CLASS – X
MATHEMATICS
CHAPTER – 6
ARITHMETIC PROGRESSION (AP)

- **Sequence**

  A succession of numbers formed according to a specific rule is called a sequence.

- Arithmetic Progression

  A sequence a_1, a_2, a_3, a_4, ..., a_n, ... is called an Arithmetic Progression(AP) if $a_{n+1} - a_n$=constant for all $n \epsilon N$.

  Or

  An arithmetic progression is a sequence in which each term other than the first is obtained by adding a fixed number to the preceding term.

- **Common Difference**

  In an AP, $\boldsymbol{a_1, a_2, a_3, \dots\dots., a_n} \dots\dots$ the value of $\boldsymbol{a_{n+1} - a_n}$ is called common difference of the AP.

- **The n^th term (or the general term) of an AP**

  Let a be the first term and d be the common difference of an AP.

  Then the AP is $a, a + d, a + 2d, a + 3d, \dots\dots\dots\dots$

  Here, $a_1 = a = a + (1 - 1)d$

  $a_2 = a + d = a + (2 - 1)d$

  $a_3 = a + 2d = a + (3 - 1)d$

  $a_4 = a + 3d = a + (4 - 1)d$

  ………………………………

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  Looking the above pattern, we can write

  $a_n = a + (n - 1)d$

  Thus, for an AP whose first term is a and the common difference is d,

  the n^{th} term (or the general term) $a_n = a + (n - 1)d$

SOLUTIONS

EXERCISE 6.1

1. Which of the following sequences are in AP? In case of AP, find the first term a and the common difference d and write the next three terms.

(i) $1^2, 2^2, 3^2, 4^2, 5^2$,...............

Solution: We have, $a_1 = 1^2, a_2 = 2^2, a_3 = 3^2, a_4 = 4^2, a_5 = 5^2$

Now, $a_2 - a_1 = 2^2 - 1^2 = 3$

$a_3 - a_2 = 3^2 - 2^2 = 5$

As $a_2 - a_1 \neq a_3 - a_2$, the given sequence is not an AP.

(ii) 8, 5, 2, −1,−4,.............

Solution: We have, $a_1 = 8, a_2 = 5, a_3 = 2, a_4 = -1, a_5 = -4$

Now, $a_2 - a_1 = 5 - 8 = -3$

$a_3 - a_2 = 2 - 5 = -3$

$a_4 - a_3 = -1 - 2 = -3$

$a_5 - a_4 = -4 - (-1) = -3$

and so on.

Here $a_{n+1} - a_n = -3$ for all $n \epsilon N$.

∴ the given sequence is an AP.

Then, $a = 8$ and $d = -3$.

And the next three terms are $-4 - 3 = -7, -7 - 3 = -10$ and $-10 - 3 = -13$

(iii) $2, \frac{5}{2}, 3, \frac{7}{2}, 4$, ...

Solution: We have, $a_1 = 2, a_2 = \frac{5}{2}, a_3 = 3, a_4 = \frac{7}{2}, a_5 = 4$

Now, $a_2 - a_1 = \frac{5}{2} - 2 = \frac{1}{2}$

$a_3 - a_2 = 3 - \frac{5}{2} = \frac{1}{2}$

$a_4 - a_3 = \frac{7}{2} - \frac{7}{2} = \frac{1}{2}$

$a_5 - a_4 = 4 - \frac{7}{2} = \frac{1}{2}$

and so on.

Here $a_{n+1} - a_n = \frac{1}{2}$ for all $n \epsilon N$.

∴ the given sequence is an AP.

Then, $a = 2$ and $d = \frac{1}{2}$

And the next three terms are $4 + \frac{1}{2} = \frac{9}{2}, \frac{9}{2} + \frac{1}{2} = 5$ and $5 + \frac{1}{2} = \frac{11}{2}$.

(iv) 0.6, 1.7, 2.8, 3.9, 5, ...

Solution: We have, $a_1 = 0.6, a_2 = 1.7, a_3 = 2.8, a_4 = 3.9,\ a_5 = 5$

Now, $a_2 - a_1 = 1.7 - 0.6 = 1.1$

$a_3 - a_2 = 2.8 - 1.7 = 1.1$

$a_4 - a_3 = 3.9 - 2.8 = 1.1$

$a_5 - a_4 = 5 - 3.9 = 1.1$

and so on.

Here $a_{n+1} - a_n = 1.1$ for all $n \epsilon N$.

∴ the given sequence is an AP.

Then, $a = 0.6$ and $d = 1.1$

And the next three terms are $5 + 1.1 = 6.1, 6.1 + 1.1 = 7.2$ and $7.2 + 1.1 = 8.3$

(v) . 3, 5, 7, 8, 10, ...

Solution: We have, $a_1 = 3, a_2 = 5, a_3 = 7, a_4 = 8,\ a_5 = 10$

Now, $a_2 - a_1 = 5 - 3 = 2$

$a_3 - a_2 = 7 - 5 = 2$

$a_4 - a_3 = 8 - 7 = 1$

As $a_2 - a_1 = a_3 - a_2 \neq a_4 - a_3$, the given sequence is not an AP.

(vi) −6, −1, 4, 9, 14, ...

Solution: We have, $a_1 = -6, a_2 = -1, a_3 = 4, a_4 = 9,\ a_5 = 14$

Now, $a_2 - a_1 = -1 - (-6) = 5$

$a_3 - a_2 = 4 - (-1) = 5$

$a_4 - a_3 = 9 - 4 = 5$

$a_5 - a_4 = 14 - 9 = 5$

and so on.

Here $a_{n+1} - a_n = 5$ for all $n \epsilon N$.

∴ the given sequence is an AP.

Then, $a = -6$ and $d = 5$

And the next three terms are $14 + 5 = 19, 19 + 5 = 24$ and $24 + 5 = 29$.

