DEPARTMENT OF EDUCATION (S)
Government of Manipur

CHAPTER-6

THE TRIANGLE AND ITS PROPERTIES

SOLUTIONS:

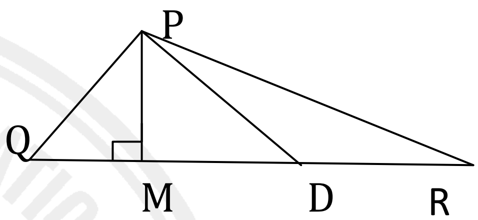
EXERCISE 6.1:

1. In △ PQR, D is the mid- point of $\overline{QR}$.

Ans: $\overline{PM}$ is <u>altitude</u>.

PD is <u>median</u>.

No, QM =MR.



2. **Draw rough sketches for the following**.

(a). Ans: In△ ABC, BE is median.

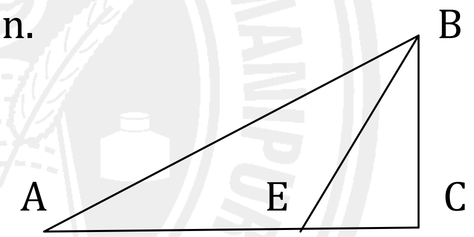
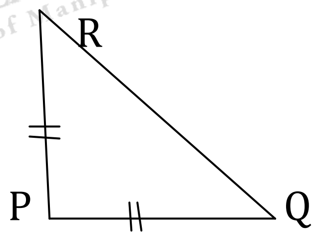
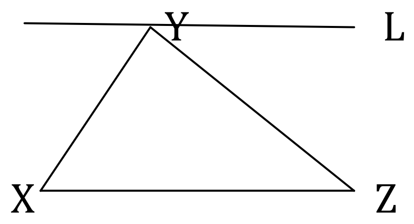


Fig (a)

(b).Ans: In△PQR, PQ and PR are altitudes of the triangle.
Here, PQR is right isosceles triangle.

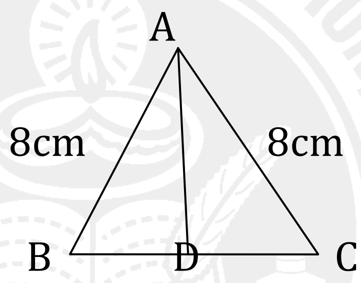


(c). Ans : In △ XYZ, YL is an altitude in the exterior of the triangle.



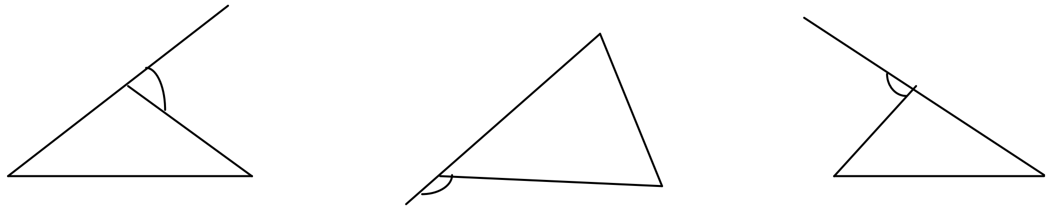
3. Verify by drawing a diagram if the median and altitude of an isosceles triangle can be same.

Ans: Here, ABC is an isosceles triangle AD is the median as well as altitude of the triangle.



THINK, DISCUSS, WRITE:

1. Ans : The other three forms of exterior angles so formed are



2. Ans : Yes, the exterior angles forms at each vertex of a triangle are equal because they have vertically opposite angles.
3. Ans : The sum of an exterior angle and its adjacent interior angle of a triangle is 180^0.

THINK, DISCUSS AND WRITE : PAGE 118

1. **What can you say about each of the interior opposite angles, when the exterior angle is**

   (i). a right angle?

   Ans : The interior opposite angles also a right angle.

   (ii**). an obtuse angle?**

   Ans : The interior opposite angle is acute angle.

