CLASS – X
MATHEMATICS
CHAPTER – 4
PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

- **The general form of a pair of linear equations in two variables**

The general form of a pair of linear equations in two variables x and y is

$$a_1x + b_1y + c_1 = 0$$

$$\text{and } \ a_2x + b_2y + c_2 = 0$$

where $a_1, b_1, c_1, a_2, b_2, c_2$ are all real numbers and $a_1{}^2 + b_1{}^2 \neq 0, a_2{}^2 + b_2{}^2 \neq 0$.

- **Solution of a Pair of Linear Equations in Two Variables**

The ordered pair (x_1, y_1) is said to be a solution of a pair of linear equations in two variables x and y, if $x = x_1, y = y_1$ satisfy both the linear equations.

- **Consistent pair:** A pair of linear equations in two variables having at least one solution is called a consistent pair.
- **Inconsistent pair:** A pair of linear equations in two variables having no solution is called an inconsistent pair.
- **Dependent pair:** A pair of linear equations is said to be a dependent pair if one equation is obtained from the other on multiplying by a constant.

Note: A dependent pair of linear equations has infinitely many solutions. So, a dependent pair of equations is also a consistent pair.

- **Graphical Representation of a Pair of Linear equations**
  1. If the graphs (lines) of a pair of linear equations intersect at one point, there is a unique solution.
  2. If the lines of a pair of linear equations are coincident, there are infinitely many solutions i.e. the pair equations is dependent.
  3. If the lines of a pair of linear equations are parallel, there is no solution i.e. the pair of linear equations is inconsistent.

Note: The graphs of a consistent pair are either intersecting lines or coincident lines.

- **Comparison of the ratios of the coefficients of a pair of linear equations**

In the pair of linear equations

$$a_1x + b_1y + c_1 = 0$$

$$\text{and } a_2x + b_2y + c_2 = 0$$

1. if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the graphs (lines) are intersecting i.e. there is a unique solution.
2. if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the lines are coincident i.e. the pair of equations has infinitely many solutions i.e. the pair of equations is dependent.
3. if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the lines are parallel i.e. the pair of equations has no solution i.e. the pair of equations is inconsistent.

Note: Condition for consistent pair is either $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

SOLUTIONS

EXERCISE 4.1

1. By comparing the ratios of the coefficients, find out whether the lines representing the following pair of linear equations intersect at one point, are parallel or coincident:

(i) $\mathbf{3x + 2y - 5 = 0}$
$\mathbf{4x - 3y + 2 = 0}$

Solution: Comparing the given pair of equations with the general form, we get

$$a_1 = 3,\ b_1 = 2,\ c_1 = -5$$

$$\text{and } a_2 = 4,\ b_2 = -3,\ c_2 = 2$$

Here, $\frac{a_1}{a_2} = \frac{3}{4}$, $\frac{b_1}{b_2} = \frac{2}{-3} = -\frac{2}{3}$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the two lines representing the given pair of equations intersect at a point.

(ii) $\mathbf{9x + 3y + 4 = 0}$
$\mathbf{18x + 6y + 8 = 0}$

Solution: Comparing the given pair of equations with the general form, we get

$$a_1 = 9,\ b_1 = 3,\ c_1 = 4$$

$$\text{and } a_2 = 18,\ b_2 = 6,\ c_2 = 8$$

Here, $\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{4}{8} = \frac{1}{2}$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the two lines representing the given pair of equations are coincident.

(iii) $\mathbf{2x + y = 3}$
$\mathbf{3x - 2y + 6 = 0}$

Solution: Writing the given pair of equations in the general form, we get

$$2x + y - 3 = 0$$
$$3x - 2y + 6 = 0$$

Here, $\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{1}{-2} = -\frac{1}{2}$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the two lines representing the given pair of equations intersect at a point.