2. Write the n^th^ term and also the first four terms of the AP whose first term a and the common difference d are given as follows:

(i) a=2, d=2

Solution: We have $a = 2, d = 2$

$$\therefore a_n = a + (n-1)d$$
$$= 2 + (n-1) \times 2$$
$$= 2 + 2n - 2$$
$$= 2n$$

The first four terms are $a_1 = a = 2$

$$a_2 = a + d = 2 + 2 = 4$$
$$a_3 = a + 2d = 2 + 2 \times 2 = 6$$
$$a_4 = a + 3d = 2 + 3 \times 2 = 8$$

(ii) $\boldsymbol{a = 6, d = -4}$

Solution: We have $a = 6, d = -4$

$$\therefore a_n = a + (n-1)d$$
$$= 6 + (n-1) \times (-4)$$
$$= 6 - 4n + 4$$
$$= 10 - 4n$$

The first four terms are $a_1 = a = 6$

$$a_2 = a + d = 6 + (4) = 6 - 4 = 2$$
$$a_3 = a + 2d = 6 + 2 \times (-4) = 6 - 8 = -2$$
$$a_4 = a + 3d = 6 + 3 \times (-4) = 6 - 12 = -6$$

(iii) $\boldsymbol{a = -10, d = -3}$

Solution: We have $a = -10, d = -3$

$$\therefore a_n = a + (n-1)d$$
$$= -10 + (n-1) \times (-3)$$
$$= -10 - 3n + 3$$
$$= -7 - 3n$$

So, $a_1 = -7 - 3 \times 1 = -10$

$$a_2 = -7 - 3 \times 2 = -13$$
$$a_3 = -7 - 3 \times 3 = -16$$
$$a_4 = -7 - 3 \times 4 = -19$$

Thus, the first four terms are $-10,\ -13, -16$ and -19.

(iv) $\boldsymbol{a = \frac{3}{2}, d = -\frac{1}{2}}$

Solution: We have $a = \frac{3}{2}, d = -\frac{1}{2}$

$$\therefore a_n = a + (n-1)d$$
$$= \frac{3}{2} + (n-1) \times (-\frac{1}{2})$$
$$= \frac{3-n+1}{2}$$
$$= \frac{4-n}{2}$$

So, $a_1 = \frac{4-1}{2} = \frac{3}{2}$

$$a_2 = \frac{4-2}{2} = 1$$
$$a_3 = \frac{4-3}{2} = \frac{1}{2}$$
$$a_4 = \frac{4-4}{2} = 0$$

Thus, the first four terms are $\frac{3}{2}$, 1, $\frac{1}{2}$ and 0.

(v) $\boldsymbol{a = 4.5, d = 0.5}$

Solution: We have $a = 4.5, d = 0.5$

$$\therefore a_n = a + (n-1)d$$
$$= 4.5 + (n-1) \times (0.5)$$
$$= 4.5 + 0.5n - 0.5$$
$$= 4 + 0.5n$$

So, $a_1 = 4 + 0.5 \times 1 = 4 + 0.5 = 4.5$

$$a_2 = 4 + 0.5 \times 2 = 4 + 1 = 5$$
$$a_3 = 4 + 0.5 \times 3 = 4 + 1.5 = 5.5$$
$$a_4 = 4 + 0.5 \times 4 = 4 + 2 = 6$$

Thus, the first four terms are 4.5, 5, 5.5 and 6.

(vi) $\boldsymbol{a = -\frac{5}{3}, d = \frac{1}{3}}$

Solution: We have $a = -\frac{5}{3}, d = \frac{1}{3}$

$$\therefore a_n = a + (n-1)d$$
$$= -\frac{5}{3} + (n-1) \times \frac{1}{3}$$
$$= \frac{-5+n-1}{3}$$
$$= \frac{-6+n}{3}$$

So, $a_1 = \frac{-6+1}{3} = \frac{-5}{3}$

$a_2 = \frac{-6+2}{3} = \frac{-4}{3}$

$a_3 = \frac{-6+3}{3} = \frac{-3}{3} = -1$

$$a_4 = \frac{-6+4}{3} = \frac{-2}{3}$$

Thus, the first four terms are $\frac{-5}{3}, \frac{-4}{3}, -1$ and $\frac{-2}{3}$.

3. Find

(i) the 11th term of the AP: 2, 5, 8, 11, 14, ---------- .

Solution: We have, $a = 2$, $d = 5 - 2 = 3$ and $n = 11$

Then, $a_{11} = 2 + (11 - 1) \times 3$ $\quad [\because a_n = a + (n-1)d]$

$= 32$

$\therefore$ the 11th term of the AP is 32.

(ii) the 15th term of the AP: $\mathbf{4, 1, -2, -5, -8}$, --------- .

Solution: We have, $a = 4$, $d = 1 - 4 = -3$ and $n = 15$

Then, $a_{15} = 4 - 3 \times (15 - 1)$ $\quad [\because a_n = a + (n-1)d]$

$= 4 - 42$

$= -38$

$\therefore$ the 15th term is -38.

(iii) the 20th term of the AP: $\mathbf{1, \frac{3}{2}, 2, \frac{5}{2}, 3}$, -----------.

Solution: We have, $a = 1$, $d = \frac{3}{2} - 1 = \frac{1}{2}$ and $n = 20$

Then, $a_{20} = 1 + (20 - 1) \times \frac{1}{2}$ $\quad [\because a_n = a + (n-1)d]$

$= 1 + \frac{19}{2}$

$= \frac{21}{2}$

$\therefore$ the 20th term of the AP is $\frac{21}{2}$.

(iv) The 18th term of the AP: $\mathbf{\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, 9\sqrt{2}}$, -------------.

Solution: We have, $a = \sqrt{2}$, $d = 3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$ and $n = 18$

Then, $a_{18} = \sqrt{2} + (18 - 1) \times 2\sqrt{2}$ $[\because a_n = a + (n-1)d]$

$= \sqrt{2} + 34\sqrt{2}$

$= 35\sqrt{2}$

$\therefore$ the 18th term of the AP is $35\sqrt{2}$.

4. (i) Is 67 a term of the AP: 5, 8, 11, 14, 17, ---------------- ?

(ii) Is -204 a term of the AP: 10, 6, 2, -2, -6, ---------------- ?

Solution:

(i) We have, $a = 5, d = 8 - 5 = 3$

If possible, let us suppose 67 is the n[th] term of the AP.

