} \in R$ called the multiplicative inverse or reciprocal of x such that $x \times \frac{1}{x} = 1$ (multiplicative identity).
11. Multiplication distributes over addition: For any real number x, y, z,
    $x(y+z) = xy + xz$

Corollaries

***1.* Cancellation law for addition**

If $x, y, z \in R$ and $x + y = x + z$, then $y = z$.

***2.* Cancellation law for multiplication**

If $x, y, z \in R, x \neq 0$ and $xy = xz$, then $y = z$.

3. For any $x \in R$, $x.0 = 0$.

***4.* For $x, y \in R, x(-y) = -xy$**

***5.* For $x, y \in R, (-x)(-y) = xy$**

***6.* If $x, y \in R$, and $xy = 0$, then $x = 0$ or $y = 0$.**

Absolute Value or Modulus of a Real Number

The absolute value or modulus of a real number x, denoted by $|x|$ is defined by

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

or

$$|x| = \begin{cases} 0, & \text{if } x = 0 \\ \text{the greater of } x \text{ or } -x, & \text{if } x \neq 0 \end{cases}$$

Some fundamental properties of absolute values of real numbers

i) $|x| \geq 0$

ii) $|-x| = |x|$

iii) $|xy| = |x||y|$

iv) $|x + y| \leq |x| + |y|$

v) $|x - y| \geq |x| - |y|$ and $|x - y| \geq |y| - |x|$

vi) $|x - y| < \delta$ if and only if $y - \delta < x < y + \delta$

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