CHAPTER 8
CIRCLES

- **Circle:** Circle is a closed figure consisting of all points which are at a constant distance (radius) from a fixed point (centre) in the plane.
- **Secant:** A line which intersects a circle at two distinct points is called a secant of the circle.
- **Tangent:** A line which intersects a circle at only one point is called a tangent to the circle.

Notes:

1. The tangent to a circle is a special case of secant, when the two endpoints of its corresponding chord coincide.
2. The common point of the tangent and the circle is called the point of contact.
3. All points of the tangent except the point of contact are exterior points of the circle.
4. There is no tangent to a circle passing through a point inside the circle.
5. There is one and only one tangent to a circle passing through a point lying on the circle.
6. There are exactly two tangents through (from) a point lying outside the circle.
7. Infinitely many tangents can be drawn to circle.

- **Theorems about tangents to a circle**

1. The tangent at any point of a circle is perpendicular to the radius through the point of contact.
2. The lengths of tangents drawn from an exterior point to a circle are equal.

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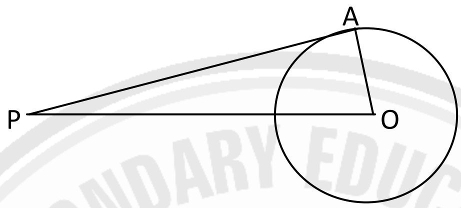
SOLUTIONS

EXERCISE 8.1

1. A point P is at a distance of 13 cm from the centre O of a circle. If the radius of the circle is 5 cm, find the length of tangent from P to the circle.

Solution:



Let O be the centre of the circle and PA be a tangent segment drawn from P to the circle.

It is given that OP = 13 cm and OA = 5 cm.

We know, ∠OAP = 90^0

Now, in the right ΔOAP, we have

$PA^2 + OA^2 = OP^2$ [by Pythagoras Theorem]

$\Rightarrow PA^2 + 5^2 = 13^2$

$\Rightarrow PA^2 + 25 = 169$

$\Rightarrow PA^2 = 169 - 25 = 144$

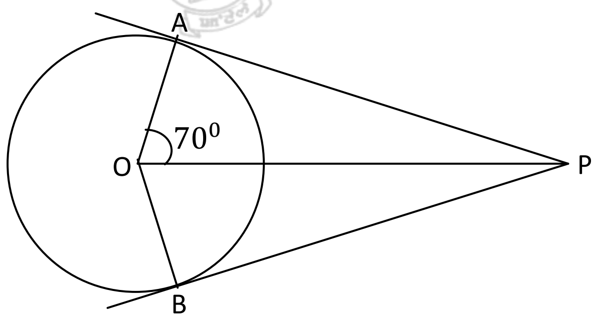
$\Rightarrow PA^2 = 12^2$

$\Rightarrow PA = 12$

∴ the length of tangent from P to the circle is 12 cm.

2. PA and PB are tangent segments drawn from an exterior point P to a circle with centre O. If $\angle AOP = 70^0$, find at what angle the two tangents are inclined to each other.

Solution:



PA and PB are two tangent segments drawn from an external point P to a circle with centre O such that $\angle AOP = 70^0$.

In ΔAOP and ΔBOP, we have

PA = PB [being tangent segments to a circle from an exterior point]

OP = OP [common side]

OA = OB [being radii of a circle]

$\therefore \Delta AOP \cong \Delta BOP$ [by SSS congruence]

$\Rightarrow \angle OPA = \angle OPB$

We have, $\angle OAP = 90^0$ [$\because$PA is a tangent segment at A]

Now, in ΔAOP, we have

$\angle OPA + \angle OAP + \angle AOP = 180^0$ [by angle sum property of triangle]

$\Rightarrow \angle OPA + 90^0 + 70^0 = 180^0$

$\Rightarrow \angle OPA + 160^0 = 180^0$

$\Rightarrow \angle OPA = 20^0$

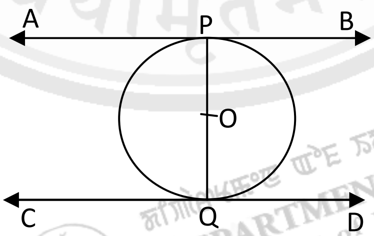
$\Rightarrow \angle OPA = \angle OPB = 20^0$

$\therefore \angle APB = \angle OPA + \angle OPB = 20^0 + 20^0 = 40^0$

Thus, the two tangents are inclined to each other at 40^0.

