CHAPTER 7
TRIANGLES

- **Congruent Figures:** Two plane figures are congruent if they have the same shape and the same size.
- **Similar Figures:** Two figures having the same shape but not necessary the same size are called similar figures.

All congruent figures are always similar but similar figures need not be congruent.

- **Similar Polygons:** Two polygons having the same number of sides are similar, if
  a) their corresponding angles are equal and
  b) their corresponding sides are in the same ratio.

SOLUTIONS

EXERCISE 7.1

1. Fill in the blanks using the correct word given in brackets:

(i) All circles are ____________ (congruent, similar).

Ans: All circles are similar.

(ii) All circles of same radii are ____________ (congruent, not congruent).

Ans: All circles of same radii are congruent.

(iii) All squares are ____________ (similar, congruent).

Ans: All squares are similar.

(iv) All ____________ triangles are similar (isosceles, equilateral).

Ans: All equilateral triangles are similar.

(v) The reduced and enlarged photographs of an object made from the same negative are ____________ (similar, congruent).

Ans: The reduced and enlarged photographs of an object made from the same negative are similar.

2. State true or false:

(i) All similar figures are congruent.

Ans: FALSE

(ii) All congruent figures are similar.

Ans: TRUE

(iii) All triangles are similar.

Ans: FALSE

(iv) All equilateral triangles are similar.

Ans: TRUE

(v) All rectangles are similar.

Ans: FALSE

(vi) All squares are not similar.

Ans: FALSE

(vii) Two photographs of the same size of the same person, one at the age of 10 years and the other at the age of 60 years are similar.

Ans: FALSE

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- **Similar Triangles:** Two triangles are similar, if
  a) their corresponding angles are equal and
  b) their corresponding sides are in the same ratio.
- **Theorem 7.1**

  Basic Proportionality Theorem (or Thale's Theorem):

  If a line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the other two sides in the same ratio.
- **Theorem 7.2**

  Converse of Basic Proportionality Theorem:

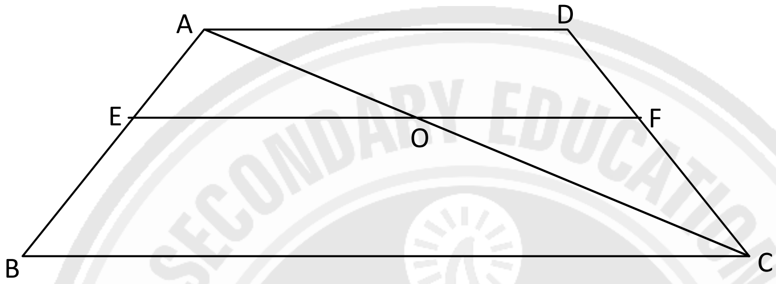
  If a line divides two sides of a triangle in the same ratio, then the line is parallel to the third side.

SOLUTIONS

EXERCISE 7.2

1. Prove that any line drawn parallel to the sides of a trapezium divides the oblique sides proportionally.

Solution:



Given: A trapezium ABCD in which AD ∥ BC. EF is drawn parallel to AD and BC intersecting the oblique sides AB and CD at E and F respectively.

To prove: $\frac{AE}{BE} = \frac{DF}{CF}$

Construction: AC is joined intersecting EF at O.

Proof: In ΔABC, EO ∥ BC

$\therefore \frac{AE}{BE} = \frac{AO}{CO}$ ------------- (1) [By Basic Proportionality Theorem)

Again in ΔADC, FO ∥ AD,

$\therefore \frac{AO}{CO} = \frac{DF}{CF}$ ------------- (2) [By Basic Proportionality Theorem)

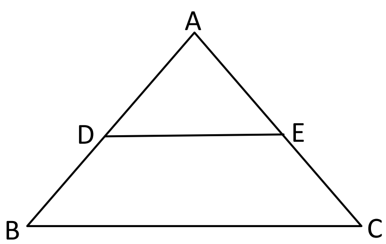
From (1) and (2), we get

$\frac{AE}{BE} = \frac{DF}{CF}$

Hence proved.

2. Prove that the line drawn from the mid-point of one side of a triangle parallel to another side bisects the third side.

Solution:



Given: A ΔABC in which D is the mid-point of side AB and DE ∥ BC meets AC at E.

To prove: DE bisects AC.

Proof: We have, AD = BD

In ΔABC, DE ∥ BC

$\therefore \frac{AD}{BD} = \frac{AE}{CE}$ [By Basic Proportionality Theorem]

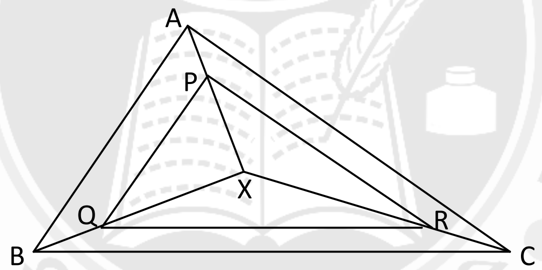
$\Rightarrow 1 = \frac{AE}{CE}$ [$\because$AD = BD]

$\Rightarrow$ AE = CE

Hence, DE bisects AC.

3. **Any point X inside a ΔABC is joined to the vertices. From a point P on AX, PQ is drawn parallel to AB meeting XB at Q and QR is drawn parallel to BC meeting XC at R. prove that PR ∥ AC.**

Solution:



Given: A ΔABC and a point X inside it. X is joined to the vertices A, B and C. P is a point on AX. PQ ∥ AB and QR ∥ BC.

To prove: PR ∥ AC

Construction: PR is joined.

Proof: In ΔAXB, PQ ∥ AB.

$\therefore \frac{XP}{PA} = \frac{XQ}{QB}$ ------------- (1) [By Basic Proportionality Theorem)

Again in ΔBXC, QR ∥ BC,

$\therefore \frac{XQ}{QB} = \frac{XR}{RC}$ ------------- (2) [By Basic Proportionality Theorem)

From (1) and (2), we have

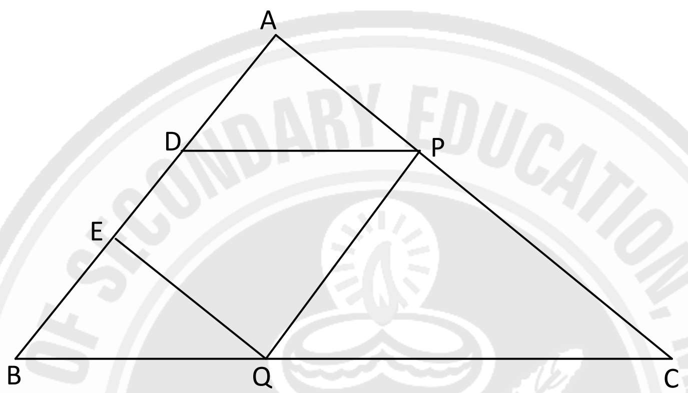
$$\frac{XP}{PA} = \frac{XR}{RC}$$

$\therefore$ PR divides the sides AX and CX of the ΔAXC in a same ratio.

Hence, by the converse of Basic Proportionality Theorem, we have PR ∥ AC.

4. **Let ABC be a triangle and D, E be two points on side AB such that AD= BE. If DP ∥ BC and EQ ∥ AC, where P and Q lies on AC and BC respectively. Prove that PQ ∥ AB.**

Solution:



Given: In ΔABC, D and E are two points on side AB such that AD= BE. DP ∥ BC and EQ ∥ AC, where P and Q lies on AC and BC respectively.

To prove: PQ ∥ AB

Proof: We have, AD= BE

$\Rightarrow AD + DE = BE + DE$

$\Rightarrow AE = BD$

In ΔABC, we have DP ∥ BC and EQ ∥ AC.

∴ By Basic Proportionality Theorem

$\frac{AD}{BD} = \frac{AP}{CP}$ -------------- (1)

and $\frac{BE}{AE} = \frac{BQ}{CQ}$

$\Rightarrow \frac{AD}{BD} = \frac{BQ}{CQ}$ ------------- (2) [∵AD= BE and AE = BD]

From(1) and (2), we get $\frac{AP}{CP} = \frac{BQ}{CQ}$

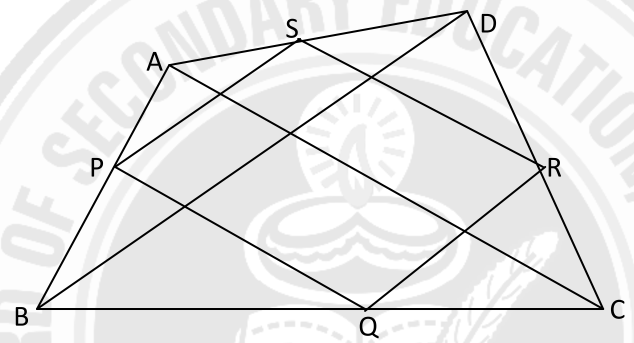
i.e. PQ divides the sides AC and BC of the ΔABC in a same ratio.

∴ PQ ∥ AB [By converse of Basic Proportionality Theorem]

Hence proved.

5. **ABCD is a quadrilateral. P, Q, R and S are the points on the sides AB, BC, CD and DA respectively such that AP : PB = AS : SD = CQ : QB = CR : RD. Prove that PQRS is a parallelogram.**

Solution:



Given: ABCD is a quadrilateral. P, Q, R and S are the points on the sides AB, BC, CD and DA respectively such that AP : PB = AS : SD = CQ : QB = CR : RD.

To prove: PQRS is a parallelogram.

Construction: AC and BD are joined.

Proof: We have, AP : PB = AS : SD and CQ : QB = CR : RD

∴ PS ∥ BD and QR ∥ BD [By converse of Basic Proportionality Theorem]

i.e. PS ∥ QR

Again, AP : PB = CQ : QB and AS : SD = CR : RD

∴ PQ ∥ AC and SR ∥ AC [By converse of Basic Proportionality Theorem]

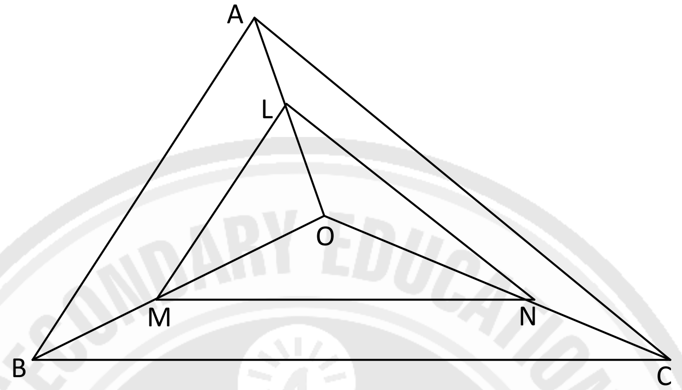
i.e. PQ ∥ SR

Now, the opposite sides of the quadrilateral PQRS are parallel.

Hence, PQRS is a parallelogram.

6. **On three line segments OA, OB and OC, points L, M, N respectively are so chosen that LM ∥ AB and MN ∥ BC but neither L, M, N nor A, B, C are collinear. Show that LN ∥ AC.**

Solution:



Given: On three line segments OA, OB and OC, points L, M, N respectively are so chosen that LM ∥ AB and MN ∥ BC but neither L, M, N nor A, B, C are collinear.