Then, $a_n = 67$

$\Rightarrow a + (n-1)d = 67$

$\Rightarrow 5 + (n-1) \times 3 = 67$

$\Rightarrow 3(n-1) = 67 - 5$

$\Rightarrow 3n - 3 = 62$

$\Rightarrow 3n = 65$

$\Rightarrow n = \frac{65}{3}$, which is impossible as number of terms cannot be a fraction.

$\therefore$ 67 is not a term of given AP.

(ii) We have, $a = 10, d = 6 - 10 = -4$

If possible, let us suppose -204 is the n[th] term of the AP.

Then, $a_n = -204$

$\Rightarrow a + (n-1)d = -204$

$\Rightarrow 10 - 4(n-1) = -204$

$\Rightarrow -4(n-1) = -204 - 10$

$\Rightarrow -4n + 4 = -214$

$\Rightarrow -4n = -218$

$\Rightarrow 2n = 109$

$\Rightarrow n = \frac{109}{2}$, which is impossible as number of terms cannot be a fraction.

$\therefore$ -204 is not a term of given AP.

5. (i) Which term of the AP: 4, 9, 14, 19, 24, -------- is 124?

(ii) Which term of the AP: 84, 80, 76, 68, --------- is 0?

Solution:

(i) We have, $a = 4, d = 9 - 4 = 5$

Let $a_n = 124$

$\Rightarrow a + (n-1)d = 124$

$\Rightarrow 4 + (n-1)5 = 124$

$\Rightarrow 5(n-1) = 124 - 4$

$\Rightarrow n - 1 = \frac{120}{5}$

$\Rightarrow n = 24 + 1$

$\Rightarrow n = 25$

$\therefore$ 124 is the 25th term of the given AP.

(ii) We have, $a = 84$, $d = 80 - 84 = -4$

Let $a_n = 0$

$\Rightarrow a + (n-1)d = 0$

$\Rightarrow 84 - 4(n-1) = 0$

$\Rightarrow -4(n-1) = -84$

$\Rightarrow n - 1 = \frac{84}{4}$

$\Rightarrow n = 21 + 1$

$\Rightarrow n = 22$

$\therefore$ 0 is the 22nd term of the given AP.

6. Find the number of terms in each of the following APs.

(i) 7, 10, 13, ---------------, 43

Solution: We have, $a = 7$, $d = 10 - 7 = 3$

Let $a_n = 43$

$\Rightarrow a + (n-1)d = 43$

$\Rightarrow 7 + (n-1)3 = 43$

$\Rightarrow 3(n-1) = 36$

$\Rightarrow n - 1 = 12$

$\Rightarrow n = 13$

$\therefore$the required number of terms of the AP is 13.

(ii) 4, 1, -2, -----------------, -86

Solution: We have,

$a = 4$, $d = 1 - 4 = -3$

Let $a_n = -86$

$\Rightarrow a + (n-1)d = -86$

$\Rightarrow 4 + (n-1) \times (-3) = -86$

$\Rightarrow -3(n-1) = -90$

$\Rightarrow n - 1 = 30$

$\Rightarrow n = 31$

$\therefore$the required number of terms of the AP is 31.

7. The n^th^ term of a sequence is 2 – 5n. Is the sequence an AP? If so, find the first term and the common difference.

Solution: We have, $a_n = 2 - 5n$

Then, $a_{n+1} - a_n = \{2 - 5(n + 1)\} - (2 - 5n)$

$= 2 - 5n - 5 - 2 + 5n$

$= -5$, which is a constant.

Hence the given sequence is an AP whose first term is $2 - 5 \times 1$ i.e. -3 and the common difference is -5.

8. Show that the sequence whose n^th^ term is $2n^2 + 3$ is not an AP.

Solution: We have, $a_n = 2n^2 + 3$

Then, $a_{n+1} - a_n = \{2(n + 1)^2 + 3\} - (2n^2 + 3)$

$= 2(n^2 + 2n + 1) + 3 - 2n^2 - 3$

$= 2n^2 + 4n + 2 - 2n^2$

$= 4n + 2$, which is not a constant.

Hence the given sequence is not an AP.

9. Find the 15^th^ term from the last term (towards the first term) of the AP: 3, 7, 11, ..., 123.

Solution: Considering the terms of the given AP in the reversed order, we have

$a = 123$ and $d = 3 - 7 = -4$

Then $a_{15} = a + (15 - 1)d$

$= 123 + 14 \times (-4)$

$= 123 - 56$

$= 67$

$\therefore$ The required 15^th^ term from the last term (towards the first term) of the given AP is 67.

10. In an AP, the 5^th^ term is 7 and the 11^th^ term is 10. Find the first term and the common difference.

Solution: Let a and d respectively be the first term and the common difference of the AP.

We have, $a_5 = 7$

$\Rightarrow a + 4d = 7$ ------------- (1)

And $a_{11} = 10$

$\Rightarrow a + 10d = 10$ ------------- (2)

Subtracting (1) from (2), we have

$6d = 3$

$\therefore d = \frac{3}{6} = \frac{1}{2}$

Substituting $d = \frac{1}{2}$ in equation (1), we have

$a + 4 \times \frac{1}{2} = 7$

$\Rightarrow a + 2 = 7$

$\Rightarrow a = 5$

$\therefore$ The required first term and the common difference of the AP are 5 and $\frac{1}{2}$ respectively.

11. The 6th and the 17th terms of an AP are 21 and 54 respectively. Find the 40th term.

Solution: Let a and d respectively be the first term and the common difference of the AP.

We have, $a_6 = 21$

$\Rightarrow a + 5d = 21$ ------------- (1)

And $a_{17} = 54$

$\Rightarrow a + 16d = 54$ ------------- (2)

Subtracting (1) from (2), we have

$11d = 33$

$\therefore d = 3$

Substituting $d = 3$ in equation (1), we have

$a + 5 \times 3 = 21$

$\Rightarrow a + 15 = 21$

$\Rightarrow a = 6$

$\therefore$ the required 40^{th} term $= a_{40} = 6 + (40 - 1) \times 3 = 123$

12. The 15^{th} term of an AP exceeds the 20^{th} term by 8. Find the common difference.

Solution: We have, $a_{15} - a_{20} = 8$

$\Rightarrow (a + 14d) - (a + 19d) = 8$

$\Rightarrow a + 14d - a - 19d = 8$

$\Rightarrow -5d = 8$

$$\therefore d = -\frac{8}{5}$$

$\therefore$ the required common difference is $-\frac{8}{5}$.