3. Prove that tangents at the ends of a diameter of a circle are parallel.

Solution:



Given: PQ is a diameter of a circle with centre O. AB and CD are tangents at P and Q respectively.

To prove: AB∥CD

Proof: We have, $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are tangents at P and Q.

$\therefore \overline{OP} \perp \overleftrightarrow{AB}$ and $\overline{OQ} \perp \overleftrightarrow{CD}$

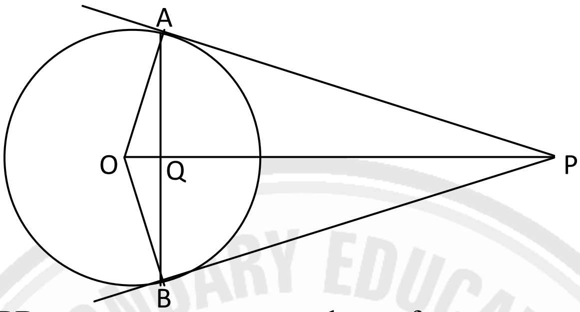
$\Rightarrow \angle OPA = \angle OQD \quad (= 90^0)$

But $\angle OPA$ and $\angle OQD$ are the alternate angles formed by PQ with two lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

$\therefore$ AB∥CD

4. If PA and PB are tangent segments drawn from an exterior point P to a circle whose centre is O, prove that OP bisects AB and hence OP⊥AB.

Solution:



Given: PA and PB are tangent segments drawn from an exterior point P to a circle with centre O. OP intersects AB at Q.

To prove: OP bisects AB and OP⊥AB.

Construction: OA and OB are joined.

Proof: In ΔAOP and ΔBOP, we have

PA = PB [being tangent segments to a circle from an exterior point]

OP = OP [common side]

OA = OB [being radii of a circle]

∴ ΔAOP≅ΔBOP [by SSS congruence]

⇒ ∠AOP = ∠BOP

⇒ ∠AOQ = ∠BOQ

In ΔAOQ and ΔBOQ, we have

OQ = OQ [common side]

∠AOQ = ∠BOQ

OA = OB [being radii of a circle]

∴ ΔAOQ≅ΔBOQ [by SAS congruence]

⇒ AQ = BQ i.e. OP bisects AB

and ∠AQO = ∠BQO

But $\angle AQO + \angle BQO = 180^0$ [being linear pair angles]

$\Rightarrow \angle AQO + \angle AQO = 180^0$

$\Rightarrow 2\angle AQO = 180^0$

$\Rightarrow \angle AQO = 90^0$

⇒ OP⊥AB

Thus, OP bisects AB and OP⊥AB.

5. Two concentric circles are of radii 6 cm and 10 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Solution: Let O be the centre of the concentric circles with radii OP = 6cm and OA = 10cm. Chord AB of the larger circle touches the smaller circle at P.

We have, AB is a tangent to the smaller circle at P. So, OP⊥AB

Also AB is a chord of the larger circle. So, OP bisects AB i.e. AB = 2.AP = 2.BP

In the right ΔAOP, we have

$AP^2 + OP^2 = OA^2$

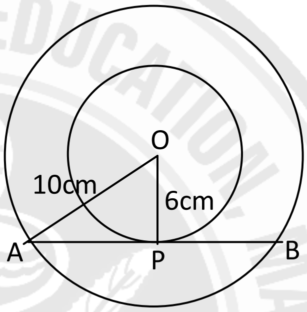
$\Rightarrow AP^2 + 6^2 = 10^2$

$\Rightarrow AP^2 + 36 = 100$

$\Rightarrow AP^2 = 64 = 8^2$

$\Rightarrow AP = 8$

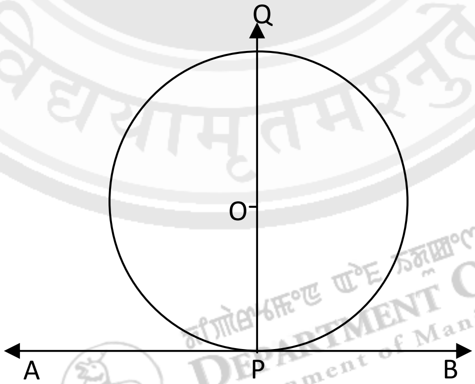
∴ AB = 2×8 = 16



Thus, the length of the chord AB of the larger circle is 16cm.

6. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Solution:



Given: AB is a tangent to a circle with centre O at a point P and PQ⊥AB

To prove: PQ passes through O.

Proof: We have, OP⊥AB [∵ tangent ⊥ radius at the point of contact]

and PQ⊥AB [given]

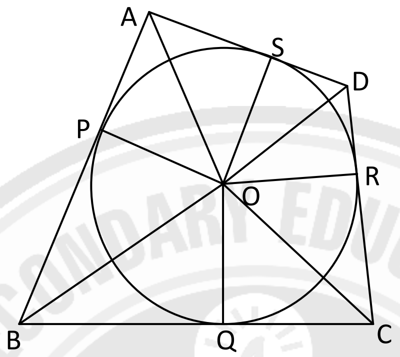
But, one and only one perpendicular can be drawn to a line through any point on the line.

So, OP and PQ must be on a same line i.e. OP and PQ are coincident.

Hence, PQ passes through O.

7. Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution:



Given: A circle with centre O touches the sides AB, BC, CD and DA of a quadrilateral ABCD at the points P, Q, R and S respectively.

To prove: $\angle AOB + \angle COD = \angle AOD + \angle BOC = 180^0$

Construction: OP, OQ, OR and OS are joined.

Proof: In ΔAOP and ΔAOS, we have

$AP = AS$ [being tangents to a circle from an exterior point]

$OA = OA$ [common side]

$OP = OS$ [radii of a circle]

$\therefore \Delta AOP \cong \Delta AOS$ [by SSS congruence]

$\Rightarrow \angle AOP = \angle AOS$

Similarly, $\angle BOP = \angle BOQ$, $\angle COQ = \angle COR$, $\angle DOR = \angle DOS$

Now, $\angle AOB + \angle COD = (\angle AOP + \angle BOP) + (\angle COR + \angle DOR)$

$= (\angle AOS + \angle BOQ) + (\angle COQ + \angle DOS)$

$= (\angle BOQ + \angle COQ) + (\angle AOS + \angle DOS)$

$= \angle AOD + \angle BOC$

But, $(\angle AOB + \angle COD) + (\angle AOD + \angle BOC) = 360^0$

$\Rightarrow (\angle AOB + \angle COD) + (\angle AOB + \angle COD) = 360^0$

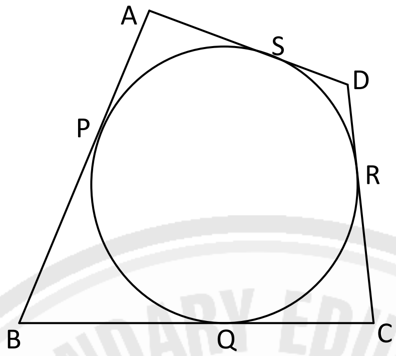
$\Rightarrow 2(\angle AOB + \angle COD) = 360^0$

$\Rightarrow \angle AOB + \angle COD = 180^0$

$\therefore \angle AOB + \angle COD = \angle AOD + \angle BOC = 180^0$

8. If a circle touches all the four sides of a quadrilateral ABCD, prove that AB + CD = BC + DA.

Solution:



Given: A circle touches the sides AB, BC, CD and DA of a quadrilateral ABCD at the points P, Q, R and S respectively.

To prove: AB + CD = BC + DA

Proof: We know, the lengths of tangents to a circle from an exterior point are equal in length.

∴ AP = AS, BP = BQ, CQ = CR and DR = DS

Now, AB + CD = (AP + BP) + (CR + DR)

= (AS + BQ) + (CQ + DS)

= (BQ + CQ) + (AS + DS)

= BC + DA

9. If ΔABC is isosceles with AB = AC. The incircle of the ΔABC touches BC at P. Prove that BP = CP.

Solution:

Given: ΔABC is isosceles with AB = AC. The incircle of ΔABC touches BC, CA and AB at P, Q and R respectively.