To prove: LN ∥ AC

Proof: In ΔAOB, LM ∥ AB

$\therefore \frac{OL}{AL} = \frac{OM}{BM}$ ------------- (1) [By Basic Proportionality Theorem)

Again, in ΔBOC, MN ∥ BC

$\therefore \frac{OM}{BM} = \frac{ON}{CN}$ ------------- (2) [By Basic Proportionality Theorem)

From (1) and (2), we get

$$\frac{OL}{AL} = \frac{ON}{CN}$$

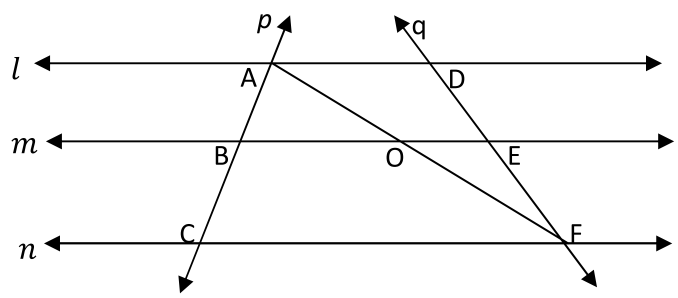
i.e. LN divides the sides OA and OC of ΔAOC in a same ratio.

∴ LN ∥ AC [By converse of Basic Proportionality Theorem]

Hence proved.

7. **Three or more parallel lines are intersected by two transversals. Prove that the intercepts made by them on the transversals are proportional.**

Solution:



Given: Three parallel lines l, m and n are intersected by two transversals p and q at A, B, C and D, E, F respectively.

To prove: $\frac{\text{AB}}{\text{BC}} = \frac{\text{DE}}{\text{EF}}$

Construction: AF is joined intersecting m at O.

Proof: In ΔACF, BO ∥ CF.

$\therefore \frac{\text{AB}}{\text{BC}} = \frac{\text{AO}}{\text{OF}}$ ------------- (1) [By Basic Proportionality Theorem)

Again in ΔAFD, OE ∥ AD,

$\therefore \frac{\text{AO}}{\text{OF}} = \frac{\text{DE}}{\text{EF}}$ ------------- (2) [By Basic Proportionality Theorem)

From (1) and (2), we have

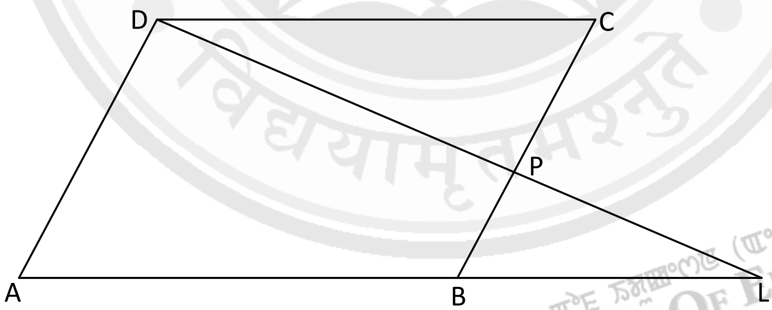
$$\frac{\text{AB}}{\text{BC}} = \frac{\text{DE}}{\text{EF}}$$

Hence proved.

8. ABCD is a parallelogram and P is a point on the side BC. DP when produced meets AB produced at L, prove that

(i) $\frac{DP}{PL} = \frac{DC}{BL}$ **(ii)** $\frac{DL}{DP} = \frac{AL}{DC}$

Solution:



Given: ABCD is a parallelogram and P is a point on the side BC. DP when produced meets AB produced at L.

To prove: **(i)** $\frac{DP}{PL} = \frac{DC}{BL}$ **(ii)** $\frac{DL}{DP} = \frac{AL}{DC}$

Proof:

(i) We have, AD∥BC i.e. AD∥BP

$\therefore \frac{\text{PL}}{\text{DP}} = \frac{\text{BL}}{\text{AB}}$ [By Basic Proportionality Theorem)

$\Rightarrow \frac{\text{DP}}{\text{PL}} = \frac{\text{AB}}{\text{BL}}$

$\therefore \frac{\text{DP}}{\text{PL}} = \frac{\text{DC}}{\text{BL}}$ [Being opposite sides of a parallelogram, AB=DC]

(ii) In ΔABL, we have $BP \parallel AD$

$\therefore \frac{PL}{DP} = \frac{BL}{AB}$ [By Basic Proportionality Theorem)

$\Rightarrow \frac{PL}{DP} + 1 = \frac{BL}{AB} + 1$

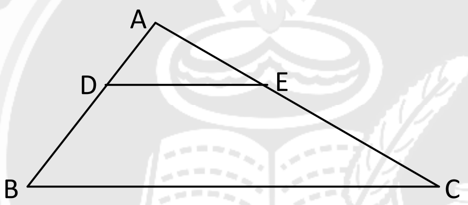
$\Rightarrow \frac{PL+DP}{DP} = \frac{BL+AB}{AB}$

$\Rightarrow \frac{DL}{DP} = \frac{AL}{AB}$

$\therefore \frac{DL}{DP} = \frac{AL}{DC}$ [Being opposite sides of a parallelogram, AB=DC]

9. In a ΔABC, D and E are points on the sides AB and AC respectively, such that $AD \times EC = AE \times DB$. DE ∥ BC.

Solution:



Given: In a ΔABC, D and E are points on the sides AB and AC respectively, such that $AD \times EC = AE \times DB$.

To prove: DE ∥ BC

Proof: We have, $AD \times EC = AE \times DB$

$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$

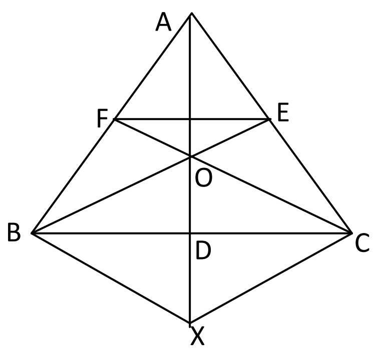
i.e. DE divides the sides AB and AC of ΔABC in a same ratio.

$\therefore$ DE ∥ BC [By converse of Basic Proportionality Theorem]

Hence proved.

10. The side BC of a triangle ABC is bisected at D. O is any point on AD. BO and CO produced meet AC and AB at E and F respectively and AD is produced to X so that D is the mid-point of OX. Prove that AO : AX = AF : AB and show that FE∥BC.

Solution:



Given: The side BC of a ΔABC is bisected at D. O is any point on AD. BO and CO produced meet AC and AB at E and F respectively and AD is produced to X so that D is the mid-point of OX.

To prove: AO : AX = AF : AB and FE ∥ BC.

Construction: BX and CX are joined.

Proof: We have, BD = CD and DO = DX

i.e. the diagonals of the quadrilateral BOCX bisect each other,

Then BOCX is a parallelogram.

∴ OC ∥ BX i.e. FO ∥ BX

Now, in the ΔABX, FO ∥ BX

$\therefore \frac{AO}{OX} = \frac{AF}{FB}$ ------------- (1) [By Basic Proportionality Theorem]

$\Rightarrow \frac{OX}{AO} = \frac{FB}{AF}$

$\Rightarrow \frac{OX}{AO} + 1 = \frac{FB}{AF} + 1$

$\Rightarrow \frac{OX+AO}{AO} = \frac{FB+AF}{AF}$

$\Rightarrow \frac{AX}{AO} = \frac{AB}{AF}$

$\Rightarrow \frac{AO}{AX} = \frac{AF}{AB}$

∴ AO : AX = AF : AB

Again, OB ∥ CX i.e. EO ∥ CX

In the ΔACX, EO ∥ CX

$\therefore \frac{AO}{OX} = \frac{AE}{CE}$ ------------- (2) [By Basic Proportionality Theorem]

From (1) and (2), we have

$\frac{AF}{FB} = \frac{AE}{CE}$

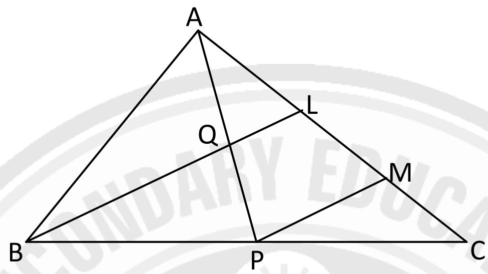
i.e. FE divides the sides AB and AC of ΔABC in a same ratio.

∴ FE ∥ BC [By converse of Basic Proportionality Theorem]

Hence proved.

11. P is the mid-point of the side BC of a triangle ABC and Q is the mid-point of AP. If BQ, when produced meets AC at L, prove that LA = $\frac{1}{3}$CA.

Solution:



Given: P is the mid-point of the side BC of a ΔABC and Q is the mid-point of AP. BQ when produced meets AC at L.

To prove: LA = $\frac{1}{3}$CA

Construction: PM ∥ BL is drawn to meet AC at M.

Proof: In ΔAPM, PM ∥ QL

$\therefore \frac{AQ}{PQ} = \frac{LA}{LM}$ [By Basic Proportionality Theorem]

$\Rightarrow 1 = \frac{LA}{LM}$ [∵Q is the mid-point of AP i.e. AQ = PQ]

$\therefore LA = LM$ ------------- (1)

In ΔBCL, PM ∥ BL

$\therefore \frac{BP}{CP} = \frac{LM}{CM}$ [By Basic Proportionality Theorem]

$\Rightarrow 1 = \frac{LM}{CM}$ [∵P is the mid-point of the side BC i.e. BP = CP]

$\therefore LM = CM$ ------------- (2)

From (1) and (2), we get

$LA = LM = CM$

We know, $LA + LM + CM = CA$

$\Rightarrow LA + LA + LA = CA$

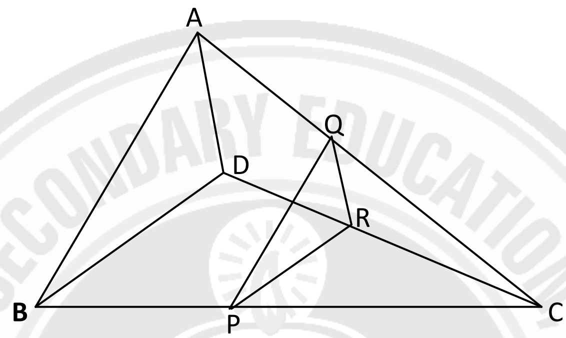
$\Rightarrow 3LA = CA$

$\therefore LA = \frac{1}{3}CA$

Hence proved.

12. **Two triangles ABC and DBC lie on the same side of BC. From a point P on BC, PQ is drawn parallel to BA and meeting AC at Q. PR is also drawn parallel to BD meeting CD at R. Prove that QR ∥ AD.**

Solution:



Given: ΔABC and ΔDBC are on the same side of BC. From a point P on BC, PQ∥BA and PR∥BD are drawn to meet AC at Q and CD at R respectively.

To prove: QR ∥ AD

Proof: In ΔABC, PQ ∥ BA

$\therefore \frac{CP}{BP} = \frac{CQ}{AQ}$ ------------- (1) [By Basic Proportionality Theorem]

In ΔDBC, PR ∥ BD

$\therefore \frac{CP}{BP} = \frac{CR}{DR}$ ------------- (2) [By Basic Proportionality Theorem]

From (1) and (2), we have

$$\frac{CQ}{AQ} = \frac{CR}{DR}$$

i.e. QR divides the sides CA and CD of ΔACD in a same ratio.

Hence, by converse of Basic Proportionality Theorem, we get QR ∥ AD.