13. How many two-digit numbers are divisible by 3?

Solution: The sequence of all the two-digit numbers divisible by 3 is 12, 15, 18, ... which is an AP.

Here, $a = 12$ and $d = 3$. The greatest two-digit number divisible by 3 is 99.

Let 99 be the n^{th} term of the AP.

Then, $a_n = 99$

$\Rightarrow a + (n-1)d = 99$

$\Rightarrow 12 + 3(n-1) = 99$

$\Rightarrow 3(n-1) = 99 - 12 = 87$

$\Rightarrow n - 1 = \frac{87}{3} = 29$

$\therefore n = 30$

$\therefore$ there are 30 two-digit numbers divisible by 3.

14. If the m^{th} term of an AP be $\frac{1}{n}$ and the n^{th} term be $\frac{1}{m}$, then show that the $(mn)^{th}$ term is 1.

Solution: Let a and d respectively be the first term and the common difference of the AP.

Then, $a_m = \frac{1}{n}$

$\Rightarrow a + (m-1)d = \frac{1}{n}$ ----------- (1)

and $a_n = \frac{1}{m}$

$\Rightarrow a + (n-1)d = \frac{1}{m}$ ----------- (2)

Subtracting (2) from (1), we have

$(m-n)d = \frac{1}{n} - \frac{1}{m}$

$\Rightarrow (m-n)d = \frac{m-n}{mn}$

$\therefore d = \frac{1}{mn}$

And, from (1), we have

$a + (m-1) \times \frac{1}{mn} = \frac{1}{n}$

$\Rightarrow a = \frac{1}{n} - \frac{m-1}{mn}$

$\Rightarrow a = \frac{m-m+1}{mn}$

$\therefore a = \frac{1}{mn}$

Now, (mn)th term= $a+(mn-1)d=\frac{1}{mn}+(mn-1)\times\frac{1}{mn}$

$$=\frac{1}{mn}(1+mn-1)$$

$$=\frac{1}{mn}\times mn$$

$$=1$$

15. If $a, b. c$ be the p^{th}, q^{th}, r^{th} terms of an AP, prove that $\boldsymbol{a(q-r)+b(r-p)+c(p-q)=0}$

Solution: Let x and y be the first term and the common difference of the AP.

Then $a=x+(p-1)y$

$b=x+(q-1)y$

$c=x+(r-1)y$

Now, $a(q-r)+b(r-p)+c(p-q)$

$=\{x+(p-1)y\}(q-r)+\{x+(q-1)y\}(r-p)+\{x+(r-1)y\}(p-q)$

$=\{(q-r)x+(q-r)(p-1)y\}+\{(r-p)x+(r-p)(q-1)y\}+\{(p-q)x+(p-q)(r-1)y\}$

$=(q-r+r-p+p-q)x+(pq-rp-q+r+qr-pq-r+p+rp-qr-p+q)y$

$=0$

Hence proved.

OR

Let x and y be the first term and the common difference of the AP.

Then $a=x+(p-1)y$ --------------- (1)

$b=x+(q-1)y$ --------------- (2)

$c=x+(r-1)y$ --------------- (3)

Subtracting equation (2) from equation (1), we have

$$a-b=(p-q)y$$

$$\Rightarrow p-q=\frac{a-b}{y}$$

Similarly, from (2) and (3), we have

$$q-r=\frac{b-c}{y}$$

And from (3) and (1), we have

$$r-p=\frac{c-a}{y}$$

Now $a(q-r)+b(r-p)+c(p-q)=\frac{a(b-c)}{y}+\frac{b(c-a)}{y}+\frac{c(a-b)}{y}$

$=\frac{a(b-c)+b(c-a)+c(a-b)}{y}$

$=\frac{ab-ca+bc-ab+ca-bc}{y}$

$=\frac{0}{y}$

$=0$

Hence proved.

16. A sum of Rs. 5000 is invested at 6% per annum simple interest. Calculate the interest at the end of each year and show that they form an AP. Also, find the interest at the end of the 25th year.

Solution: We have,

Interest at the end of the first year, $a_1=\text{Rs.}\frac{5000\times6\times1}{100}=\text{Rs.}300$

Interest at the end of the second year, $a_2=\text{Rs.}\frac{5000\times6\times2}{100}=\text{Rs.}600$

Interest at the end of the third year, $a_3=\text{Rs.}\frac{5000\times6\times3}{100}=\text{Rs.}900$

Interest at the end of the fourth year, $a_4=\text{Rs.}\frac{5000\times6\times4}{100}=\text{Rs.}1200$

and so on.

Now, $a_2-a_1=\text{Rs.}600-\text{Rs.}300=\text{Rs.}300$

$a_3-a_2=\text{Rs.}900-\text{Rs.}600=\text{Rs.}300$

$a_4-a_3=\text{Rs.}1200-\text{Rs.}900=\text{Rs.}300$ and so on.

Here, $a_2-a_1=a_3-a_2=a_4-a_3=\text{Rs.}300=$ a constant.

Thus the interests at the end of each year form an AP.

Interest at the end of the 25th year, $a_{25}=300+(25-1)\times300$

$=7500$ (in Rs.)

17. In an auditorium, the seats are so arranged that there are 8 seats in the first row, 11 seats in the second, 14 seats in the third etc. thereby increasing the number of seats by 3 every next row. If there are 50 seats in the last row, how many rows of seats are there in the auditorium?

Solution: We have, the number of seats in the first row, second row, third row, fourth row etc. are 8, 11, 14, 17, ... , 50 etc. which form an AP.

Here, $a=8, d=3$

and $a_n = 50$

$\Rightarrow a + (n-1)d = 50$

$\Rightarrow 8 + 3(n-1) = 50$

$\Rightarrow 3(n-1) = 42$

$\Rightarrow n - 1 = 14$

$\Rightarrow n = 15$

$\therefore$ there are 15 rows of seats in the auditorium.

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- **Sum of the first n terms of an AP**

Let a and d be the first term and the common difference of an AP.