To prove: BP = CP

Proof: We know, the lengths of tangents to a circle from an exterior point are equal in length.

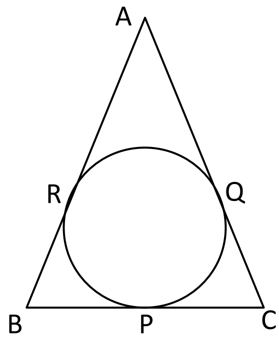
∴ AR = AQ, BP = BR and CP = CQ

We have, AB = AC [given]

⇒ AR + BR = AQ + CQ

⇒ BR = CQ [∵AR = AQ]

∴ BP = CP



10. Prove that the parallelogram circumscribing a circle is a rhombus.

Solution:

Given: A circle touches the sides AB, BC, CD and DA of a parallelogram ABCD at the points P, Q, R and S respectively.

To prove: ABCD is a rhombus.

Proof: We know, the lengths of tangents to a circle from an exterior point are equal in length.

∴ AP = AS, BP = BQ, CQ = CR and DR = DS

Also, AB = CD and BC = DA ---------- (1) [being opposite sides of a parallelogram]

Now, 2.AB = AB +AB

= AB + CD

= (AP + BP) + (CR + DR)

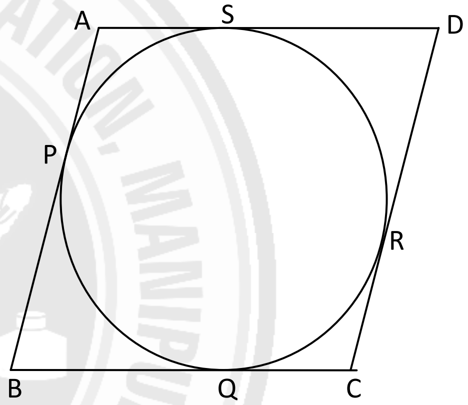
= (AS + BQ) + (CQ + DS)

= (BQ + CQ) + (AS + DS)

= BC + DA

= 2.BC

i.e. AB = BC ---------- (2)



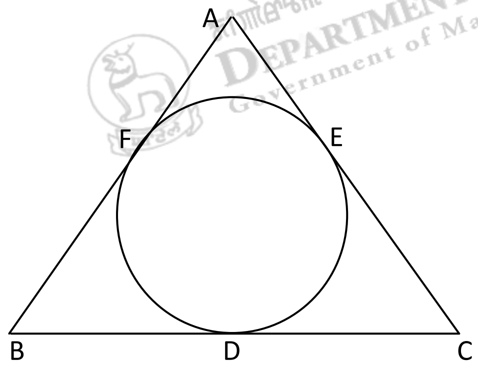
From (1) and (2), we get

AB = BC = CD = DA

Hence, ABCD is a rhombus.

11. The incircle of a ΔABC touches the sides BC, CA and AB at D, E and F respectively. Show that AF + BD + CE = AE + BF + CD = $\frac{1}{2}$ (perimeter of ΔABC).

Solution:



Given: The incircle of a ΔABC touches the sides BC, CA and AB at D, E and F respectively.

To prove: AF + BD + CE = AE + BF + CD = $\frac{1}{2}$ (perimeter of ΔABC)

Proof: We know, the lengths of tangents to a circle from an exterior point are equal in length.

∴ AF = AE, BF = BD and CD = CE

Now, $\frac{1}{2}$ (perimeter of ΔABC) = $\frac{1}{2}$ (AB + BC + CA)

= $\frac{1}{2}$ (AF + BF + BD + CD + CE + AE)

= $\frac{1}{2}$ (AF + BD + BD + CE + CE + AF)

= $\frac{1}{2}$ (2AF + 2BD + 2CE)

= $\frac{1}{2}$ ×2(AF + BD + CE)

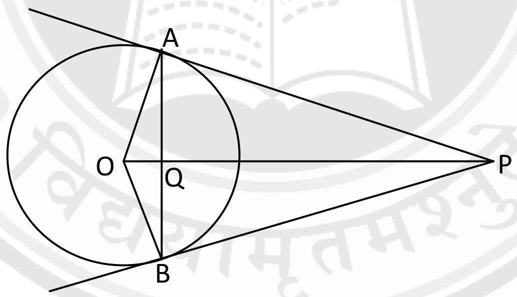
= AF + BD + CE

= AE + BF + CD

Hence, AF + BD + CE = AE + BF + CD = $\frac{1}{2}$ (perimeter of ΔABC)

12. If PA and PB are tangent segments drawn from an external point P to a circle with centre O, prove that ∠OAB = $\frac{1}{2}$∠APB.

