CHAPTER: 13

EXPONENT AND POWERS

SOLUTIONS:

Exercise 13.1

Q1. Find the value of

(i) 2^6
$= 2^6$
$= 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 4 \times 4 \times 4$
$= 16 \times 4$
$= 64$

(ii) 9^3
$= 9^3$
$= 9 \times 9 \times 9$
$= 81 \times 9$
$= 729$

(iii) 11^2
$= 11^2$
$= 11 \times 11$
$= 121$

(iv) 5^4
$= 5^4$
$= 5 \times 5 \times 5 \times 5$
$= 25 \times 25$
$= 625$

Q2. Express the following in exponential form

(i) $6 \times 6 \times 6 \times 6$
$= 6 \times 6 \times 6 \times 6$
$= 6^4$

(ii) $t \times t$
$= t \times t$
$= t^2$

(iii) $b \times b \times b \times b$
$= b \times b \times b \times b$
$= b^4$

(iv) $5 \times 5 \times 7 \times 7 \times 7$
$= 5 \times 5 \times 7 \times 7 \times 7$
$= 5^2 \times 7^3$

(v) $2 \times 2 \times a \times a$
$= 2 \times 2 \times a \times a$
$= 2^2 \times a^2$

(VI) $a \times a \times a \times c \times c \times c \times c \times d$
$=(a \times a \times a) \times (c \times c \times c \times c) \times d$
$= a^3 \times c^4 \times d$

Q3. Express each of the following numbers using exponential notational.

(i) 512
$= 512$
$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^9$

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

(ii) 343
$= 7 \times 7 \times 7$
$= 7^3$

7	343
7	49
	7

(iii) 729
$= 729$
$= 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$= 3^6$

3	729
3	243
3	81
3	27
3	9
	3

(iv) 3125
$= 5 \times 5 \times 5 \times 5 \times 5$
$= 5^5$

5	3125
5	625
5	125
5	25
	5

Q4. Identify the greater number, wherever possible in each of the following.

(i) 4^3 or 3^4

Sol^n: 4^3 or 3^4
$4^3 = 4 \times 4 \times 4 = 64$
$3^4 = 3 \times 3 \times 3 \times 3 = 81$
Clearly, $81 > 64$
$\therefore 4^3 < 3^4$

(ii) 5^3 or 3^5

Sol^n: 5^3 or 3^5

$5^3 = 5 \times 5 \times 5 = 125$

$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

Clearly, 243 > 125

$\therefore 5^3 < 3^5$

(iii) 2^8 or 8^2

Sol^n: 2^8 or 8^2

$2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$

$8^2 = 8 \times 8 = 64$

Clearly, 256 > 64

$\therefore 2^8 > 8^2$

(iv) 100^2 or 2^{100}

Sol^n: 100^2 or 2^{100}

$100^2 = 100 \times 100 = 10{,}000$

$2^{100} = 2^{100}$ [cannot write because number is very big]

Clearly, $2^{100} > 100^2$

(v) 2^{10} or 10^2

Sol^n: 2^{10} or 10^2

$2^{10} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 4 \times 4 \times 4 \times 4 \times 4$

$= 16 \times 16 \times 4$

$= 256 \times 4$

$= 1024$

$10^2 = 10 \times 10$

$= 100$

Clearly, 1024 > 100

$\therefore 2^{10} > 10^2$

Q5. Express each of the following as product of powers of their factors

(i) 648

Sol^n: 648

Prime factors of 648 $= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

$= 2^3 \times 3^4$

2	648
2	324
2	162
3	81
3	27
3	9
	3

(ii) 405

Sol^{n}: 405

Prime factors of $405 = 5 \times 3 \times 3 \times 3 \times 3$

$= 5 \times 3^4$

5	405
3	81
3	27
3	9
	3

(iii) 540

Sol^{n}: 540

Prime factors of $540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$= 2^2 \times 3^3 \times 5$