Then the AP is

$$a, a+d, a+2d, \dots \dots . a+(n-2)d, a+(n-a)d, \dots$$

Let S_n denotes the sum of the first n terms of the AP.

Then $S_n = a + (a+d) + (a+2d) + \dots + \{a+(n-2)d\} + \{a+(n-1)d\}$

And $S_n = \{a+(n-1)d\} + \{a+(n-2)d\} + \dots . + (a+2d) + (a+d) + a$

Adding the above relations, we have

$2.S_n = \{2a+(n-1)d\} + \{2a+(n-1)d\} + \dots . +$ to n terms

$\Rightarrow 2.S_n = n\{2a+(n-1)d\}$

$\Rightarrow S_n = \frac{n}{2}\{2a+(n-1)d\}$ -------- (i)

$\Rightarrow S_n = \frac{n}{2}\{a + a + (n-1)d\}$

$\therefore S_n = \frac{n}{2}(a + a_n)$

$= \frac{n}{2}(a + l)$ ----------- (ii)

Thus, for an AP whose first term is a and the common difference is d,

the sum of the first n terms, $S_n = \frac{n}{2}(a + a_n)$

$= \frac{n}{2}(a + l)$

SOLUTIONS

EXERCISE 6.2

1. Find the sum of the following APs:

(i) 40, 36, 32, 28, --------------, to 15 terms

Solution: Here, $a = 40$ and $d = 36 - 40 = -4$

And, we have $S_n = \frac{n}{2}[2a + (n-1)d]$

Putting $n = 15$, we have

the required sum $= S_{15} = \frac{15}{2}[2 \times 40 + (15 - 1) \times (-4)]$

$= \frac{15}{2}(80 - 56)$

$= \frac{15}{2} \times 24$

$= 15 \times 12$

$= 180$

(ii) 3, 8, 13, 18, --------------, to 12 terms

Solution: Here, $a = 3$ and $d = 8 - 3 = 5$

And, we have $S_n = \frac{n}{2}[2a + (n-1)d]$

Putting $n = 12$, we have

the required sum $= S_{12} = \frac{12}{2}[2 \times 3 + (12 - 1) \times 5]$

$= 6[6 \times 55] = 6 \times 61$

$= 366$

(iii) $-2, -\frac{3}{2}, -1, -\frac{1}{2}$, --------------, to 25 terms

Solution: Here, $a = -2$ and $d = -\frac{3}{2} - (-2) = \frac{1}{2}$

And, we have $S_n = \frac{n}{2}[2a + (n-1)d]$

Putting $n = 25$, we have

the required sum $= S_{25} = \frac{25}{2}[2 \times (-2) + (25 - 1) \times \frac{1}{2}]$

$= \frac{25}{2} \times [-4 + 24 \times \frac{1}{2}]$

$= \frac{25}{2} \times [-4 + 12]$

$= \frac{25}{2} \times 8 = 25 \times 4$

$= 100$

(iv) 4.5, 4.3, 4.1, 3.9, --------------, to 14 terms

Solution: Here, $a = 4.5$ and $d = 4.3 - 4.5 = -0.2$

And, we have $S_n = \frac{n}{2}[2a + (n-1)d]$

Putting $n = 14$, we have

the required sum $= S_{14} = \frac{14}{2}[2 \times 4.5 + (14 - 1) \times (-0.2)]$

$= 7 \times [9 + 13 \times (-0.2)]$

$= 7 \times [9 - 2.6]$

$= 7 \times 6.4$

$= 44.8$

2. Find the sum of all terms of the following finite APs:

(i) 63, 61, 59, --------------, 35

Solution: Here, $a = 63$ and $d = 61 - 63 = -2$

Let there be n terms in the AP.

Then, $a_n = 35$

$\Rightarrow a + (n-1)d = 35$

$\Rightarrow 63 + (n-1) \times (-2) = 35$

$\Rightarrow -2(n-1) = 35 - 63$

$\Rightarrow -2(n-1) = 35 - 63$

$\Rightarrow -2(n-1) = -28$

$\Rightarrow n - 1 = 14$

$\therefore n = 15$

$\therefore$ the required sum $= S_{15} = \frac{15}{2}[63 + 35]$ $\quad [\because S_n = \frac{n}{2}[a + l]$

$= \frac{15}{2} \times 98$

$= 15 \times 49$

$= 735$

(ii) 2, 5, 8, --------------, 182

Solution: Here, $a = 2$ and $d = 5 - 2 = 3$

Let there be n terms in the AP.

Then, $a_n = 182$

$\Rightarrow a + (n-1)d = 182$

$\Rightarrow 2 + (n-1) \times 3 = 182$

$\Rightarrow 3(n-1) = 182 - 2$

$\Rightarrow 3(n-1) = 180$

$\Rightarrow n-1 = 60$

$\therefore n = 61$

$\therefore$ the required sum $= S_{61} = \frac{61}{2}[2+182]$ $\quad [\because S_n = \frac{n}{2}[a+l]$

$= \frac{61}{2} \times 184$

$= 61 \times 92$

$= 5612$

(iii) $\mathbf{-10, -2}$**, 6, 14, --------------, 78**

Solution: Here, $a = -10$ and $d = -2-(-10) = -2+10 = 8$

Let there be n terms in the AP.

Then, $a_n = 78$

$\Rightarrow a+(n-1)d = 78$

$\Rightarrow -10+(n-1)\times 8 = 78$

$\Rightarrow 8(n-1) = 78+10$

$\Rightarrow 8(n-1) = 88$

$\Rightarrow n-1 = 11$

$\therefore n = 12$

$\therefore$ the required sum $= S_{12} = \frac{12}{2}[-10+78]$ $\quad [\because S_n = \frac{n}{2}(a+l)]$

$= 6 \times 68$

$= 408$

(iv) $\mathbf{2, \frac{5}{3}, \frac{4}{3}, 1}$**, --------------,** $\mathbf{-6}$

Solution: Here, $a = 2$ and $d = \frac{5}{3} - 2 = \frac{-1}{3}$

Let there be n terms in the AP.