   (iii). **an acute angle?**

   Ans: The interior opposite angle is obtuse angle.
2. **Can the exterior angle of a triangle be a straight angle?**

   Ans : No, the sum of three angles of a triangle is 180^0.

TRY THESE:

1. Soln: Let x^0 be one of the interior opposite angle, then

   $25^0+x^0 = 70^0$

   $\Rightarrow x^0 = 70^0-25^0$

   $=45^0$

2. Soln : We know that the sum of the two interior opposite angles = exterior angles
i.e. $60^0 + 80^0 = 140^0$.

3. The exterior angle be 90^0 and the interior opposite angles be 45^0.

EXERCISE 6.2:

1. **Find the value of the unknown exterior angles in the following diagrams**.

(i). Ans : We know that the sum of two exterior opposite angles = the sum of the exterior angle.

Then, $50^0 + 70^0 = x$.

⇨ $120^0 = x$

Therefore $x = 120^0$.

(ii). $x = 65^0 + 45^0 = 110^0$

Therefore $x = 110^0$.

(iii). $x = 30^0 + 40^0$

Therefore $x = 70^0$

(iv). $x = 60^0 + 60^0 = 120^0$.

(v). $x = 50^0 + 50^0 = 100^0$.

(vi) $x = 30^0 + 60^0 = 90^0$

2. **Find the value of the unknown interior angle x in the following figures.**

Soln : (i). $50^0 + x = 115^0$

$\Rightarrow x = 115^0 - 50^0$

$\Rightarrow x = 65^0$.

(ii). $70^0 + x = 100^0$

$\Rightarrow x = 100^0 - 70^0$

$\Rightarrow x = 30^0$

(iii). $x + 90^0 = 125^0$

$\Rightarrow x = 125^0 - 90^0$

$\Rightarrow x = 35^0$

(iv). $x + 60^0 = 120^0$

$\Rightarrow x = 120^0 - 60^0$

$\Rightarrow x = 60^0$

(v). $x + 30^0 = 80^0$

$\Rightarrow x = 80^0 - 30^0$

$\Rightarrow x = 50^0$

(vi). $x + 35^0 = 75^0$

$\Rightarrow x = 75^0 - 35^0$

$\Rightarrow x = 40^0$

EXERCISE 6.3

Find the value of the unknown x in the following diagram.

(i). Soln : We know that the sum of three angles of atriangle is 180^0 , then ,

$x+50^0+60^0=180^0$

- ⇨ $x+ 110^0 = 180^0$
- ⇨ $x= 180^0 - 110^0$

$= 70^0$

(ii). $x+90^0+30^0=180^0$

- ⇨ $x+ 120^0 = 180^0$
- ⇨ $x= 180^0 - 120^0$

$= 60^0$

(iii). $x+30^0+110^0=180^0$

- ⇨ $x+ 140^0 = 180^0$

$x= 180^0 - 140^0 = 40^0$

(iv). $x+50^0+x=180^0$

- ⇨ $2x = 180^0-50^0$
- ⇨ $2 x= 130^0$
- ⇨ $x = 130^0/2 = 65^0$.

(v). $x+ x +x = 180^0$

- ⇨ $3x = 180^0$
- ⇨ $x= 180^0/3$
- ⇨ $x = 60^0$.

(vi). x+2x +900= 1800

⇨ $3x= 180^0 - 90^0 = 90^0$

⇨ $x \quad = 90^0/3= 30^0$.

2. **Find the value of the unknown x and y in the following diagram**.

(i) We have,

$50^0+x = 120^0$

$=>x = 120^0$ - C= 70^0

Then,

$50^0+70^0+y = 180^0$ [Sum of three angles of triangle]

⇨ $y+ 120^0 =180^0$

⇨ $y = 180^0 - 120^0$

$= 60^0$

(ii) Here,

$80^0 = y$ [vertically opposite angle]

Then,

$x+ 50^0+80^0 = 180^0$ [Sum of three angles of triangle]

⇨ $x+ 130^0 =180^0$

⇨ $y = 180^0 - 130^0 = 50^0$.