2	540
2	270
3	135
3	45
3	15
	5

(iv) 3600

Sol^{n}: 3600

Prime factors of $3600 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

$= 2^4 \times 3^2 \times 5^2$

2	3600
2	1800
2	900
2	450
5	225
5	45
3	9
	3

Q6. Simplify

(i) 2×10^3

Sol^{n}: 2×10^3

$= 2 \times 10 \times 10 \times 10$

$= 2000$

(ii) $7^2 \times 2^2$

Sol^{n}: $7^2 \times 2^2$

$= 7 \times 7 \times 2 \times 2$

$= 49 \times 4$

$= 196$

(iii) $2^3 \times 5$

Sol^{n}: $2^3 \times 5$

$= 2 \times 2 \times 2 \times 5$

$= 8 \times 5$

$= 40$

(iv) 3×4^4

Sol^n: 3×4^4

$= 3 \times 4 \times 4 \times 4 \times 4$

$= 3 \times 256$

$= 768$

(v) 0×10^2

Sol^n: 0×10^2

$= 0 \times 10 \times 10$ [any number multiply by 0 is always 0]

$= 0$

(vi) $5^2 \times 3^3$

Sol^n: $5^2 \times 3^3$

$= 5 \times 5 \times 3 \times 3 \times 3$

$= 25 \times 27$

$= 675$

(vii) $2^4 \times 3^2$

Sol^n: $2^4 \times 3^2$

$= 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$= 4 \times 4 \times 9$

$= 144$

(viii) $3^2 \times 10^4$

Sol^n: $3^2 \times 10^4$

$= 3 \times 3 \times 10 \times 10 \times 10 \times 10$

$= 9 \times 10000$

$= 90000$

Q8. Compare the following numbers

(i) 2.7×10^{12} and 1.5×10^8

Sol^n: $2.7 \times 10^{12} = 27 \times 10^{11}$

$1.5 \times 10^8 = 15 \times 10^7$

Here, $27 > 15$

$10^{11} > 10^7$

So, $2.7 \times 10^{12} > 1.5 \times 10^8$

(ii) 4×10^{14} and 3×10^{17}

Sol^{n}: 4×10^{14} and 3×10^{17}
Here, $4 > 3$, but in 3×10^{17} powers is 17 and in 4×10^{14} powers is 14
So, $17 > 14$
$\therefore 3 \times 10^{17} > 4 \times 10^{14}$
Or $4 \times 10^{14} < 3 \times 10^{17}$

Q7. Simplify

(i) $(-4)^3$

Sol^{n}: $= (-4)^3$
$= (-4) \times (-4) \times (-4)$
$= -64$ $[(-1)^{\text{odd numbers}} = -1]$

(ii) $(-3) \times (-2)^3$

Sol^{n}: $= (-3) \times (-2) \times (-2) \times (-2)$ $[(-1)^{\text{odd numbers}} = -1]$
$= (-3) \times (-8)$ $[- \times - = +]$
$= 24$

(iii) $(-3)^2 \times (-5)^2$

Sol^{n}: $= (-3) \times (-3) \times (-5) \times (-5)$ $[(-1)^{\text{even numbers}} = 1]$
$= 9 \times 25$
$= 225$

(iv) $(-2)^3 \times (-10)^3$

Sol^{n}: $= (-2)^3 \times (-10)^3$
$= (-2) \times (-2) \times (-2) \times (-10) \times (-10) \times (-10)$
$= -8 \times -1000$ $[(-1)^{\text{odd numbers}} = -1]$
$= 8000$ $[- \times - = +]$

Exercise 13.2

Q1. Using laws of exponents simplify and write the answer in exponential form

(i) $3^2 \times 3^4 \times 3^8$

Sol^n: $= 3^2 \times 3^4 \times 3^8$
$= 3^{(2+4+8)}$ $[a^m \times a^n = a^{m+n}]$
$= 3^{14}$