Solution:



Given: PA and PB are tangent segments drawn from an external point P to a circle with centre O. AB intersects OP at Q.

To prove: ∠OAB = $\frac{1}{2}$∠APB

Construction: OB is joined.

Proof: We know, OA⊥PA and OB⊥PB [∵ tangent ⊥ radius at the point of contact]

⇒ ∠OAP = ∠OBP = 90^0

and ∠OAB = ∠OBA [∵OA = OB]

In the quadrilateral OAPB, we have

∠AOB + ∠OAP + ∠APB + ∠OBP = 360^0

⇒ ∠AOB + 90^0 + ∠ APB + 90^0 = 360^0

⇒ ∠AOB + ∠ APB = 180^0 ---------- (1)

In ΔAOB, we have

$\angle AOB + \angle OAB + \angle OBA = 180^0$ [by angle sum property of triangle]

$\Rightarrow \angle AOB + \angle OAB + \angle OAB = 180^0$

$\Rightarrow \angle AOB + 2\angle OAB = 180^0$ ---------- (2)

From (1) and (2), we have

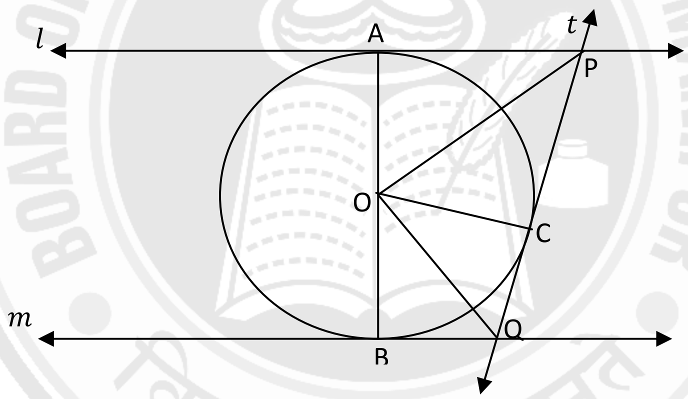
$\angle AOB + 2\angle OAB = \angle AOB + \angle APB \ (= 180^0)$

$\Rightarrow 2\angle OAB = \angle APB$

$\therefore \angle OAB = \frac{1}{2}\angle APB$

13. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

Solution:



Given: l and m are two parallel tangents of a circle with centre O at A and B. Another tangent t at C intersects l and m at P and Q respectively.

To prove: $\angle POQ = 90^0$

Construction: OA, OB and OC are joined.

Proof: In ΔAOP and ΔCOP, we have

OA = OC [radii of a circle]

OP = OP [common side}

PA = PC [being tangents to a circle from an exterior point]

$\therefore \Delta AOP \cong \Delta COP$ [by SSS congruence]

$\Rightarrow \angle OPC = \angle OPA = \frac{1}{2}\angle APC$

$\Rightarrow \angle OPQ = \frac{1}{2}\angle APQ$

Similarly, $\angle OQP = \frac{1}{2}\angle BQP$

We know, $\angle APQ + \angle BQP = 180^0$ [$\because$ sum of the interior angles on the same side of a transversal is 180^0]

$\Rightarrow \frac{1}{2}\angle APQ + \frac{1}{2}\angle BQP = \frac{1}{2} \times 180^0$

$\Rightarrow \angle OPQ + \angle OQP = 90^0$

In ΔPOQ, we have

$\angle POQ + \angle OPQ + \angle OQP = 180^0$ [by angle sum property of triangle]

$\Rightarrow \angle POQ + 90^0 = 180^0$

$\therefore \angle POQ = 90^0$

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