(iii) Here,

$x= 50^0 + 60^0$ [vertically opposite angle]

Then,

$x+ 50^0+80^0 = 180^0$ [Sum of two interior angles =exterior angles of triangle]

⇨ $x= 110^0$

Then,

$y+50^0 + 60^0 = 180^0$

⇨ $y + 110^0 = 180^0$

⇨ $y = 180^0- 110^0 = 70^0$

(iv) $x = 60^0$ [vertically opposite angle]

Then,

$x + y + 30^0 = 180^0$

⇨ $60^0 + y + 30^0 = 180^0$

$y + 90^0 = 180^0$

⇨ $y = 180^0 - 90^0 = 90^0$.

(v) $y = 90^0$ [vertically opposite angle]

Then,

$y + x + x = 180^0$

⇨ $90^0 + 2x = 180^0$

⇨ $2x = 180^0 - 90^0$

⇨ $x = 90^0/2 = 45^0$.

(vi) From the figure , we see that

$x = y$ [vertically opposite angles]

Then,

$x + x + x = 180^0$

⇨ $3x = 180^0$

⇨ $x = 180^0/3 = 60^0$.

Hence, $x = y = 60^0$.

TRY THESE

1. Soln : Let x be the third angle , then

$x + 30^0 + 80^0 = 180^0$

⇨ $x + 110^0 = 180^0$

⇨ $x = 180^0 - 110^0 = 70^0$.

2. Soln : Let x be the equal angle , then

$x+x + 80^0 = 180^0$

⇨ $2x+ 80^0 = 180^0$

$2x = 180^0 - 80^0$

$X = 100^0/2 = 50^0$.

3. : Let x be the ratio of an angle , then

$x+2x + x = 180^0$

⇨ $4x = 180^0$

$x = 180^0 /4$

$x= 45^0$.

i.e. $45^0 : 90^0 : 45^0$.

<u>THINK, DISCUSS, WRITE:</u>

1. Ans : No, we cannot have a triangle with two right angles.
2. Ans : No, we cannot have a triangle with two obtuse angles.
3. Ans Yes, we can have a triangle with two acute angles.
4. Ans : No, we cannot have a triangle with all the three angles greater than 60^0.
5. Ans : Yes, we can have a triangle with all the three angles equal to 60^0, it is equilateral triangle.
6. Ans : No, we cannot have a triangle with all the three angles less than 60^0.

TRY THESE

Find angle x in each figure.

(i) $x = 40^0$. [Base angles opposite to equal side are equal].

(ii) $x = 90^0$ [right angle].

(iii) $X = 50^0$.

(iv)

$x + x + 100^0 = 180^0$

⇨ $2x + 100^0 = 180^0$

⇨ $2x = 180^0 - 100^0$

⇨ $2x = 80^0 = x = 80^0 /2 = 40^0$.

(v)

$x + x + 90^0 = 180^0$

⇨ $2x + 90^0 = 180^0$

⇨ $2x = 180^0 - 90^0$

⇨ $2x = 90^0 = x = 90^0 /2 = 45^0$.

(vi)

$x + 40^0 + 40^0 = 180^0$

⇨ $x + 80^0 = 180^0$

⇨ $x = 180^0 - 80^0 = 100^0$.

(vii) $x + x = 120^0$

⇨ $2x = 120^0$

⇨ $X = 120^0 /2 = 60^0$.

(viii) $x + x = 110^0$

⇨ $2x = 110^0$

$x = 110^0 /2 = 55^0$.

(ix) $X = 30^0$ [vertically opposite angles].

1. Find the angles x and y in each figure.

   (I) X=y are two base angles opposite to equal sides.Then,

   $x + x + y = 180^0$

   ⇨ $120^0 + y = 180^0$

   ⇨ $y = 180^0 - 120^0$

   ⇨ $y = 90^0$.

   Hence,

   $x = 180^0 - 2y$

   ⇨ $180^0 - 120^0 = 60^0$.