(ii) $6^{15} \div 6^{10}$

Sol^n: $= 6^{15} \div 6^{10}$
$= 6^{(15-10)}$ $[a^m \div a^n = a^{m-n}]$
$= 6^5$

(iii) $a^3 \times a^2$

Sol^n: $= a^3 \times a^2$
$= a^{(3+2)}$
$= a^5$

(iv) $7^x \times 7^2$

sol^n: $= 7^x \times 7^2$
$= 7^{x+2}$

(v) $(5^2)^3 \div 5^3$

Sol^n: $= 5^{2\times3} \div 5^3$ $[(a^m)^n = a^{m\times n}]$
$= 5^6 \div 5^3$
$= 5^{(6-3)}$
$= 5^3$

(vi) $2^5 \times 5^5$

Sol^n: $= 2^5 \times 5^5$
$= (2 \times 5)^5$ $[a^m \times b^m = (ab)^m]$
$= 10^5$

(vii) $a^4 \times b^4$

sol^n: $= a^4 \times b^4$
$= (a\times b)^4$ $[a^m \times b^m = (ab)^m]$

(viii) $(3^4)^3$

sol^n: $= (3^4)^3$
$= 3^{4\times3}$
$= 3^{12}$

(ix) $(2^{20} \div 2^{15}) \times 2^3$

sol^n: $= 2^{(20-15)} \times 2^3$
$= 2^5 \times 2^3$
$= 2^{5+3}$
$= 2^8$

(x) $8^t \div 8^2$

Sol^n: $= 8^t \div 8^2$
$= 8^{t-2}$

Q2. Simplify and expres each of the following in exponential form:

(i) $\dfrac{2^3 \times 3^4 \times 4}{3\times32}$

Sol^n: $\dfrac{2^3 \times 3^4 \times 4}{3\times32}$

$= \dfrac{2^3 \times 3^4 \times 2^2}{3\times2^5}$

$= \dfrac{2^{3+2} \times 3^4}{3\times2^5}$

$= \dfrac{2^5 \times 3^4}{3\times2^5}$

$= 2^5 \div 2^5 \times 3^4 \div 3^1$

$= 2^{(5-5)} \times 3^{(4-1)}$

$= 2^0 \times 3^3$

$= 1 \times 3^3$ [$2^0 = 1$, any number except 0, raised to the power 0 is 1]

$= 3^3$

2	32
2	16
2	8
2	4
	2

(ii) $\{(5^2)^3 \times 5^4\} \div 5^7$

Sol^n: $= \{(5^2)^3 \times 5^4\} \div 5^7$
$= \{5^{2\times3} \times 5^4\} \div 5^7$
$= (5^6 \times 5^4) \div 5^7$
$= 5^{6+4} \div 5^7$
$= 5^{10} \div 5^7$
$= 5^{10-7}$
$= 5^3$

(iii) $25^4 \div 5^3$

Sol^n: $= 25^4 \div 5^3$
$= (5^2)^4 \div 5^3$
$= 5^{2\times4} \div 5^3$
$= 5^8 \div 5^3$
$= 5^{8-3}$
$= 5^5$

(iv) $\dfrac{3\times7^2\times11^8}{21\times11^3}$

Sol^n: $= \dfrac{3\times7^2\times11^8}{21\times11^3}$
$= \dfrac{3\times7^2\times11^8}{3\times7\times11^3}$
$= 3^1\div3^1\times7^2\div7^1\times11^8\div11^3$
$= 3^{(1-1)}\times7^{(2-1)}\times11^{(8-3)}$
$= 3^0\times7^1\times11^5$
$= 1\times7\times11^5$
$= 7\times11^5$

(v) $\dfrac{3^7}{3^4\times3^3}$

Sol^n: $= \dfrac{3^7}{3^4\times3^3}$
$= 3^7\div3^{(4+3)}$
$= 3^7\div3^7$
$= 3^{(7-7)}$
$= 3^0$ [$3^0 = 1$, any number except 0, raised to the power 0 is 1]
$= 1$

(vi) $2^0+3^0+4^0$

Sol^n: $= 2^0+3^0+4^0$ [$2^0 = 1$, any number except 0, raised to the power 0 is 1]
$= 1+1+1$
$= 3$