Then, $a_n = -6$

$\Rightarrow a+(n-1)d = -6$

$\Rightarrow 2+(n-1)\times\left(-\frac{1}{3}\right) = -6$

$\Rightarrow -\frac{1}{3}(n-1) = -6-2$

$\Rightarrow -\frac{1}{3}(n-1) = -8$

$\Rightarrow n-1 = 24$

$\therefore n = 25$

$\therefore$ the required sum $= S_{25} = \frac{25}{2}[2-6]$ $\qquad [\because S_n = \frac{n}{2}[a+l]$

$= \frac{25}{2} \times (-4)$

$= 25 \times (-2)$

$= -50$

3. How many terms of the AP: 1, 6, 11, 16, ... must be taken so that their sum is 148.

Solution: Here, $a = 1$ and $d = 6 - 1 = 5$

Let n be the number of terms to be taken so that

$S_n = 148$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 148$

$\Rightarrow \frac{n}{2}[2 \times 1 + (n-1) \times 5] = 148$

$\Rightarrow n[2 + 5(n-1)] = 296$

$\Rightarrow n[2 + 5n - 5] = 296$

$\Rightarrow n[5n - 3] = 296$

$\Rightarrow 5n^2 - 3n - 296 = 0$

$\Rightarrow 5n^2 - (40 - 37)n - 296 = 0$

$\Rightarrow 5n^2 - 40n + 37n - 296 = 0$

$\Rightarrow 5n(n-8) + 37(n-8) = 0$

$\Rightarrow (n-8)(5n-37) = 0$

$\Rightarrow n - 8 = 0$ or $5n - 37 = 0$

$\Rightarrow n = 8$ or $n = \frac{37}{5}$ (neglected as no. of terms should be a natural number)

$\therefore$ the required number of terms so that the sum is 148 is 8.

4. Find the number of terms of the AP: 32, 28, 24, 20, ... of which the sum is 132. Explain the double answer.

Solution: Here, $a = 32$ and $d = 28 - 32 = -4$

Let n be the number of terms to be taken so that

$S_n = 132$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 132$

$\Rightarrow \frac{n}{2}[2 \times 32 + (n-1) \times (-4)] = 132$

$\Rightarrow \frac{n}{2} \times 2[32 - 2(n-1)] = 132$

$\Rightarrow n[32 - 2n + 2] = 132$

$\Rightarrow n[-2n + 34] = 132$

$\Rightarrow -2n^2 + 34n - 132 = 0$

$\Rightarrow n^2 - 17n + 66 = 0$

$\Rightarrow n^2 - (6 + 11)n + 66 = 0$

$\Rightarrow n^2 - 6n - 11n + 66 = 0$

$\Rightarrow n(n - 6) - 11(n - 6) = 0$

$\Rightarrow (n - 6)(n - 11) = 0$

$\Rightarrow n - 6 = 0$ or $n - 11 = 0$

$\Rightarrow n = 6$ or $n = 11$

$\therefore$ the required number of terms so that the sum is 132 is either 6 or 11.

The double answer is explained by the fact that the sum of the terms from 7^{th} to 11^{th} terms is 0 and thus the sum of the first 6 and the first 11 terms are the same, each being 132.

5. Find the sum of

(i) the first n natural numbers.

Solution: The sequence of the first n natural numbers is: 1, 2, 3, ...,n which is an AP.

Here, $a = 1$ and $l = n$

$\therefore$ the required sum $= S_n = \frac{n}{2}(a + l)$

$= \frac{n}{2}(1 + n)$

$= \frac{n(n+1)}{2}$

(ii) the first 200 natural numbers.

Solution: The sequence of the first 200 natural numbers is : 1, 2, 3, ... ,200 which is an AP.

Here, $a = 1$ and $l = n = 200$

$\therefore$ the required sum $= S_n = \frac{n}{2}[a + l]$

$= \frac{200}{2}[1 + 200]$

$= 100 \times 201$

$= 20100$

(iii) the first 100 odd natural numbers.

Solution: The sequence of the first 100 odd natural numbers is : 1, 3, 5, ... which is an AP.

Here, $a = 1, d = 3 - 1 = 2$ and $n = 100$

$\therefore$ the required sum $= S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{100}{2}[2 \times 1 + (100 - 1) \times 2]$

$= 50 \times (2 + 99 \times 2)$

$= 50 \times (2 + 198)$

$= 50 \times (2 + 198)$

$= 50 \times 200$

$= 10000$

(iv) the first 100 even natural numbers.

Solution: The sequence of the first 100 even natural numbers is : 2, 4, 6, ...
which is an AP.

Here, $a = 2$, $d = 4 - 2 = 2$ and $n = 100$

$\therefore$ the required sum $= S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{100}{2}[2 \times 2 + (100 - 1) \times 2]$

$= \frac{100}{2} \times 2(2 + 99)$

$= 100 \times 101$

$= 10100$

6. Find the sum of all odd numbers between 100 and 200.

Solution: The sequence of all odd numbers between 100 and 200 is :

101, 103, 105, ..., 199 which is an AP.

Here, $a = 101$, $d = 103 - 101 = 2$

And $l = a_n = 199$

$\Rightarrow a + (n-1)d = 199$

$\Rightarrow 101 + (n-1) \times 2 = 199$

$\Rightarrow 2(n-1) = 98$

$\Rightarrow n - 1 = 49$

$\Rightarrow n = 50$

$\therefore$ the required sum $= S_n = \frac{n}{2}[a + l]$

$= \frac{50}{2}[101 + 199]$

$= 25 \times 300$

$= 7500$

7. Find the sum of the first 25 multiples of 6.

Solution: The sequence of the first 25 multiples of 6 is:

6, 12, 18, ..., 150 which is an AP.

Here, $a = 6$, $l = 150$ and $n = 25$

$\therefore$ the required sum $= S_{25} = \frac{25}{2}[6 + 150]$ $\quad [\because S_n = \frac{n}{2}(a + l)]$

$= \frac{25}{2} \times 156 = 25 \times 78$

$= 1950$

8. Find the sum of all multiples of 7 between 100 and 300.

Solution: The sequence of all multiples of 7 between 100 and 300 is :

105, 112, 119, ..., 294 which is an AP.