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- **Theorem 7.3**

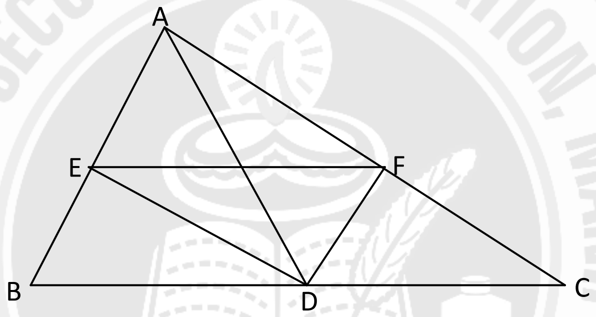
The internal bisector of an angle of a triangle divides the opposite sides internally in the ratio of the other two sides.

SOLUTIONS

EXERCISE 7.3

1. AD is a median of ΔABC. The bisectors of ∠ADB and ∠ADC meet AB and AC at E and F respectively. Prove that EF∥BC.

Solution:



Given: AD is a median of ΔABC. The bisectors of ∠ADB and ∠ADC meet AB and AC at E and F respectively.

To prove: EF∥BC

Proof: We have, AD is a median of ΔABC.

$\therefore BD = CD$

In ΔABD, DE is the internal bisector of ∠ADB.

$\therefore \frac{AE}{BE} = \frac{AD}{BD}$ ------------- (1)

In ΔADC, DF is the internal bisector of ∠ADC.

$\therefore \frac{AF}{CF} = \frac{AD}{CD}$

$\Rightarrow \frac{AF}{CF} = \frac{AD}{BD}$ ------------- (2) $[\because CD = BD]$

From (1) and (2), we get

$\frac{AE}{BE} = \frac{AF}{CF}$

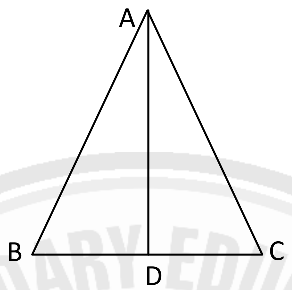
i.e. EF divides AB and AC of ΔABC in a same ratio.

∴ EF∥BC [by converse of Basic Proportionality Theorem]

Hence proved.

2. If the bisector of an angle of a triangle bisects the opposite side, prove that the triangle is isosceles.

Solution:



Given: A ΔABC in which the bisector AD of $\angle A$ bisects the opposite side BC.

To prove: ΔABC is isosceles.

Proof: In ΔABC, AD bisects $\angle A$ and the side BC.

$$\therefore BD = CD$$

And $\frac{AB}{AC} = \frac{BD}{CD}$

$$\Rightarrow \frac{AB}{AC} = 1$$

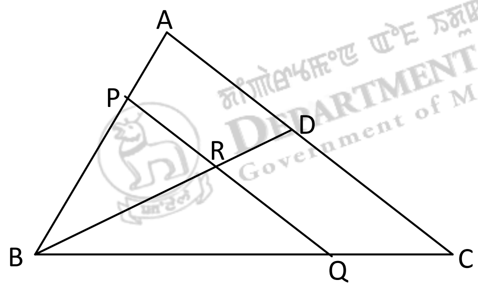
$$\Rightarrow AB = AC$$

Hence, ΔABC is isosceles.

3. In ΔABC, the bisector of $\angle B$ meets AC at D. A line PQ is drawn parallel to AC meeting AB, BC and BD at P, Q and R respectively. Show that

(i) PR × BQ=QR × BP (ii) AB × CQ=BC × AP

Solution:



Given: In ΔABC, the bisector of $\angle B$ meets AC at D. A line PQ is drawn parallel to AC meeting AB, BC and BD at P, Q and R respectively.

To prove: (i) PR × BQ=QR × BP (ii) AB × CQ=BC × AP

Proof: (i) In ΔBPQ, BR bisects $\angle B$.

$$\therefore \frac{BQ}{BP} = \frac{QR}{PR} \Rightarrow PR \times BQ = QR \times BP$$

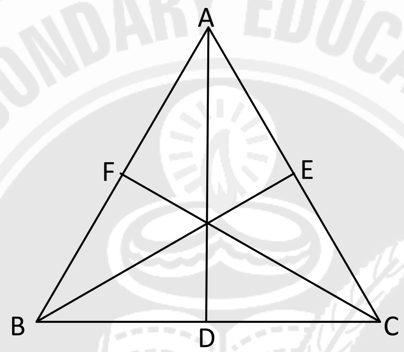
(ii) In ΔABC, $PQ \parallel AC$

$$\therefore \frac{AB}{AP} = \frac{BC}{CQ} \quad \text{[by Basic Proportionality Theorem]}$$

$$\Rightarrow AB \times CQ = BC \times AP$$

4. If the medians of a triangle are the bisectors of the corresponding angles of the triangle, prove that the triangle is equilateral.

Solution:



Given: ΔABC in which the medians AD, BE and CF are the bisectors of the corresponding angles $\angle A$, $\angle B$ and $\angle C$ respectively.

To prove: ΔABC is an equilateral.

Proof: We have AD, BE and CF are the medians of ΔABC.

$$\therefore BD = CD, CE = AE \text{ and } AF = BF$$

Also, AD and BE are the bisectors of $\angle A$, $\angle B$ and $\angle C$ respectively.

$$\therefore \frac{BD}{CD} = \frac{AB}{AC}$$

$$\Rightarrow 1 = \frac{AB}{AC}$$

$$\Rightarrow AB = AC \text{ ------------- (1)}$$

$$\frac{CE}{AE} = \frac{BC}{AB}$$

$$\Rightarrow 1 = \frac{BC}{AB}$$

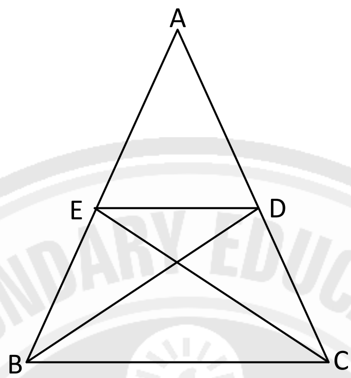
$$\Rightarrow AB = BC \text{ ------------- (2)}$$

From (1) and (2), we get $AB = BC = AC$

Hence, ΔABC is an equilateral.

5. The bisectors of the angles ∠B and ∠C of a triangle ABC meet the opposite sides at D and E respectively. If ED∥BC, prove that the triangle is isosceles.

Solution:



Given: In ΔABC, the bisectors of ∠B and ∠C meet the opposite sides at D and E respectively and ED∥BC.

To prove: ΔABC is isosceles.

Proof: BD is the internal bisector of ∠B.

$$\therefore \frac{AD}{CD} = \frac{AB}{BC}$$

CE is the internal bisector of ∠C.

$$\therefore \frac{AE}{BE} = \frac{AC}{BC}$$

But, in ΔABC, ED∥BC

$$\therefore \frac{AE}{BE} = \frac{AD}{CD} \text{ [by Basic Proportionality Theorem]}$$

$$\Rightarrow \frac{AC}{BC} = \frac{AB}{BC}$$

$$\Rightarrow AB = AC$$

Hence, ΔABC is isosceles.

- **Criteria for similarity of triangles**
  - **Theorem 7.4 (AAA similarity)**

    If the corresponding angles of two triangles are equal, then the triangles are similar.

Corollary: (AA similarity)

If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.

  - **Theorem 7.5 (SSS similarity)**

    If the corresponding sides of two triangles are in the same ratio, then the triangles are similar.

Definitions:

1. Two triangles are similar if the corresponding angles are equal.
2. Two triangles are similar if the corresponding sides are in the same ratio.

  - **Theorem 7.6 (SAS similarity)**

    If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio, the triangles are similar.

  - **Theorem 7.7**

    If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.

SOLUTIONS

EXERCISE 7.4

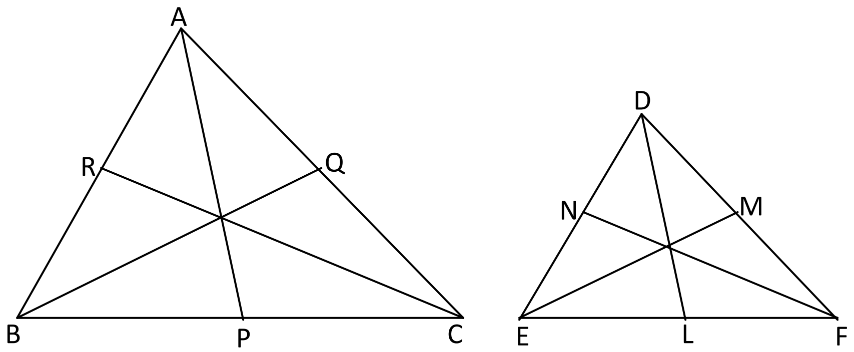
1. If two triangles are similar, prove that the corresponding

(i) medians are proportional.

(ii) altitudes are proportional.

Solution:

(i)



Given: $\Delta ABC \sim \Delta DEF$. AP, BQ, CR and DL, EM and FN are the corresponding medians of the triangles ABC and DEF.

To prove: $\frac{AP}{DL} = \frac{BQ}{EM} = \frac{CR}{FN}$

Proof: We have, $\frac{AB}{DE} = \frac{BC}{EF}$ [$\because \Delta ABC \sim \Delta DEF$]

$\Rightarrow \frac{AB}{DE} = \frac{2.BP}{2.EL}$

$\Rightarrow \frac{AB}{DE} = \frac{BP}{EL}$

In ΔABP and ΔDEL, we get

$\frac{AB}{DE} = \frac{BP}{EL}$

and $\angle B = \angle E$ [$\because \Delta ABC \sim \Delta DEF$]

$\therefore \Delta ABP \sim \Delta DEL$ [by SAS similarity]

$\Rightarrow \frac{AB}{DE} = \frac{AP}{DL}$ ------------- (1)

Similarly, $\Delta BCQ \sim \Delta EFM$

$\Rightarrow \frac{BC}{EF} = \frac{BQ}{EM}$ ------------- (2)

and $\Delta ACR \sim \Delta DFN$

$\Rightarrow \frac{AC}{DF} = \frac{CR}{FN}$ ------------- (3)

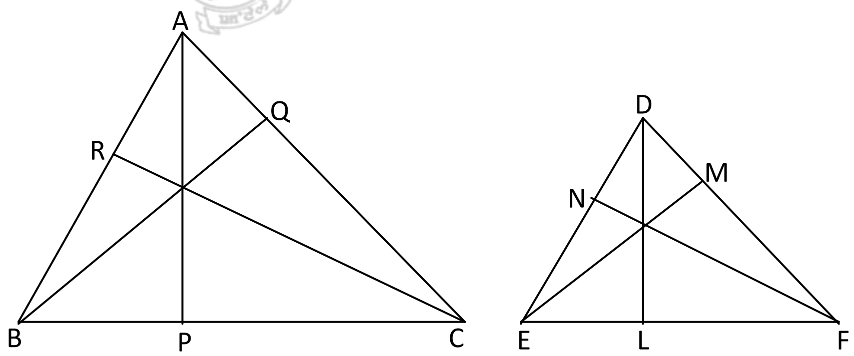
But $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ ------------- (4) [$\because \Delta ABC \sim \Delta DEF$]

From (1), (2), (3) and (4), we have

$\frac{AP}{DL} = \frac{BQ}{EM} = \frac{CR}{FN}$

Hence proved.

(ii)



Given: $\Delta ABC \sim \Delta DEF$. AP, BQ, CR and DL, EM and FN are the corresponding altitudes of the triangles ABC and DEF.