(vii) $2^0\times3^0\times4^0$

Sol^n: $= 2^0\times3^0\times4^0$ [$2^0 = 1$, any number except 0, raised to the power 0 is 1]
$= 1\times1\times1$
$= 1$

(viii) $(3^0+2^0)\times5^0$

Sol^n: $= (3^0+2^0)\times5^0$
$= (1+1)\times1$
$= 2\times1$
$= 2$

(ix) $\dfrac{2^8\times a^5}{4^2\times a^3}$

Sol^n: $= \dfrac{2^8\times a^5}{4^2\times a^3}$
$= \dfrac{2^8\times a^5}{(2^2)^3\times a^3}$
$= \dfrac{2^8\times a^5}{2^6\times a^3}$
$= 2^8\div2^6\times a^5\div a^3$
$= 2^{(8-6)}\times a^{(5-3)}$
$= 2^2\times a^2$
$= (2a)^2$

(x) $(a^5\div a^3)\times a^8$

Sol^n: $=(a^5\div a^3)\times a^8$
$= (a^{5-3})\times a^8$
$= a^2\times a^8$
$= a^{(2+8)}$
$= a^{10}$

(xi) $\dfrac{4^5\times a^8b^3}{4^5\times a^5b^2}$

Sol^n: $= \dfrac{4^5\times a^8b^3}{4^5\times a^5b^2}$
$= 4^5\div4^5\times a^8\div a^5\times b^3\div b^2$
$= 4^{(5-5)}\times a^{(8-5)}\times b^{(3-2)}$
$= 4^0\times a^3\times b^1$
$= 1\times a^3\times b$
$= a^3b$

(xii) $(2^3\times2)^2$

Sol^n: $= (2^3\times2)^2$
$= (2^3\times2^1)^2$
$= (2^{3+1})^2$
$= (2^4)^2$
$= 2^{4\times2}$
$= 2^8$

Q3. Say true or false and justify your answer

(i) $10\times10^{11} = 100^{11}$

Sol^{n}: $10\times10^{11} = 100^{11}$
$\Rightarrow 10^{(1+11)} = (10\times10)^{11}$
$\Rightarrow 10^{12} = (10^{2})^{11}$
$\Rightarrow 10^{12} = 10^{2\times11}$
$\Rightarrow 10^{12} = 10^{22}$
$\Rightarrow 10^{12}$ not equal to 10^{22}
False

(ii) $2^{3}>5^{2}$

Sol^{n}: $= 2^{3}>5^{2}$
$= 2\times2\times2 > 5\times5$
$= 8>25$
8 is not greater than 25
False.

(iii) $2^{3}\times3^{2}=6^{5}$

Sol^{n}: $\Rightarrow 2^{3}\times3^{2}=6^{5}$
$\Rightarrow 2^{3}\times3^{2}=(2\times3)^{5}$
$\Rightarrow 2^{3}\times3^{2}= 2^{5}\times3^{5}$
$2^{3}\times3^{2}$is not equal to $2^{5}\times3^{5}$
False

(iv) $3^{0}= (1000)^{0}$

Sol^{n}: $\Rightarrow 3^{0}= (1000)^{0}$
$\Rightarrow 1 = 1$ [any no. except 0 raised to the power 0 is 1]
True.

Q4. Express each of the following as product of prime factors only in exponential form

(i) 108×192

Sol^{n}: $= 108\times192$
$= (2\times2\times3\times3\times3) \times (2\times2\times2\times2\times2\times3)$
$= 2^{2}\times3^{3}\times2^{6}\times3^{1}$
$= 2^{2}\times2^{6}\times3^{3}\times3^{1}$
$= 2^{2+6}\times3^{3+1}$
$= 2^{8}\times3^{4}$