Here, $a = 105$, $d = 112 - 105 = 7$

and $l = a_n = 294$

$\Rightarrow a + (n - 1)d = 294$

$\Rightarrow 105 + (n - 1) \times 7 = 294$

$\Rightarrow 7(n - 1) = 294 - 105$

$\Rightarrow 7(n - 1) = 189$

$\Rightarrow n - 1 = \frac{189}{7}$

$\Rightarrow n - 1 = 27$

$\Rightarrow n = 28$

$\therefore$ the required sum $= S_{28} = \frac{28}{2}[105 + 294]$ $\quad [\because S_n = \frac{n}{2}(a + l)]$

$= 14 \times 399$

$= 5586$

9. If the 12th term of an AP is -13 and the sum of the first four terms is 24, find the sum of the first 15 terms.

Solution: We have, $a_{12} = -13$

$\Rightarrow a + 11d = -13$

$\Rightarrow a = -13 - 11d$ .... (1)

and $S_4 = 24$

$\Rightarrow \frac{4}{2}[2a + (4 - 1)d] = 24$ $[\because S_n = \frac{n}{2}\{2a + (n - 1)d\}]$

$\Rightarrow 2[2a + 3d] = 24$

$\Rightarrow 2a + 3d = 12$

$\Rightarrow a = \frac{12 - 3d}{2}$ .... (2)

From (1) & (2), we have

$$-13 - 11d = \frac{12-3d}{2}$$

$$\Rightarrow 12 - 3d = -26 - 22d$$

$$\Rightarrow 22d - 3d = -26 - 12$$

$$\Rightarrow 19d = -38$$

$$\Rightarrow d = \frac{-38}{19}$$

$$\Rightarrow d = -2$$

Substituting $d = -2$ in (2), we have

$$a = \frac{12-3\times(-2)}{2} = \frac{12+6}{2} = \frac{18}{2} = 9$$

$\therefore$ the required sum $= S_{15}$

$$= \frac{15}{2}[2 \times 9 + (15-1) \times (-2)] \quad [\because S_n = \frac{n}{2}\{2a + (n-1)d\}]$$

$$= \frac{15}{2}[18 - 14 \times 2]$$

$$= \frac{15}{2} \times (18 - 28)$$

$$= \frac{15}{2} \times (-10)$$

$$= 15 \times (-5)$$

$$= -75$$

10. Find the sum of the first 22 terms of an AP whose 8th and 16th terms are respectively 37 and 85.

Solution: We have, $a_8 = 37$

$$\Rightarrow a + 7d = 37 \text{ .... (1)}$$

And $a_{16} = 85$

$$\Rightarrow a + 15d = 85 \text{ .... (2)}$$

Subtracting equation (1) from equation (2), we have

$$8d = 48$$

$$\Rightarrow d = 6$$

Substituting $d = 6$ in (1), we have

$$a + 7 \times 6 = 37$$

$$\Rightarrow a + 42 = 37$$

$$\Rightarrow a = 37 - 42$$

$$\Rightarrow a = -5$$

$\therefore$ the required sum $= S_{22}$

$$= \frac{22}{2}[2 \times (-5) + (22-1) \times 6] \quad [\because S_n = \frac{n}{2}\{2a + (n-1)d\}]$$

$$= 11[-10 + 21 \times 6]$$

$$= 11 \times (-10 + 126)$$

$$= 11 \times 116$$

$$= 1276$$

11. The sum of the first 7 terms if an AP is 10 and that of the next 7 terms is 17. Find the first term and the common difference of the AP.

Solution: We have, $S_7 = 10$

$$\Rightarrow \frac{7}{2}[2a + (7-1)d] = 10 \quad [\because S_n = \frac{n}{2}\{2a + (n-1)d\}]$$

$$\Rightarrow \frac{7}{2}[2a + 6d] = 10$$

$$\Rightarrow \frac{7}{2} \times 2[a + 3d] = 10$$

$$\Rightarrow 7[a + 3d] = 10$$

$\Rightarrow 7a + 21d = 10$ .... (1)

and $S_{7+7} = 10 + 17$

$$\Rightarrow S_{14} = 27$$

$$\Rightarrow \frac{14}{2}[2a + 13d] = 27 \quad [\because S_n = \frac{n}{2}\{2a + (n-1)d\}]$$

$$\Rightarrow 7[2a + 13d] = 27$$

$\Rightarrow 14a + 91d = 27$ .... (2)

Multiplying equation (1) by 2 and subtracting from equation (2), we have

$$(14a + 91d) - (14a + 42d) = 27 - 20$$

$$\Rightarrow 14a + 91d - 14a - 42d = 7$$

$$\Rightarrow 49d = 7$$

$$\Rightarrow d = \frac{7}{49} = \frac{1}{7}$$

Substituting $d = \frac{1}{7}$ in equation (1), we have

$$7a + 21 \times \frac{1}{7} = 10$$

$$\Rightarrow 7a + 3 = 10$$

$$\Rightarrow 7a = 10 - 3$$

$$\Rightarrow 7a = 7$$

$$\Rightarrow a = 1$$

$\therefore$ the required first term is 1 and the common difference is $\frac{1}{7}$.

12. Find the sum of the first 50 terms of an AP whose nth term is $3 - 2n$.

Solution: We have, $a_n = 3 - 2n$

Then $a = a_1 = 3 - 2 \times 1 = 3 - 2 = 1$

and $l = a_{50} = 3 - 2 \times 50 = 3 - 100 = -97$

$\therefore$ the required sum of the first 50 terms of the AP $= S_{50} = \frac{n}{2}(a + l)$

$= \frac{50}{2}(1 - 97)$

$= 25 \times (-96)$

$= -2400$

13. Find the first term and the common difference of an AP if the sum of the first n terms is $\frac{n(5n+7)}{12}$.

Solution: We have, $S_n = \frac{n(5n+7)}{12}$

Then, $S_1 = \frac{1\times(5\times1+7)}{12} = \frac{12}{12} = 1$

$S_2 = \frac{2\times(5\times2+7)}{12} = \frac{2\times17}{12} = \frac{17}{6}$

Let a and d respectively be the first term and the common difference of the AP.