   (II) Here, $x = 45^0$

   Then, $y = x + 90^0$

   $= 45^0 + 90^0 = 135^0$.

   (III) Here, $x + 92^0 = y$

   Then, $x + 92^0 + x = 180^0$

   ⇨ $2x = 180^0 - 92^0 = 88^0$

   $x = 88^0/2 = 44^0$

   Then, $y = x + 92^0$

   $= 44^0 + 92^0$

   $= 136^0$.

NOTE: Sum of the length of any two sides of a triangle is greater than the length of the third side.

EXERCISE 6.4

1. Soln:

   (i) Here, the sides of the triangle are 2cm, 3cm and 5cm

   Then,

   (2 + 3)cm = 5cm which is equal to the third side of the triangle. So, it is not possible.

(ii) Soln: Sum of the smaller two sides of a triangle

(3+6)cm = 9cm > 7cm

So, it is possible.

(iii) Soln:Sum of to side of a triangle

(3+2)cm = 5cm<6cm

So, it is possible.

2. Soln:
   (i) Yes, OP+OR>PQ
   (ii) Yes, OQ+OR>QR
   (iii) YES, OR+OP>RP. Because O is the point in the interior of the trianglePQR.

3. Soln: In two triangle, ABM and AMC

   BM=MC, AB>AM,AM=AC

   So, AB+BC+CA>2AM.

4. Soln: Yes, AB+BC+CD+DA>AC+BD

   This is because sum of all sides of a quadrilateral is greater than sum of the two diagonals.

5. Soln:Yes, AB+BC+CD+DA<2(AC+BD)

   ie, Thrice the sum of two diagonals is greater thanthe sum of all sides of a quadrilaterals.

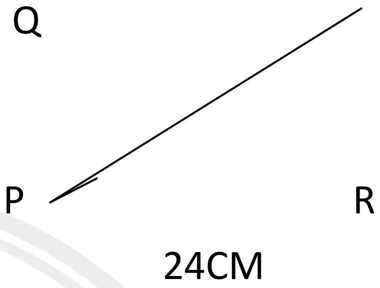
6. Soln: Here, sum of all sides of two triangle = (12+15)cm=27cm and their difference is (15-12)cm= 3cm.

   So, the length of the third sides of the triangle is between 3cm to 27cm.

EXERCISE 6.5

1. Soln : In the right angled triangle PQR right angle at P. Then PQ = 10cm, PR= 24, and QR = x.

   By Pythagoras property



   $X^2 = 10^2 + 24^2$

   ⇨ $X^2 = 10 \times 10 + 24 \times 24$

   ⇨ X = 100 + 576 = 676

   ⇨ $x = \pm\sqrt{676}$

   ⇨ $\pm\sqrt{2 X 2 X 13 X 13}$

   ⇨ $\sqrt{2^2 \times 13^2}$

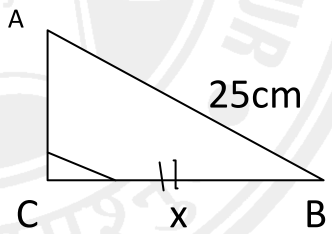
   ⇨ $\sqrt{(2 \times 13)}$

   = 26cm

2. Soln : Here, AB 25cm, AC = 7cm

   Let x be the side of BC. Then by Pythagoras Property

   We get,



   $x^2 + 7^2 = 25^2$

   ⇨ $x^2 + 49 = 625$

   ⇨ $x^2 = 625 – 49$

   ⇨ $x^2 = 576$

   ⇨ $x = \sqrt{576}$

   ⇨ $x = \sqrt{2^2 \times 2^2 \times 2^2 \times 3^2}$

   = 2 x 2 x 2 x 3.

   = 24

   Therefore, x is 24cm.

3. Here, length of ladder = 15cm is placing against wall and height of window id 12cm from the foot of the wall. Then let x cm be the distance from the foot of ladder from the wall.