2	108
2	54
3	27
3	9
	3

2	192
2	96
2	48
2	24
2	12
2	6
	3

(ii) 270

Sol^{n}: $= 270$

$= 2\times3\times3\times3\times5$

$= 2\times3^{3}\times5$

2	270
3	90
3	30
2	10
	5

(iii) 729×64

Sol^{n}: $= (3\times3\times3\times3\times3\times3)\times(2\times2\times2\times2\times2\times2)$

$= 3^{6}\times2^{6}$

3	729
3	243
3	81
3	27
3	9
	3

2	64
2	32
2	16
2	8
2	4
	2

(iv) 768

Sol^{n}: $=768$

$= 2\times2\times2\times2\times2\times2\times2\times2\times3$

$= 2^{8}\times3$

2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
	3

Q5. Simplify:

(i) $= \frac{(2^5)^2 \times 7^3}{8^3 \times 7}$

Sol^n: $= \frac{(2^5)^2 \times 7^3}{8^3 \times 7}$

$= \frac{2^{5\times2} \times 7^3}{(2^3)^3 \times 7}$

$= \frac{2^{10} \times 7^3}{2^9 \times 7}$

$= 2^{10} \div 2^9 \times 7^3 \div 7^1$ $[a^m \div a^n = a^{m-n}]$

$= 2^{(10-9)} \times 7^{(3-1)}$

$= 2^1 \times 7^2$

$= 2 \times 49$

$= 98$

(ii) $\frac{25 \times 5^2 \times t^8}{10^3 \times t^4}$

Sol^n. $\frac{25 \times 5^2 \times t^8}{10^3 \times t^4}$

$= \frac{5^2 \times 5^2 \times t^8}{(2\times5)^3 \times t^4}$

$= \frac{5^{(2+2)} \times t^8}{2^3 \times 5^3 \times t^4}$

$= \frac{5^4 \times t^8}{2^3 \times 5^3 \times t^4}$

$= \frac{1}{2^3} \times 5^4 \div 5^3 \times t^8 \div t^4$ $[a^m \div a^n = a^{m-n}]$

$= \frac{1}{2^3} \times 5^{4-3} \times t^{8-4}$

$= \frac{1}{2} \times 5^1 \times t^4$

$= \frac{5 \times t^4}{2^3}$

$= \frac{5t^4}{8}$

(iii) $\frac{3^5 \times 10^5 \times 25}{5^7 \times 6^5}$

Soln: $\frac{3^5 \times 10^5 \times 25}{5^7 \times 6^5}$

$= \frac{3^5 \times (2\times5)^5 \times 5^2}{5^7 \times (2\times3)^5}$

$= \frac{3^5 \times 2^5 \times 5^5 \times 5^2}{2^5 \times 3^5 \times 5^7}$

$= \frac{2^5 \times 3^5 \times 5^{5+2}}{2^5 \times 3^5 \times 5^7}$

$= \frac{2^5 \times 3^5 \times 5^7}{2^5 \times 3^5 \times 5^7}$

$= 2^5 \div 2^5 \times 3^5 \div 3^5 \times 5^7 \div 5^7$

$= 2^{(5-5)} \times 3^{(5-5)} \times 2^{(7-7)}$

$= 2^0 \times 3^0 \times 5^0$ [$\because 2^0=1$

$= 1 \times 1 \times 1$ $3^0=1$

$= 1$ $5^0=1$]

Exercise 13.3

Q1. Write the following numbers in the expanded forms:
279404, 3006194, 2806196, 120719, 20068

Sol^{n}:

279404 $= 2x10000 + 7x10000 + 9x1000 + 4x100 + 0x10 + 4x1$
$= 2x10^5 + 7x10^4 + 9x10^3 + 4x10^3 + 4x10^2 + 0x10^1 + 4x10^0$

3006194 $= 3x1000000 + 0x100000 + 0x10000 + 6x1000 + 1x100 + 9x10 + 4x1$
$= 3x10^6 + 0x10^5 + 0x10^4 + 6x10^3 + 1x10^2 + 9x10^1 + 4x10^0$