To prove: $\frac{AP}{DL} = \frac{BQ}{EM} = \frac{CR}{FN}$

Proof: In ΔABP and ΔDEL, we get

$\angle APB = \angle DLE \quad (= 90^0)$

and $\angle B = \angle E \quad [\because \Delta ABC \sim \Delta DEF]$

$\therefore \Delta ABP \sim \Delta DEL \quad$ [by AA similarity]

$\Rightarrow \frac{AB}{DE} = \frac{AP}{DL}$ ------------- (1)

Similarly, $\Delta BCQ \sim \Delta EFM$

$\Rightarrow \frac{BC}{EF} = \frac{BQ}{EM}$ ------------- (2)

and $\Delta ACR \sim \Delta DFN$

$\Rightarrow \frac{AC}{DF} = \frac{CR}{FN}$ ------------- (3)

But $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ ------------- (4) $[\because \Delta ABC \sim \Delta DEF]$

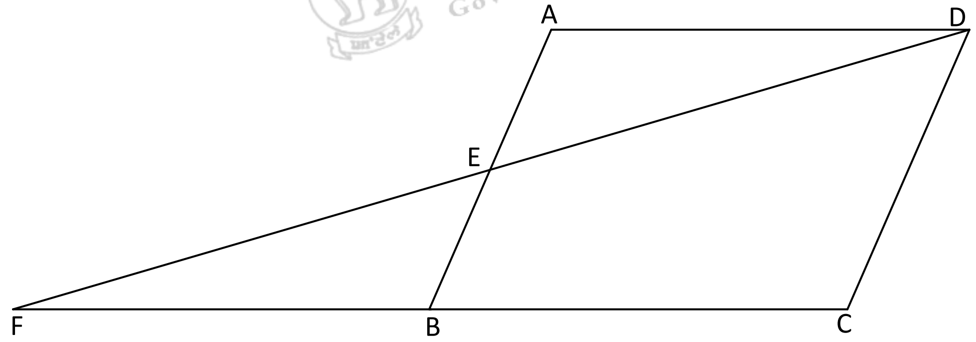
From (1), (2), (3) and (4), we have

$\frac{AP}{DL} = \frac{BQ}{EM} = \frac{CR}{FN}$

Hence proved.

2. ABCD is a parallelogram and E is the mid-point of AB. If F is the point of intersection of $\overleftrightarrow{DE}$ and $\overleftrightarrow{BC}$, prove that BC = BF.

Solution:



Given: ABCD is a parallelogram and E is the mid-point of AB. F is the point of intersection of $\overleftrightarrow{DE}$ and $\overleftrightarrow{BC}$.

To prove: BC = BF

Proof: In ΔAED and ΔBEF, we have

$\angle DAE = \angle FBE$ [being pair of alternate angles]

and $\angle AED = \angle BEF$ [being vertically opposite angles]

$\therefore \Delta AED \sim \Delta BEF$ [by AA similarity]

Then, $\frac{AE}{BE} = \frac{AD}{BF}$

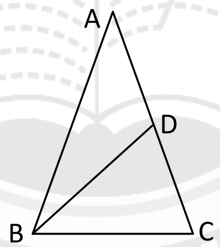
$\Rightarrow 1 = \frac{AD}{BF}$ [$\because AE = BE$]

$\Rightarrow AD = BF$ [being opposite sides of a parallelogram, AD = BC]

$\therefore BC = BF$

3. ABC is an isosceles triangle in which AB = AC and D is a point on AC such that $BC^2 = AC.CD$. Prove that BC = BD.

Solution:



Given: ABCD is an isosceles triangle in which AB = AC and D is a point on AC such that $BC^2 = AC.CD$.

To prove: BC = BD

Proof: We have, $BC^2 = AC.CD$

$\Rightarrow BC.BC = AC.CD$

$\Rightarrow \frac{BC}{CD} = \frac{AC}{BC}$

Thus, in ΔABC and ΔBDC, we have

$\frac{BC}{CD} = \frac{AC}{BC}$

and $\angle C = \angle C$ [common angle]

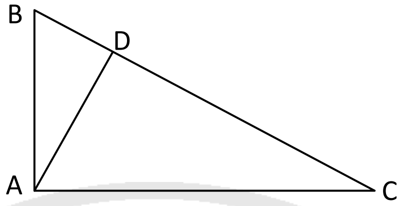
$\therefore \Delta ABC \sim \Delta BDC$ [by AA similarity]

$\Rightarrow \frac{AB}{BD} = \frac{AC}{BC}$

$\therefore BC = BD$ [$\because AB = AC$]

4. If in a triangle ABC, AD⊥BC and $AD^2 = BD.DC$, prove that ∠BAC = 90^0.

Solution:



Given: In ΔABC, AD⊥BC and $AD^2 = BD.DC$.

To prove: ∠BAC = 90^0

Proof: We have, $AD^2 = BD.DC$

$\Rightarrow AD.AD = BD.DC$

$\Rightarrow \frac{BD}{AD} = \frac{AD}{DC}$

Thus, in ΔBDA and ΔADC, we have

$\frac{BD}{AD} = \frac{AD}{DC}$

and ∠BDA = ∠ADC [= 90^0]

∴ ΔBDA~ΔADC [by SAS similarity]

⇒ ∠BAD = ∠ACD

In ΔADC, we have

∠ADC + ∠ACD + ∠CAD = 180^0 [by angle sum property of triangle]

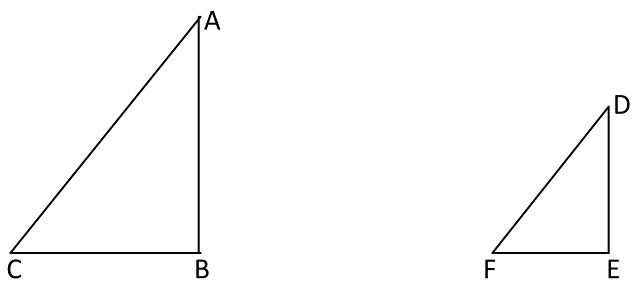
⇒ 90^0 + ∠ACD + ∠CAD = 180^0 [being AD⊥BC, ∠ADC = 90^0]

⇒ ∠ BAD + ∠ CAD = 90^0

∴∠BAC = 90^0

5. Find the height of a vertical tower which casts a shadow of length 36m at the time when the shadow of a vertical post of length 5m is 3 m.

Solution:



Let AB and DE respectively be the vertical tower and the vertical post. BC and EF be their respective shadows.

Then, BC = 36 m, DE = 5 m and EF = 3 m

In ΔABC and ΔDEF, we have

$\angle ABC = \angle DEF \quad [= 90^0]$

and $\angle ACB = \angle DFE$ (being measured at the same altitude of the sun)

$\therefore \Delta BDA \sim \Delta ADC$ [by AA similarity]

$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF}$

$\Rightarrow \frac{AB}{5} = \frac{36}{3}$

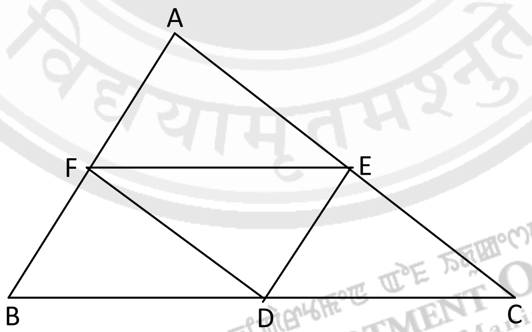
$\Rightarrow \frac{AB}{5} = 12$

$\Rightarrow AB = 60 \text{ m}$

Hence, the height of the tower is 60 m.

6. Prove that the line segment joining the mid-points of the sides of a triangle form four triangles, each of which is similar to the original triangle.

Solution:



Given: ΔABC in which D, E and F are the mid-points of the sides BC, CA and AB respectively.

To Prove: Each of the triangles AFE, FBD, EDC and DEF is similar to ΔABC.

Proof: We have, F and E are the mid-points of AB and AC.

$\therefore FE \parallel BC$ [by mid-point theorem]

$\Rightarrow \angle AFE = \angle ABC$ and $\angle AEF = \angle C$ [being corresponding angles]

$\therefore \Delta AFE \sim \Delta ABC$ [by AA similarity]

Similarly, ΔFBD~ΔABC and ΔEDC~ΔABC

Again, DF∥CA and DE∥AB i.e. DF∥EA and DE∥FA

∴ AFDE is a parallelogram.

⇒∠EDF = ∠A

Similarly, BDEF is also a parallelogram.

⇒∠DEF = ∠B

Now, in ΔDEF and ΔABC, we have

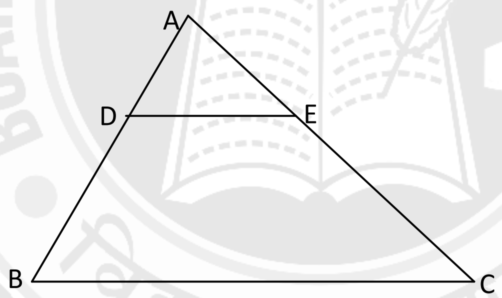
∠EDF = ∠A and ∠DEF = ∠B

∴ ΔDEF~ΔABC [by AA similarity]

Thus, each of the triangles AFE, FBD, EDC and DEF is similar to ΔABC.

7. In a ΔABC, DE is parallel to base BC with D on AB and E on AC. If $\frac{AD}{DB} = \frac{2}{3}$, find $\frac{BC}{DE}$.

Solution:



We have, $\frac{AD}{DB} = \frac{2}{3}$ i.e. $\frac{DB}{AD} = \frac{3}{2}$ and DE∥BC

∴ ∠ABC = ∠ADE and ∠ACB = ∠AED [being corresponding angles]

Then, ΔABC ~ΔADE [by AA similarity]

$$\therefore \frac{BC}{DE} = \frac{AB}{AD}$$

$$\Rightarrow \frac{BC}{DE} = \frac{AD+DB}{AD}$$

$$\Rightarrow \frac{BC}{DE} = \frac{AD}{AD} + \frac{DB}{AD}$$

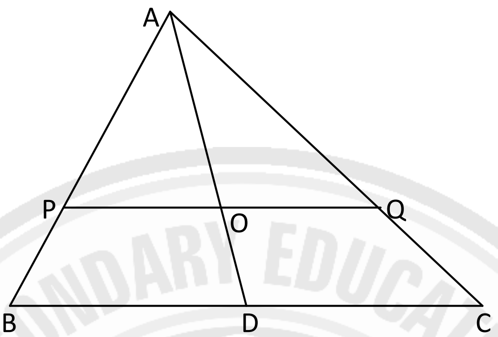
$$\Rightarrow \frac{BC}{DE} = 1 + \frac{3}{2}$$

$$\Rightarrow \frac{BC}{DE} = \frac{2+3}{2}$$

$$\therefore \frac{BC}{DE} = \frac{5}{2}$$

8. In a ΔABC, P and Q are points on AB and AC respectively such that $PQ \parallel BC$. Prove that median AD bisects PQ.

Solution:



Given: In ΔABC, P and Q are points on AB and AC respectively such that $PQ \parallel BC$ and median AD intersects PQ at O.