   By Pythagoras Property, we get

   $x^2 + 12^2 = 15^2$

   ⇨ $X^2 + (12 \times 12) = 15 \times 15$

   ⇨ $X^2 + 144 = 225$

   ⇨ $X^2 = 225 - 144$

   ⇨ $X^2 = 81$

   ⇨ $X = \sqrt{9 \times 9}$

   $= \sqrt{9^2}$

   Therefore, x = 9cm.

4. Which of the following can be the sides of a right triangle?

   (i) 2.5cm, 6.5cm, 6cm.

   Soln : we have, $(2.5)^2 + 6^2 = (6.5)^2$

   ⇨ $6.25 + 36 = 42.25$

   ⇨ $42.25 = 42.25$

   This can be the sides of right triangle.

   (ii) Here,

   2cm, 2cm, 5cm are the sides of the triangle then,

   By Pythagoras Property, we get

   $2^2 + 2^2 = 5^2$

   => $4 + 4 = 25$

   => $8 \neq 25$

   Which cannot be the sides of a right triangle.

(iii) Here,

1.5cm, 2cm, 2.5cm are the sides of the triangle then,

By Pythagoras Property, we get

$(1.5)^2 + 2^2 = (2.5)^2$

$\Rightarrow 2.25 + 4 = 6.25$

$\Rightarrow 6.25 = 6.25$

Which can be the sides of a right triangle.

5. Soln:

Let xm be the top of the tree touches to the ground.

By Pythagoras Property, we get

$X^2 = 12^2 + 5^2$

$= 144 + 25$

$= 169$

⇨ $x = \sqrt{169}$

⇨ $\sqrt{13 \times 13} = 13$ cm

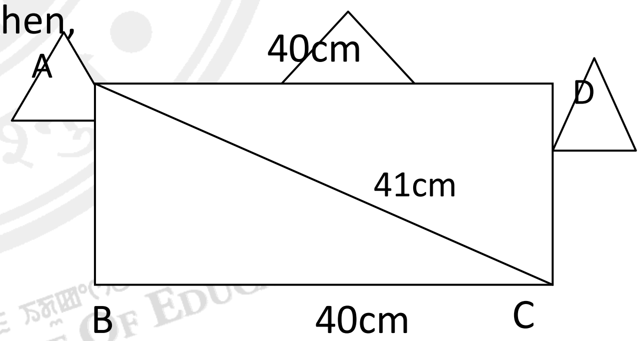
The original height of the tree = AB + AC = 13 +5 = 18m.

6. In the $\triangle$PQR, QR is the longest side then,

(i) $PQ^2 + QR^2 = RP^2$, which is false

(ii) $PQ^2 + RP^2 = QR^2$, which is true.

(iii) $RP^2 + QR^2 = PQ^2$, which is false.



7. Soln : Here,

AB = DC opposite sides of a rectangle.

ABC ≅ ADC

In ABC,

BC=40cm, AC= 41cm, AND AB= xcm.

Then, by Pythagoras Property, we get

$AB^2 + BC^2 = AC^2$

⇨ $X^2 + 40^2 = 41^2$

⇨ $X^2 + 1600 = 1681$

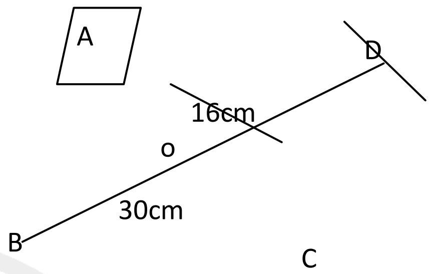
⇨ $X^2 = 1681 - 1600 = 81.$

⇨ $X = \sqrt{81} = \sqrt{9 \times 9} = 9cm$

Hence, $AB = 9cm.$

8. Soln : ABCD is rhombus in which AC and BD are diagonals

Then,



AO = 16/2 = 8

BO = 30/2 = 15

$AB^2 = BO^2 + AO^2$

⇨ $AB^2 = 15^2 + 8^2$

⇨ $AB^2 = 289$

⇨ $AB = \sqrt{289} = 17$

Therefore, perimeter of the rhombus is AB + BC + CD + DA

= 17+17+17+17 = 68cm.