$a = a_1 = S_1 = 1$

and $a + (a + d) = S_2$

$\Rightarrow 1 + 1 + d = \frac{17}{6}$

$\Rightarrow d = \frac{17}{6} - 2$

$\Rightarrow d = \frac{17-12}{6}$

$\Rightarrow d = \frac{5}{6}$

$\therefore$ the required first term is 1 and the common difference is $\frac{5}{6}$.

14. In an AP, if the sum of the first m terms is equal to n and that of the first n terms is equal to m, then prove that the sum of the first $(m + n)$ terms is $-(m + n)$.

Solution: We have, $S_m = n$

$\Rightarrow \frac{m}{2}[2a + (m - 1)d] = n$

$\Rightarrow 2a + (m - 1)d = \frac{2n}{m}$ --------------- (1)

and $S_n = m$

$\Rightarrow \frac{n}{2}[2a + (n - 1)d] = m$

$\Rightarrow 2a + (n - 1)d = \frac{2m}{n}$ ----------------- (2)

Subtracting equation (2) from equation (1), we get

$$\{2a + (m-1)d\} - \{2a + (n-1)d\} = \frac{2n}{m} - \frac{2m}{n}$$

$$\Rightarrow 2a + (m-1)d - 2a - (n-1)d = \frac{2(n^2 - m^2)}{mn}$$

$$\Rightarrow (m - 1 - \text{n} + 1)d = \frac{2(n^2 - m^2)}{mn}$$

$$\Rightarrow (m - \text{n})d = \frac{-2(m^2 - n^2)}{mn}$$

$$\Rightarrow d = \frac{-2(m+\text{n})(m-\text{n})}{mn(m-\text{n})}$$

$$\Rightarrow d = \frac{-2(m+\text{n})}{mn}$$

Then, the sum of the first $(m+n)$ terms $= S_{(m+n)}$

$$= \frac{m+n}{2}[2a + (m+n-1)d]$$

$$= \frac{m+n}{2}[2a + (m-1)d + nd]$$

$$= \frac{m+n}{2}\left[\frac{2n}{m} + \frac{-2n(m+\text{n})}{mn}\right]$$

$$= \frac{m+n}{2}\left[\frac{2n}{m} + \frac{-2(m+\text{n})}{m}\right]$$

$$= \frac{m+n}{2}\left[\frac{2n - 2(m+\text{n})}{m}\right]$$

$$= \frac{m+n}{2}\left[\frac{2n - 2m - 2n}{m}\right]$$

$$= \frac{m+n}{2}\left[\frac{-2m}{m}\right]$$

$$= \frac{m+n}{2} \times (-2)$$

$$= -(m+n)$$

15. If the sum of the first n, 2n, 3n terms of an AP are S_1, S_2 *and* S_3 respectively. Show that $S_3 = 3(S_2 - S_1)$.

Solution: We have, $S_1 = S_n = \frac{n}{2}[2a + (n-1)d]$

$$S_2 = S_{2n} = \frac{2n}{2}[2a + (2n-1)d]$$

$$S_3 = S_{3n} = \frac{3n}{2}[2a + (3n-1)d]$$

Now $3(S_2 - S_1) = 3[\frac{2n}{2}\{2a + (2n-1)d\} - \frac{n}{2}\{2a + (n-1)d\}$

$= \frac{3n}{2}[2\{2a + (2n-1)d\} - \{2a + (n-1)d\}]$

$= \frac{3n}{2}[4a + (4n-2)d - 2a - (n-1)d]$

$= \frac{3n}{2}[(4-2)a + (4n-2-n+1)d]$

$= \frac{3n}{2}[2a + (3n-1)d]$

$= S_3$

Hence shown.

16. A man repays a debt of Rs. 4860 by paying Rs. 30 in the first month and then increases the amount by Rs. 15 every month. How long will it take him to clear the debt (Assume that no interest is charged)?

Solution: We have, the amounts of money (in Rs.) that the man repays the debt in the first month, second month, third month etc. are 30, 45, 60 etc. which form an AP.

Here, $a = 30$ and $d = 15$.

Now, the total amount to be repaid (in Rs.) $= 4860$

i.e. $S_n = 4860$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 4860$

$\Rightarrow \frac{n}{2}[2 \times 30 + 15(n-1)] = 4860$

$\Rightarrow \frac{n}{2}(60 + 15n - 15) = 4860$

$\Rightarrow \frac{n}{2}(15n + 45) = 4860$

$\Rightarrow \frac{15n}{2}(n+3) = 4860$

$\Rightarrow n(n+3) = 4860 \times \frac{2}{15}$

$\Rightarrow n^2 + 3n = 324 \times 2$

$\Rightarrow n^2 + 3n = 648$

$\Rightarrow n^2 + 3n - 648 = 0$

$\Rightarrow n^2 + (27-24)n - 648 = 0$

$\Rightarrow n^2 + 27n - 24n - 648 = 0$

$\Rightarrow n(n+27) - 24(n+27) = 0$

$\Rightarrow (n-24)(n+27) = 0$

$\Rightarrow n - 24 = 0$ or $n + 27 = 0$

$\Rightarrow n = 24$ or $n = -27$

As number of terms cannot be negative, $n = -27$ is neglected.

∴ the required time taken to clear the debt is 24 months i.e. 2 years.

17. A manufacturer of radio sets produced 600 units in the third year and 700 units in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find the production in the first year and also the total production in 8 years.

Solution: As the production increases uniformly by a fixed number every year, productions of radio sets each year form an AP.

Here, $a_3 = 600$

$\Rightarrow a + 2d = 600$ ----------------- (1)

and $a_7 = 700$

$\Rightarrow a + 6d = 700$ ----------------- (2)

Subtracting equation (1) from equations (2), we have

$4d = 100$

$\Rightarrow d = 25$

Substituting $d = 25$ in equation (1), we have

$a + 2 \times 25 = 600$

$\Rightarrow a = 600 - 50$

$\Rightarrow a = 550$

∴ the required production in the first year = 550 units

and total production in 8 years = $S_8 = \frac{n}{2}[2a + (n-1)d]$

$= \frac{8}{2}[2 \times 550 + (8-1) \times 25]$

$= 4 \times (1100 + 175)$

$= 4 \times 1275$

$= 5100$ units.

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