To prove: AD bisects PQ i.e. OP = OQ

Proof: We have, BD = CD [$\because$ AD is a median of ΔABC]

In ΔAOP and ΔADB, we have

$\angle AOP = \angle ADB$ and $\angle APO = \angle ABD$ [being corresponding angles]

$\therefore \Delta APO \sim \Delta ABD$ [by AA similarity]

$\Rightarrow \frac{OA}{AD} = \frac{OP}{BD}$ ------------- (1)

Similarly, $\Delta AOQ \sim \Delta ADC$

$\Rightarrow \frac{OA}{AD} = \frac{OQ}{CD}$ ------------- (2)

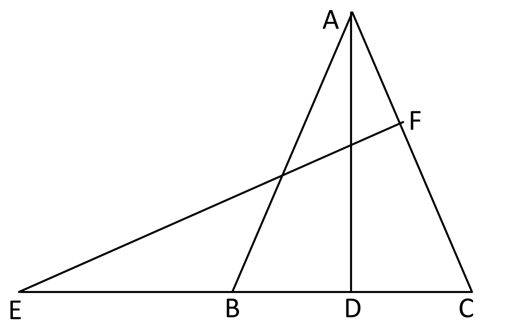
From (1) and (2), we get

$\frac{OP}{BD} = \frac{OQ}{CD}$

$\therefore$ OP = OQ [$\because$ BD = CD]

9. ABC is an isosceles triangle in which AB = AC. AD is drawn perpendicular to BC. From a point E on CB produced, EF is drawn perpendicular to AC. Prove that $\Delta ADC \sim \Delta ECF$.

Solution:



Given: ABC is an isosceles triangle in which AB = AC. AD is drawn perpendicular to BC. From a point E on CB produced, EF is drawn perpendicular to AC.

To prove: $\Delta ADC \sim \Delta ECF$

Proof: In ΔADC and ΔECF, we have

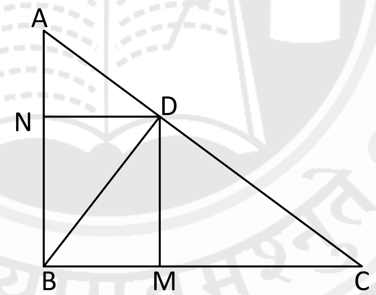
$\angle ADC = \angle EFC$ $(= 90^0)$

and $\angle C = \angle C$ [common angle]

$\therefore \Delta ADC \sim \Delta ECF$ [by AA similarity]

10. ABC is a right triangle right angled at B and D is the foot of the perpendicular drawn from B on AC. If DM⊥BC and DN⊥AB where M, N lie on BC, AB respectively, prove that (i) $DM^2 = DN \times MC$ (ii) $DN^2 = DM \times AN$

Solution:



Given: ABC is a right triangle right angled at B and BD⊥AC also DM⊥BC and DN⊥AB.

To prove: (i) $DM^2 = DN \times MC$ (ii) $DN^2 = DM \times AN$

Proof: We have, $\angle MBN = \angle BMD = \angle BND = 90^0$

$\therefore \angle MDN = 90^0$

So, BMDN is a rectangle.

$\therefore BM = DN$ and $DM = BN$

We know, $\angle DBN + \angle DBM = 90^0$ $[\because \angle ABC = 90^0]$

and $\angle DBM + \angle DCM = 90^0$ $[\because \angle BDC = 90^0]$

Then, $\angle DBN + \angle DBM = \angle DBM + \angle DCM$ $(= 90^0)$

$\Rightarrow \angle DBN = \angle DCM$

Similarly, $\angle DBM = \angle DAN$

(i) In Δ DCM and Δ DBN, we have

$\angle DNB = \angle DMC \ (= 90^0)$

and $\angle DBN = \angle DCM$

$\therefore \Delta DBN \sim \Delta DCM$ [by AA similarity]

$\Rightarrow \frac{DN}{DM} = \frac{BN}{MC}$

$\Rightarrow \frac{DN}{DM} = \frac{DM}{MC}$

$\Rightarrow DM^2 = DN \times MC$

(ii) In ΔDBM and ΔDAN, we have

$\angle DMB = \angle DNA \ (= 90^0)$

and $\angle DBM = \angle DAN$

$\therefore \Delta DBM \sim \Delta DAN$ [by AA similarity]

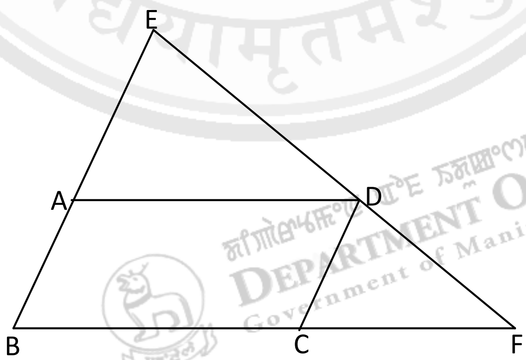
$\Rightarrow \frac{DM}{DN} = \frac{BM}{AN}$

$\Rightarrow \frac{DM}{DN} = \frac{DN}{AN}$

$\Rightarrow DN^2 = DM \times AN$

11. Through the vertex D of parallelogram ABCD, a line is drawn to intersect the sides BA produced and BC produced at E and F respectively. Prove that $\frac{AD}{AE} = \frac{BF}{BE} = \frac{CF}{CD}$.

Solution:



Given: Through the vertex D of a parallelogram ABCD, a line is drawn to intersect the sides BA produced and BC produced at E and F respectively.

To prove: $\frac{AD}{AE} = \frac{BF}{BE} = \frac{CF}{CD}$

Proof: We have AD∥BC and DC∥AB [being opposite sides of a parallelogram]

i.e. AD∥BF and DC∥EB

Now, in ΔAED and ΔBEF, we have

$\angle EAD = \angle EBF$ and $\angle EDA = \angle EFB$ [being pairs of corresponding angles]

$\therefore \Delta\, AED \sim \Delta\, BEF$ [by AA similarity]

$$\Rightarrow \frac{AD}{BF} = \frac{AE}{BE}$$

$$\Rightarrow AD \times BE = AE \times BF$$

$$\Rightarrow \frac{AD}{AE} = \frac{BF}{BE} \text{ ------------- (1)}$$

Again, in ΔCDF and ΔBEF, we have

$\angle FDC = \angle FEB$ and $\angle FCD = \angle FBE$ [being pairs of corresponding angles]

$\therefore \Delta\, CDF \sim \Delta\, BEF$ [by AA similarity]

$$\Rightarrow \frac{CD}{BE} = \frac{CF}{BF}$$

$$\Rightarrow CD \times BF = CF \times BE$$

$$\Rightarrow \frac{BF}{BE} = \frac{CF}{CD} \text{ ------------- (2)}$$

Combining (1) and (2), we get

$$\frac{AD}{AE} = \frac{BF}{BE} = \frac{CF}{CD}$$

12. The perimeters of two similar triangles ABC and PQR are respectively, 72 cm and 48 cm. If PQ = 20 cm, find AB.

Solution: We have $\Delta ABC \sim \Delta PQR$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = k \text{ (say)}$$

$\Rightarrow AB = k.PQ$, $BC = k.QR$ and $AC = k.PR$

And $\dfrac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} = \dfrac{72}{48}$

$$\Rightarrow \frac{AB+BC+AC}{PQ+QR+PR} = \frac{3}{2}$$

$$\Rightarrow \frac{k.PQ+k.QR+k.PR}{PQ+QR+PR} = \frac{3}{2}$$

$$\Rightarrow \frac{k(PQ+QR+PR)}{PQ+QR+PR} = \frac{3}{2}$$

$$\Rightarrow k = \frac{3}{2}$$

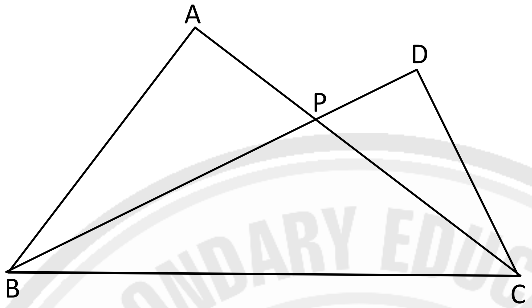
Then, $AB = \frac{3}{2} \times 20$ cm

$\Rightarrow AB = 3 \times 10$ cm

$\therefore AB = 30$ cm

13. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that AP × PC = BP × PD.

Solution:



Given: Two right triangles ABC and DBC are on the same hypotenuse BC and on the same side of BC. AC and BD intersect at P.

To prove: AP × PC = BP × PD

Proof: In ΔAPB and ΔDPC, we have

$\angle BAC = \angle CDB\ (= 90^0)$

and $\angle APB = \angle DPC$ [being vertically opposite angles]

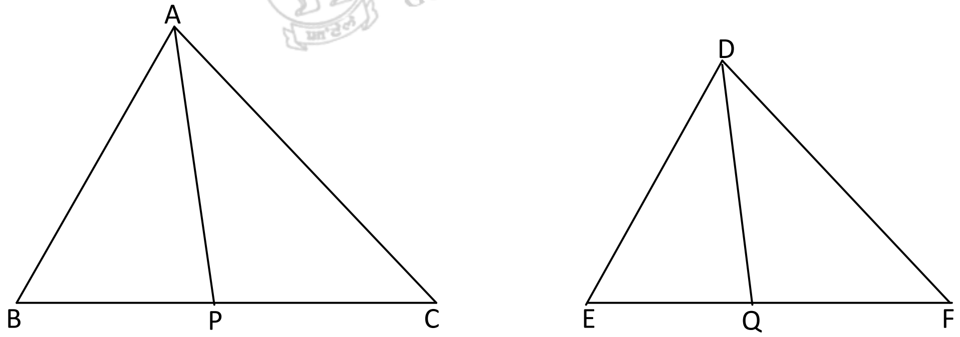
$\therefore \Delta APB \sim \Delta DPC$ [by AA similarity]

$\Rightarrow \frac{AP}{PD} = \frac{BP}{PC}$

$\therefore AP \times PC = BP \times PD$

14. If one angle of a triangle is equal to one angle of another triangle and the bisectors of these equal angles divide the opposite sides in the same ratio, prove that the triangles are similar.

Solution:



Given: ΔABC and ΔDEF in which $\angle A = \angle D$. AP and DQ are the bisectors of $\angle A$ and $\angle D$ respectively, such that $\frac{BP}{CP} = \frac{EQ}{FQ}$.

To prove: $\Delta ABC \sim \Delta DEF$

Proof: We have AP and DQ are the bisectors of $\angle A$ and $\angle D$.

$\therefore \frac{BP}{CP} = \frac{AB}{AC}$ and $\frac{EQ}{FQ} = \frac{DE}{DF}$

But $\frac{BP}{CP} = \frac{EQ}{FQ}$ [given]

$\Rightarrow \frac{AB}{AC} = \frac{DE}{DF}$

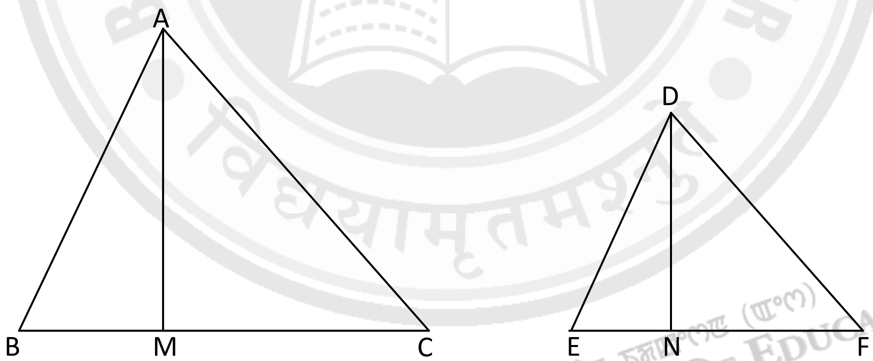
Now, in ΔAPB and ΔDPC, we have

$\frac{AB}{AC} = \frac{DE}{DF}$ and $\angle A = \angle D$

$\therefore \Delta ABC \sim \Delta DEF$ [by SAS similarity]

15. If two triangles are equiangular, prove that the ratio of the corresponding sides is same as the ratio of the corresponding altitudes.