2806196 $= 2x1000000 + 8x100000 + 0x10000 + 6x10000 + 1x100 + 9x10 + 6x1$
$= 2x10^6 + 8x10^5 + 0+10^4 + 6x10^3 + 1x10^2 + 9x10^1 + 6x10^0$

120719 $= 1x100000 + 2x10000 + 0x100 + 7x100 + 1x10 + 9x1$
$= 1x10^5 + 2x10^4 + 0x10^3 + 7x10^2 + 1x10^1 + 9x10^0$

20068 $= 2x10000 + 0x1000 + 0x100 + 6x10 + 8x1$
$= 2x10^4 + 0x10^3 + 0x10^2 + 6x10^1 + 8x10^0$

Q2. Find the number from each of the following expanded forms.

(a) $8x10^4 + 6x10^3 + 0x10^2 + 4x10^1 + 5x10^0$
$= 8x10000 + 6x1000 + 0x100 + 4x10 + 5x1$
$= 80000 + 6000 + 000 + 40 + 5$
$= 86045$

(b) $4x10^5 + 5x10^3 + 3x10^2\ 2x10^0$
$= 4x100000 + 0+5x1000 + 3x100 + 0+2x1$
$= 400000 + 0+5000 + 300 + 0 + 2$
$= 405302$

(c) $3x10^4 + 7x10^2 + 5x10^0$
$= 3x10000 + 0 + 7x100 + 0 + 5x1$
$= 30000 + 0 + 700 + 0 + 5$
$= 30705$

(d) $5x10^5 + 2x10^2 + 3x10^1$
$= 9x100000 + 0x10000 + 0x1000 + 2x100 + 3x10 + 0x10^0$
$= 900000 + 0 + 0 + 200 + 30 + 0$
$= 900230$

Q3. Express the following numbers in standard form:

(i) 5,00,00,000

= 5,00,00,000

= 5x1,00,00,000

= $5x10^7$

(ii) 70,00,000

= 70,00,000

= 7x1000000

= $7x10^6$

(iii) 3,18,65,00,000

= 3 18 65 00 000

= 3.1865 x 1000000000

= 3.1865×10^9

(iv) 3,90878

= 390878

= 3.90878 x 100000

= 3.90878×10^5

(v) 39087.8

= 39087.8

= 3.90878 x 10000

= 3.90878×10^4

(vi) 3908.78

= 3908.78

= 3.90878 x 1000

= 3.90878×10^3

Q4. Express the number appearing in the following statements in standard form.

(a) The distance between Earth and Moon is 384,000,000m

384,000,000 = (3.84x100000000) m

= 3.84×10^8m

(b) Speed of light in vacuum is 300,000,000 m/s

300,000,000 = (3 x 100,000,000) m/s

=$3x\ 10^8$ m/s

(c) Diameter of Earth is 1,27,56,000 m

1,27,56,000 = (1.2756 x 10,000,000) m

= 1.2756×10^7m

(d) Diameter of sum is 1,400,000,000 m

1,400,000,000 = (1.4 x 1,000,000,000) m

= 1.4×10^9m

(e) In a galaxy there are on an average 100,000,000,000 stars

100,000,000,000 = 1 x 100,000,000,000

= 1×10^{11} stars

(f) The universe is estimated to be about 12,000,000,000

12,000,000,000 = 1.2 x 10,000,000,000

= 1.2×10^{10} years old.

(g) The distance of the sun from the centre of the Milky way Galaxy is estimated to be

300,000,000,000,000,000,000 = 3 x 100,000,000,000,000,000,000

= 3×10^{20}m

(h) 60,230,000,000,000,000,000,000 molecules are contained in a drop of water weighing 1.8 gm

60,230,000,000,000,000,000,000 = 6.023 x 10,000,000,000,000,000,000,000

= 6.023×10^{22}

(i) The Earth has 1,535,000,000 cubic km of sea water

1,353,000,000 = 1.353 x 1,000,000,000

= 1.353×10^9 km^3

(j) The population of India was about 1,027,000,000 in March 2001

1,027,000,000 = 1.027 x 1,000,000,000

= 1.027×10^9