Solution:



Given: ΔABC and ΔDEF in which $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$. AM and DN are corresponding altitudes of ΔABC and ΔDEF.

To prove: $\frac{AB}{DE} = \frac{AM}{DN}$

Proof: In ΔABM and ΔDEN, we have

$\angle B = \angle E$ (given)

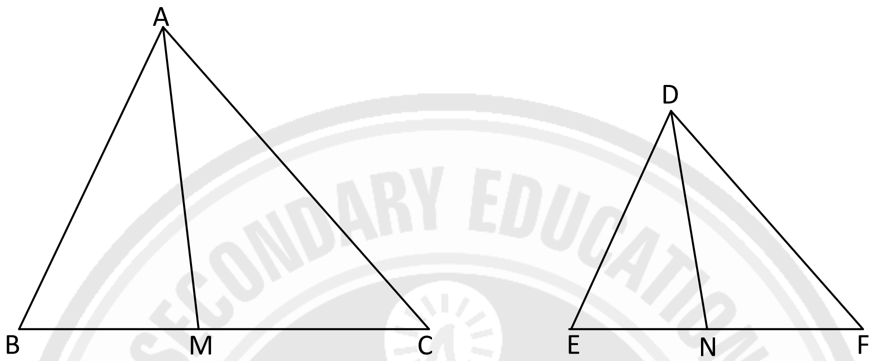
and $\angle AMB = \angle DNE$ $(= 90^0)$

$\therefore \Delta ABM \sim \Delta DEN$ [by AA similarity]

$\Rightarrow \frac{AB}{DE} = \frac{AM}{DN}$

16. If two triangles are equiangular, prove that the ratio of the corresponding sides is same as the ratio of the corresponding angle bisector segments.

Solution:



Given: ΔABC and ΔDEF in which $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$. AM and DN are the bisectors of $\angle A$ and $\angle D$ respectively.

To prove: $\frac{AB}{DE} = \frac{AM}{DN}$

Proof: In ΔABM and ΔDEN, we have

$\angle B = \angle E$ (given)

and $\angle BAM = \angle EDN$ $[\because \frac{1}{2}\angle A = \frac{1}{2}\angle D]$

$\therefore \Delta ABM \sim \Delta DEN$ [by AA similarity]

$\Rightarrow \frac{AB}{DE} = \frac{AM}{DN}$

➢ Areas of Similar Triangles

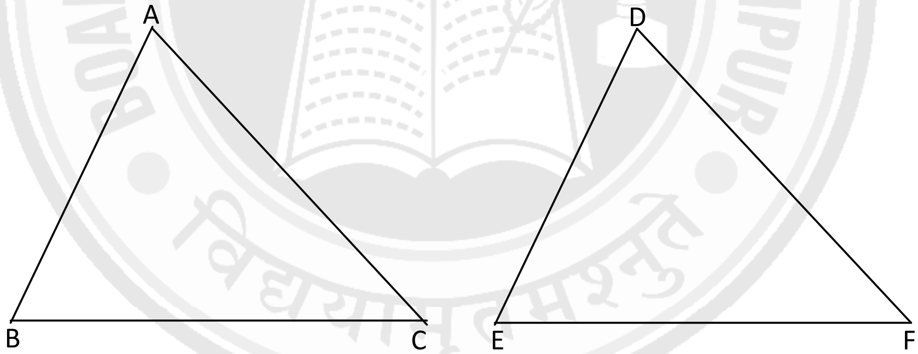
Theorem 7.8 The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

SOLUTIONS

EXERCISE 7.5

1. If the areas of two similar triangles are equal, prove that they are congruent.

Solution:



Given: $\Delta ABC \sim \Delta DEF$ and $ar(\Delta ABC) = ar(\Delta DEF)$

To prove: $\Delta ABC \cong \Delta DEF$

Proof: We have $\Delta ABC \sim \Delta DEF$

Then, $\dfrac{ar(\Delta DEF)}{ar(\Delta ABC)} = \dfrac{AB^2}{DE^2} = \dfrac{BC^2}{EF^2} = \dfrac{AC^2}{DF^2}$

$\Rightarrow 1 = \dfrac{AB^2}{DE^2} = \dfrac{BC^2}{EF^2} = \dfrac{AC^2}{DF^2}$

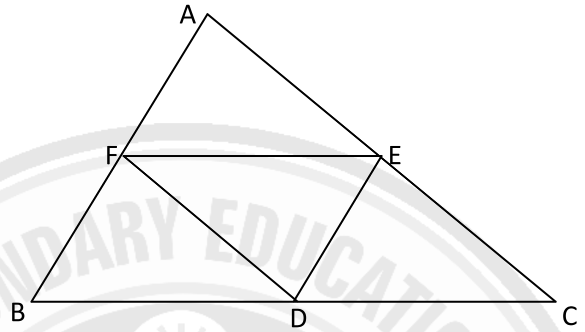
$\Rightarrow AB^2 = DE^2, BC^2 = EF^2, AC^2 = DF^2$

$\Rightarrow AB = DE, BC = EF, AC = DF$

$\therefore \Delta ABC \cong \Delta DEF$ [by SSS congruence]

2. If D, E, F are respectively the mid-points of the sides BC, CA, AB of a ΔABC, prove that $\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$

Solution:



Given: D, E, F are respectively the mid-points of the sides BC, CA, AB of a ΔABC.

To prove: $\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$

Proof: We have D, E, F are respectively the mid-points of the sides BC, CA, AB.

$\therefore DE = \frac{1}{2}AB, EF = \frac{1}{2}BC$ and $DF = \frac{1}{2}CA$. [by mid-point theorem]

i.e. $\frac{DE}{AB} = \frac{1}{2}, \frac{EF}{BC} = \frac{1}{2}$ and $\frac{DF}{CA} = \frac{1}{2}$

i.e. $\frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{CA}$

$\therefore \Delta ABC \sim \Delta DEF$ [by SSS similarity]

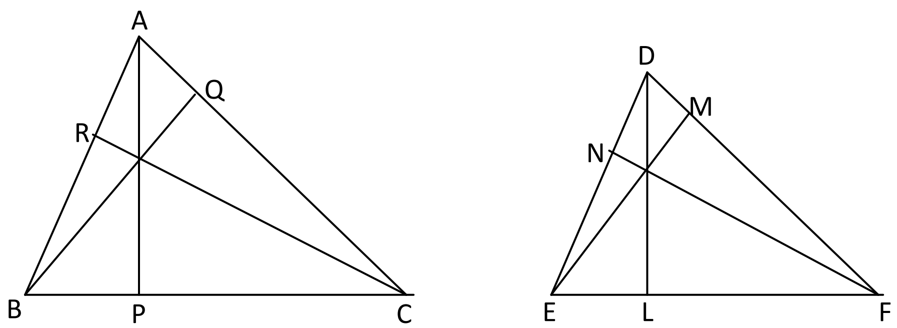
Then, $\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{DE^2}{AB^2} = \frac{EF^2}{BC^2} = \frac{DF^2}{AC^2}$

$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \left(\frac{DE}{AB}\right)^2 = \left(\frac{1}{2}\right)^2$

$\therefore \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$

3. Prove that the areas of two similar triangles are in the ratio of the square of the corresponding altitudes.

Solution:



Given: AP, BQ, CR and DL, EM, FN are the altitudes of two similar triangles ABC and DEF.

To prove: $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AP^2}{DL^2} = \frac{BQ^2}{EM^2} = \frac{CR^2}{FN^2}$

Proof: In ΔABP and ΔDEL, we have

$\angle B = \angle E$ [$\because \Delta ABC \sim \Delta DEF$]

and $\angle APB = \angle DLE$ $(= 90^0)$

$\therefore \Delta ABP \sim \Delta DEL$ [by AA similarity]

$\Rightarrow \frac{AB}{DE} = \frac{AP}{DL}$

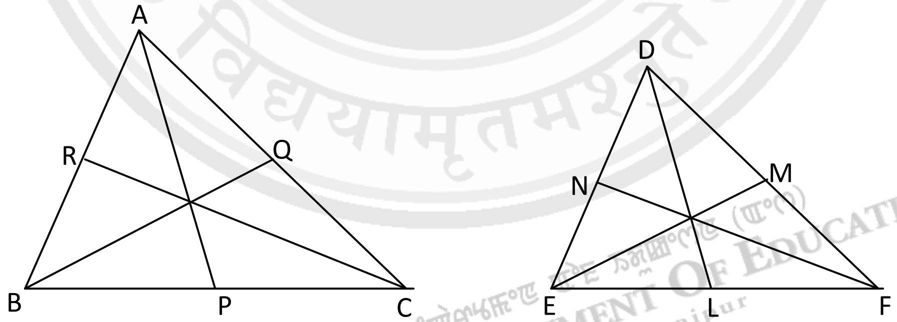
Similarly, $\frac{BC}{EF} = \frac{BQ}{EM}$ and $\frac{AC}{DF} = \frac{CR}{FN}$

We know, $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$ [$\because \Delta ABC \sim \Delta DEF$]

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AP^2}{DL^2} = \frac{BQ^2}{EM^2} = \frac{CR^2}{FN^2}$

4. Prove that the areas of two similar triangles are in the ratio of the square of the corresponding medians.

Solution:



Given: AP, BQ, CR and DL, EM, FN are the medians of two similar triangles ABC and DEF.

To prove: $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AP^2}{DL^2} = \frac{BQ^2}{EM^2} = \frac{CR^2}{FN^2}$

Proof: In ΔABP and ΔDEL, we have

$\frac{AB}{DE} = \frac{BC}{EF}$ [$\because \Delta ABC \sim \Delta DEF$]

$\Rightarrow \frac{AB}{DE} = \frac{\frac{1}{2}.BC}{\frac{1}{2}.EF}$

$\Rightarrow \frac{AB}{DE} = \frac{BP}{EL}$

and $\angle B = \angle E$ [$\because \Delta ABC \sim \Delta DEF$]

$\therefore \Delta ABP \sim \Delta DEL$ [by SAS similarity]

$\Rightarrow \frac{AB}{DE} = \frac{AP}{DL}$

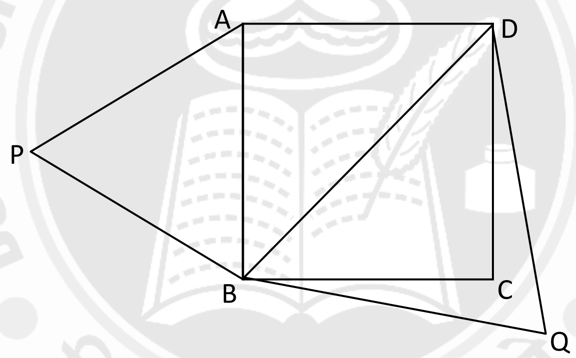
Similarly, $\frac{BC}{EF} = \frac{BQ}{EM}$ and $\frac{AC}{DF} = \frac{CR}{FN}$

We know, $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$ [$\because \Delta ABC \sim \Delta DEF$]

$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AP^2}{DL^2} = \frac{BQ^2}{EM^2} = \frac{CR^2}{FN^2}$

5. Prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonal.

Solution:



Given: ABCD is a square. ABP and BDQ are equilateral triangles.

To prove: $ar(\Delta ABP) = \frac{1}{2} \times ar(\Delta BDQ)$

Proof: In the right ΔABD, we have

$AB^2 + AD^2 = BD^2$ [by Pythagoras Theorem]

$\Rightarrow AB^2 + AB^2 = BD^2$

$\Rightarrow 2.AB^2 = BD^2$

We know equilateral triangles are similar.

So, $\Delta ABP \sim \Delta BDQ$

$\Rightarrow \frac{ar(\Delta ABP)}{ar(\Delta BDQ)} = \frac{AB^2}{BD^2}$

$\Rightarrow \frac{ar(\Delta ABP)}{ar(\Delta BDQ)} = \frac{AB^2}{2.AB^2}$

$\Rightarrow \frac{ar(\Delta ABP)}{ar(\Delta BDQ)} = \frac{1}{2}$

$\therefore ar(\Delta ABP) = \frac{1}{2} \times ar(\Delta BDQ)$

- **Theorem 7.9 (Pythagoras Theorem)**

In a right triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

- **Theorem 7.10 (Converse of Pythagoras Theorem)**

In a triangle, if the square of one side is equal to the sum of the squares of the remaining two, the angle opposite to the first side is a right angle.

SOLUTIONS

EXERCISE 7.6

1. Sides of triangles are given below. Determine which of them are right triangles.

(i) 3cm, 4cm, 5cm (ii) 5cm, 12cm, 13cm (iii) 4cm, 5cm, 7cm

(iv) 8cm, 11cm, 15cm (v) 9cm, 40cm, 41cm

Solution: (i) Let AB = 3cm, BC = 4cm and CA = 5cm

We have, $AB^2 = 3^2 = 9$; $BC^2 = 4^2 = 16$ and $CA^2 = 5^2 = 25$

Here, $AB^2 + BC^2 = CA^2$

Thus, ABC is a right triangle.

(ii) We have,

$$5^2 + 12^2 = 25 + 144 = 169 = 13^2$$

Hence, the triangle is a right triangle.

(iii) We have, $4^2 + 5^2 = 16 + 25 = 41 \neq 7^2$

Hence, the given triangle is not a right triangle.

(iv) We have,

$$8^2 + 11^2 = 64 + 121 = 185 \neq 15^2$$

Hence, the triangle is not a right triangle.

(v) We have,

$$9^2 + 40^2 = 81 + 1600 = 1681 = 41^2$$

Hence, the triangle is a right triangle.

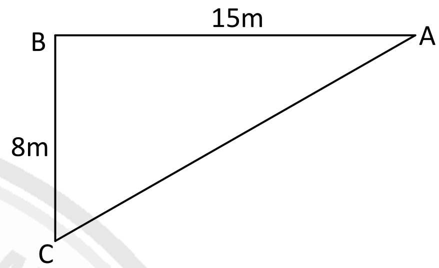
2. A man goes 15m due west and then 8m due north. Find his distance from the starting point.

Solution:

Let A and C respectively be the starting point and the final point. B be the point to the west of A.

Then, AB = 15m, BC = 8m and $\angle B = 90^0$.

In the right ΔABC, we have



$AC^2 = AB^2 + BC^2$ [by Pythagoras Theorem]

$\Rightarrow AC^2 = 15^2 + 8^2$

$\Rightarrow AC^2 = 225 + 64$

$\Rightarrow AC^2 = 289$

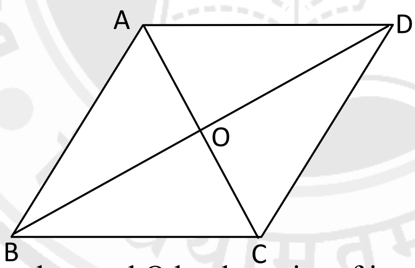
$\Rightarrow AC^2 = 17^2$

$\therefore AC = 17$

Thus, the distance of the man from the starting point is 17m.

3. The length of a side of a rhombus is 5cm and the length of one of its diagonals is 6cm. Find the length of the other diagonal.

Solution:



Let ABCD be the rhombus and O be the point of intersection of the diagonals.

Then, AB = BC = CD = DA = 5cm and AC = 6cm.

We know the diagonals of a rhombus bisect each other at right angles.

So, $OA = OC = \frac{1}{2} \times 6cm = 3cm$, $OB = OD = \frac{1}{2} \times BD$ i.e. BD = 2.OB = 2.OD

Now, in the right ΔAOB, we have

$OB^2 + OA^2 = AB^2$ [by Pythagoras Theorem]

$\Rightarrow OB^2 + 3^2 = 5^2$

$\Rightarrow OB^2 + 9 = 25$

$\Rightarrow OB^2 = 16$

$\Rightarrow OB^2 = 4^2$

$\Rightarrow OB = 4$

$\therefore BD = 2\times 4 = 8$

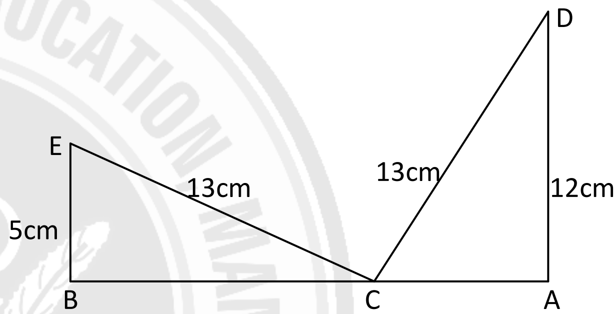
Thus, the length of the other diagonal is 8cm.

4. A ladder 13m long reaches a window which is 12m above the ground on one side of a street keeping its foot at the same point, the ladder is turned to other side of the street and it just reaches a window 5m high. Find the width of the street.

Solution: Let AB be the width of the street. Let CD and CE respectively be the first and the second positions of the ladder.

Then, CD = CE = 13m, AD = 12m, BE = 5m and ∠A = ∠B = 90^0

In the right ΔACD, we have



$$AC^2 + AD^2 = CD^2 \text{ [By Pythagoras Theorem]}$$
$$\Rightarrow AC^2 + 12^2 = 13^2$$
$$\Rightarrow AC^2 + 144 = 169$$
$$\Rightarrow AC^2 = 25$$
$$\Rightarrow AC^2 = 5^2$$
$$\Rightarrow AC = 5$$

In the right ΔBCE, we have

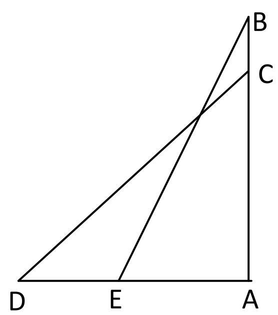
$$BC^2 + BE^2 = CE^2 \text{ [By Pythagoras Theorem]}$$
$$\Rightarrow BC^2 + 5^2 = 13^2$$
$$\Rightarrow BC^2 + 25 = 169$$
$$\Rightarrow BC^2 = 144$$
$$\Rightarrow BC^2 = 12^2$$
$$\Rightarrow BC = 12$$

Now, AC + BC = 5 + 12 = 17

Thus, the width of the street is 17m.

5. A ladder reaches 1m below the top of a vertical wall when its foot is at a distance of 6m from the wall. When the foot is shifted 2m nearer the wall, the ladder just reaches the top of the wall; find the height of the wall.

Solution:



Let AB be the wall; CD and BE respectively be the initial and the final positions of the ladder.

Then, AD = 6m, DE = 2m, AE = 6m – 2m = 4m, BC = 1m, BE = CD and $\angle A = 90^0$

In the right ΔACD, we have

$$CD^2 = AD^2 + AC^2 \text{ [By Pythagoras Theorem]}$$

$$\Rightarrow CD^2 = 6^2 + (AB - BC)^2$$

$$\Rightarrow CD^2 = 36 + (AB - 1)^2$$

$$\Rightarrow CD^2 = 36 + AB^2 - 2.AB + 1^2$$

$$\Rightarrow CD^2 = AB^2 - 2.AB + 37 \quad \text{------------- (1)}$$

In the right ΔABE, we have

$$BE^2 = AE^2 + AB^2 \text{ [By Pythagoras Theorem]}$$

$$\Rightarrow BE^2 = 4^2 + AB^2$$

$$\Rightarrow BE^2 = 16 + AB^2$$

$$\Rightarrow CD^2 = AB^2 + 16 \quad \text{------------- (2) } [\because BE = CD]$$

From (1) and (2), we have

$$AB^2 + 16 = AB^2 - 2.AB + 37$$

$$\Rightarrow 2.AB = 37 - 16$$

$$\Rightarrow 2.AB = 21$$

$$\Rightarrow AB = \frac{21}{2}$$

$$\Rightarrow AB = 10.5$$

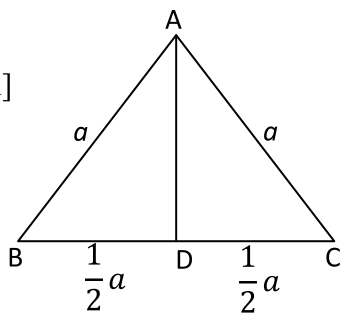
Thus, the height of the wall is 10.5m.

6. Find the length of the altitude and area of an equilateral triangle having ‘a’ as the length of a side.

Solution: ABC is an equilateral triangle, in which length of each side is ‘a’ and AD is an altitude.

Then, AB = BC = CA = a, BD = CD = $\frac{1}{2}a$ and AD⊥BC.

In the right ΔABD, we have



$$AD^2 + BD^2 = AB^2 \quad \text{[By Pythagoras Theorem]}$$

$$\Rightarrow AD^2 + \left(\frac{1}{2}a\right)^2 = a^2$$

$$\Rightarrow AD^2 + \frac{a^2}{4} = a^2$$

$$\Rightarrow AD^2 = a^2 - \frac{a^2}{4}$$

$$\Rightarrow AD^2 = \frac{4a^2 - a^2}{4}$$

$$\Rightarrow AD^2 = \frac{3a^2}{4}$$

$$\Rightarrow AD = \sqrt{\frac{3a^2}{4}}$$

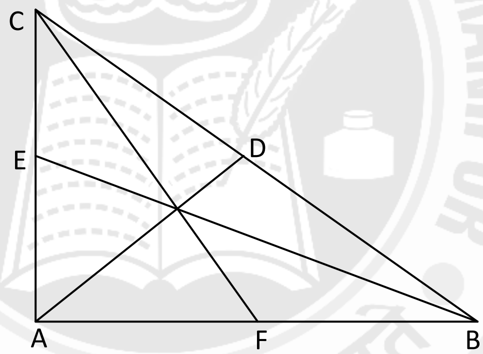
$$\Rightarrow AD = \frac{\sqrt{3}}{2}a$$

Thus, the length of the altitude is $\frac{\sqrt{3}}{2}a$.

Also, the area of the equilateral $\Delta ABC = \frac{1}{2} \times BC \times AD = \frac{1}{2}a.\frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{4}a^2$

7. If D, E, F are the mid-points of the sides BC, CA, AB of a right ΔABC (rt. $\angle$ed at A) respectively, prove that $3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$.

Solution:



Given: D, E, F are the mid-points of the sides BC, CA, AB respectively of a right ΔABC right angled at A.

To prove: $3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$

Proof: As $\angle BAC = 90^0$, the circle drawn with centre D and diameter BC will pass through A, B and C.

$$\therefore AD = BD = CD = \frac{1}{2}BC \quad (= \text{radius})$$

$$\Rightarrow AD^2 = \left(\frac{1}{2}BC\right)^2$$

$$\Rightarrow AD^2 = \frac{BC^2}{4}$$

$$\Rightarrow 4.AD^2 = BC^2 \quad \text{------------- (1)}$$

In the right ΔABE, we have

$$BE^2 = AE^2 + AB^2 \text{ [By Pythagoras Theorem]}$$

$$\Rightarrow BE^2 = \left(\frac{1}{2}AC\right)^2 + AB^2$$

$$\Rightarrow BE^2 = \frac{AC^2}{4} + AB^2$$

$$\Rightarrow 4BE^2 = AC^2 + 4AB^2 \quad \text{------------- (2)}$$

In the right ΔACF, we have

$$CF^2 = AF^2 + AC^2 \text{ [By Pythagoras Theorem]}$$

$$\Rightarrow CF^2 = \left(\frac{1}{2}AB\right)^2 + AC^2$$

$$\Rightarrow CF^2 = \frac{AB^2}{4} + AC^2$$

$$\Rightarrow 4CF^2 = AB^2 + 4AC^2 \quad \text{------------- (3)}$$

Adding (1), (2) and (3), we get

$$4AD^2 + 4BE^2 + 4CF^2 = BC^2 + AC^2 + 4AB^2 + AB^2 + 4AC^2$$

$$\Rightarrow 4(AD^2 + BE^2 + CF^2) = BC^2 + AC^2 + 4AB^2 + AB^2 + 4AC^2$$

$$= BC^2 + 5AB^2 + 5AC^2$$

$$= BC^2 + 2(AB^2 + AC^2) + 3(AB^2 + AC^2)$$

$$= BC^2 + 2.BC^2 + 3(AB^2 + AC^2)$$

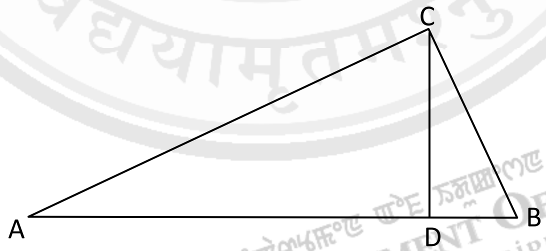
$$= 3BC^2 + 3(AB^2 + AC^2)$$

$$= 3(AB^2 + BC^2 + AC^2)$$

8. In a right triangle ABC right angled at C, if p is the length of the perpendicular segment drawn from C upon AB, then prove that

(i) $ab = pc$ (ii) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$, where a = BC, b = CA and c = AB.

Solution:



Given: ABC is a right triangle right angled at C and CD⊥AB such that BC = a, CA = b, AB = c and CD = p.

To prove: (i) $ab = pc$ (ii) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$

Proof: (i) In ΔABC and ΔACD, we have

$\angle A = \angle A$ (common angle)

and $\angle ACB = \angle ADC$ $(= 90^0)$

$\therefore \Delta ABC \sim \Delta ACD$ [by AA similarity]

$$\Rightarrow \frac{AB}{CA} = \frac{BC}{CD}$$

$$\Rightarrow CA \times BC = AB \times CD$$

$$\Rightarrow b \times a = c \times p$$

$$\Rightarrow ab = pc$$

(ii) We have, $ab = pc$

$$\Rightarrow c = \frac{ab}{p}$$

In ΔABC, we have

$$CA^2 + BC^2 = AB^2 \text{ [by Pythagoras Theorem]}$$

$$\Rightarrow b^2 + a^2 = c^2$$

$$\Rightarrow b^2 + a^2 = \left(\frac{ab}{p}\right)^2$$

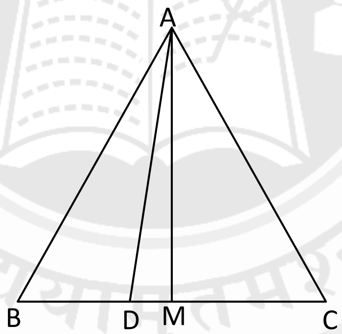
$$\Rightarrow b^2 + a^2 = \frac{a^2b^2}{p^2}$$

$$\Rightarrow \frac{b^2}{a^2b^2} + \frac{a^2}{a^2b^2} = \frac{1}{p^2}$$

$$\therefore \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2}$$

9. In an equilateral triangle ABC the side BC is trisected at D. Prove that $9AD^2 = 7AB^2$.

Solution:



Given: ABC is an equilateral triangle and BC is trisected at D.

To prove: $9AD^2 = 7AB^2$

Construction: Altitude AM is drawn i.e. AM$\perp$BC is drawn.

Proof: We have AB = BC = CA [being the sides of an equilateral triangle]

$$BD = \frac{1}{3}BC = \frac{1}{3}AB \text{ and } BM = \frac{1}{2}BC = \frac{1}{2}AB$$

$$\therefore DM = BM - BD = \frac{1}{2}AB - \frac{1}{3}AB = \frac{3.AB - 2.AB}{6} = \frac{AB}{6}$$

In ΔABM, we have

$$AM^2 + BM^2 = AB^2 \text{ [by Pythagoras Theorem]}$$

$$\Rightarrow AM^2 = AB^2 - BM^2$$

$$\Rightarrow AM^2 = AB^2 - \left(\frac{1}{2}\text{ AB}\right)^2$$

$$\Rightarrow AM^2 = AB^2 - \frac{AB^2}{4}$$

$$\Rightarrow AM^2 = \frac{4.AB^2 - AB^2}{4}$$

$$\Rightarrow AM^2 = \frac{3.AB^2}{4} \text{ ------------- (1)}$$

In ΔADM, we have

$$AM^2 + DM^2 = AD^2 \text{ [by Pythagoras Theorem]}$$

$$\Rightarrow AM^2 = AD^2 - DM^2$$

$$\Rightarrow AM^2 = AD^2 - \left(\frac{AB}{6}\right)^2$$

$$\Rightarrow AM^2 = AD^2 - \frac{AB^2}{36} \text{ ------------- (2)}$$

From (1) and (2), we have

$$\frac{3.AB^2}{4} = AD^2 - \frac{AB^2}{36}$$

$$\Rightarrow AD^2 = \frac{3.AB^2}{4} + \frac{AB^2}{36}$$

$$\Rightarrow AD^2 = \frac{27.AB^2 + AB^2}{36}$$

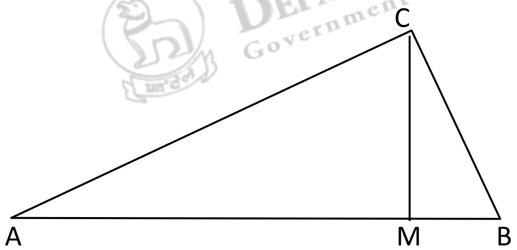
$$\Rightarrow AD^2 = \frac{28.AB^2}{36}$$

$$\Rightarrow AD^2 = \frac{7.AB^2}{9}$$

$$\therefore 9AD^2 = 7.AB^2$$

10. If A be the area of a right triangle and *b* one of the sides containing the right angle, prove that the length of the altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^4+4A^2}}$.

Solution:



Given: ABC is a right triangle right angled at C such that ar(ΔABC) = A and AC = b. CM is the altitude on the hypotenuse AB.

To prove: CM = $\frac{2Ab}{\sqrt{b^4+4A^2}}$

Proof: We have, ar(ΔABC) = A

$$\Rightarrow \frac{1}{2} \times AC \times BC = A$$

$$\Rightarrow \frac{1}{2} \times b \times BC = A$$

$$\Rightarrow BC = \frac{2A}{b}$$

In ΔABC, we have

$$AB^2 = AC^2 + BC^2 \text{ [by Pythagoras Theorem]}$$

$$\Rightarrow AB^2 = b^2 + \left(\frac{2A}{b}\right)^2$$

$$\Rightarrow AB^2 = b^2 + \frac{4A^2}{b^2}$$

$$\Rightarrow AB^2 = \frac{b^4+4A^2}{b^2}$$

$$\Rightarrow AB = \sqrt{\frac{b^4+4A^2}{b^2}}$$

$$\Rightarrow AB = \frac{\sqrt{4A^2+b^4}}{b}$$

Again, ar(ΔABC) = A

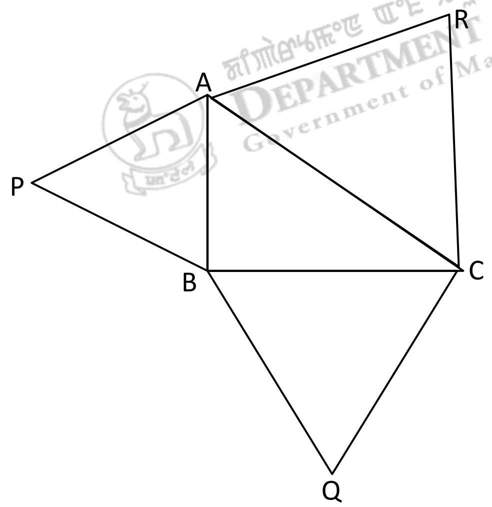
$$\Rightarrow \frac{1}{2} \times AB \times CM = A$$

$$\Rightarrow \frac{1}{2} \times \frac{\sqrt{4A^2+b^4}}{b} \times CM = A$$

$$\Rightarrow CM = \frac{2Ab}{\sqrt{4A^2+b^4}}$$

11. Prove that the equilateral triangles described on the two sides of a right angled triangle are together equal to the equilateral triangle on the hypotenuse in terms of their area.

Solution:



Given: ABP, BCQ and ACR are equilateral triangles described on the sides AB, BC and AC respectively of a right triangle ABC right angled at B.

To prove: ar(ΔABP) + ar(ΔBCQ) = ar(ΔACR)

Proof: In ΔABC, we have

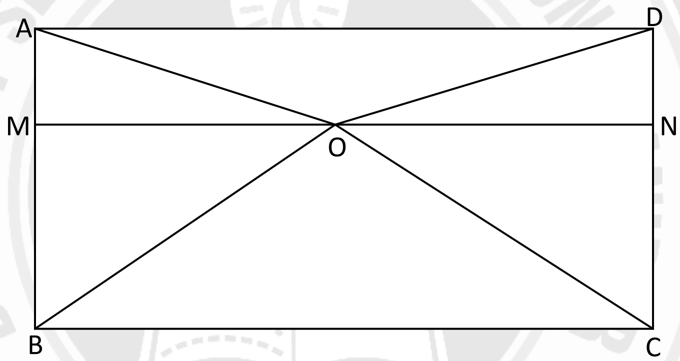
$$AB^2 + BC^2 = AC^2 \quad \text{[by Pythagoras Theorem]}$$

$$\Rightarrow \frac{\sqrt{3}}{4}AB^2 + \frac{\sqrt{3}}{4}BC^2 = \frac{\sqrt{3}}{4}AC^2 \quad \text{[Multiplying both sides by } \tfrac{\sqrt{3}}{4}\text{]}$$

∴ ar(ΔABP) + ar(ΔBCQ) = ar(ΔACR) [∵area of an equilateral $\Delta = \frac{\sqrt{3}}{4}a^2$]

12. If O is any point in the interior of a rectangle ABCD, prove that $OA^2 + OC^2 = OB^2 + OD^2$.

Solution:



Given: O is a point in the interior of a rectangle ABCD.

To prove: $OA^2 + OC^2 = OB^2 + OD^2$

Construction: MN∥BC is drawn through O.

Proof: We know, AMND and MBCN are also rectangles.

∴ AM = DN and BM = CN [being opposite sides of a parallelogram]

We have, $OA^2 = AM^2 + OM^2$

$OB^2 = BM^2 + OM^2$

$OC^2 = CN^2 + ON^2$

And $OD^2 = DN^2 + ON^2$

Now, $OA^2 + OC^2 = (AM^2 + OM^2) + (CN^2 + ON^2)$

$= (DN^2 + OM^2) + (BM^2 + ON^2)$

$= (BM^2 + OM^2) + (DN^2 + ON^2)$

$= OB^2 + OD^2$

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