(iv) $\mathbf{6x - 2y + 5 = 0}$
$\mathbf{3x - y + 9 = 0}$

Solution: Comparing the given pair of equations with the general form, we get

$$\frac{a_1}{a_2} = \frac{6}{3} = 2, \qquad \frac{b_1}{b_2} = \frac{-2}{-1} = 2 \text{ and } \frac{c_1}{c_2} = \frac{5}{9}$$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the two lines representing the given pair of equations are parallel.

(v) $\mathbf{\frac{3}{2}x + \frac{5}{3}y = 7}$

$\mathbf{9x + 10y = 14}$

Solution: Writing the given pair of equations in the general form, we get

$$\frac{3}{2}x + \frac{5}{3}y - 7 = 0$$
$$9x + 10y - 14 = 0$$

Here, $\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{3}{2} \times \frac{1}{9} = \frac{1}{6}, \quad \frac{b_1}{b_2} = \frac{\frac{5}{3}}{10} = \frac{5}{3} \times \frac{1}{10} = \frac{1}{6}$ and $\frac{c_1}{c_2} = \frac{-7}{-14} = \frac{1}{2}$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the two lines representing the given pair of equations are parallel.

(vi) $\mathbf{x + y = 5}$
$\mathbf{3x - 5y = -9}$

Solution: Writing the given pair of equations in the general form, we get

$$x + y - 5 = 0$$
$$3x - 5y + 9 = 0$$

Here, $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{1}{-5} = -\frac{1}{5}$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the two lines representing the given pair of equations intersect at a point.

2. By comparing the ratios of the coefficients, find out whether the following pairs of linear equations are consistent or not:

(i) $\mathbf{2x + y - 3 = 0}$

$\mathbf{3x + 4y + 6 = 0}$

Solution: Comparing the given pair of equations with the general form, we get

$$a_1 = 2, b_1 = 1, c_1 = -3$$

and $a_2 = 3, b_2 = 4, c_2 = 6$

Here, $\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{1}{4}$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the given pair of equations is consistent with a unique solution.

(ii) $\mathbf{3x + 4y = 12}$

$\mathbf{6x + 8y = 24}$

Solution: Writing the given pair of equations in the general form, we get

$$3x + 4y - 12 = 0$$

$$6x + 8y - 24 = 0$$

Here, $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{4}{8} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-12}{-24} = \frac{1}{2}$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence, the given pair of equations is consistent with infinitely many solutions.

(iii) $\mathbf{3x - y + 5 = 0}$

$\mathbf{4x + 3y = 11}$

Solution: Writing the given pair of equations in the general form, we get

$$3x - y + 5 = 0$$

$$4x + 3y = 11$$

Here, $\frac{a_1}{a_2} = \frac{3}{4}, \frac{b_1}{b_2} = \frac{-1}{3}$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence, the given pair of equations is consistent with a unique solution.

(iv) $\mathbf{2x - 3y = 8}$

$\mathbf{4x - 6y = 9}$

Solution: Writing the given pair of equations in the general form, we get

$$2x - 3y - 8 = 0$$

$$4x - 6y - 9 = 0$$

Here, $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$, $\quad \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence the given pair of equations is inconsistent.

(v) $\mathbf{\frac{2}{3}x + 2y = 8}$

$\mathbf{x + 3y = 2}$

Solution: Writing the given pair of equations in the general form, we get

$$\frac{2}{3}x + 2y - 8 = 0$$

$$x + 3y - 2 = 0$$

Here, $\frac{a_1}{a_2} = \frac{\frac{2}{3}}{1} = \frac{2}{3}$, $\quad \frac{b_1}{b_2} = \frac{2}{3}$ and $\frac{c_1}{c_2} = \frac{-8}{-2} = 4$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence the given pair of equations is inconsistent.

3. Solve the following pair of equations graphically:

(i) $\mathbf{x - 3y + 6 = 0}$

$\mathbf{x - 3y - 12 = 0}$

Solution: $x - 3y + 6 = 0$ --------------- (1)

$x - 3y - 12 = 0$ --------------- (2)

Some values of x and y satisfying equations (1) are given in table (I).

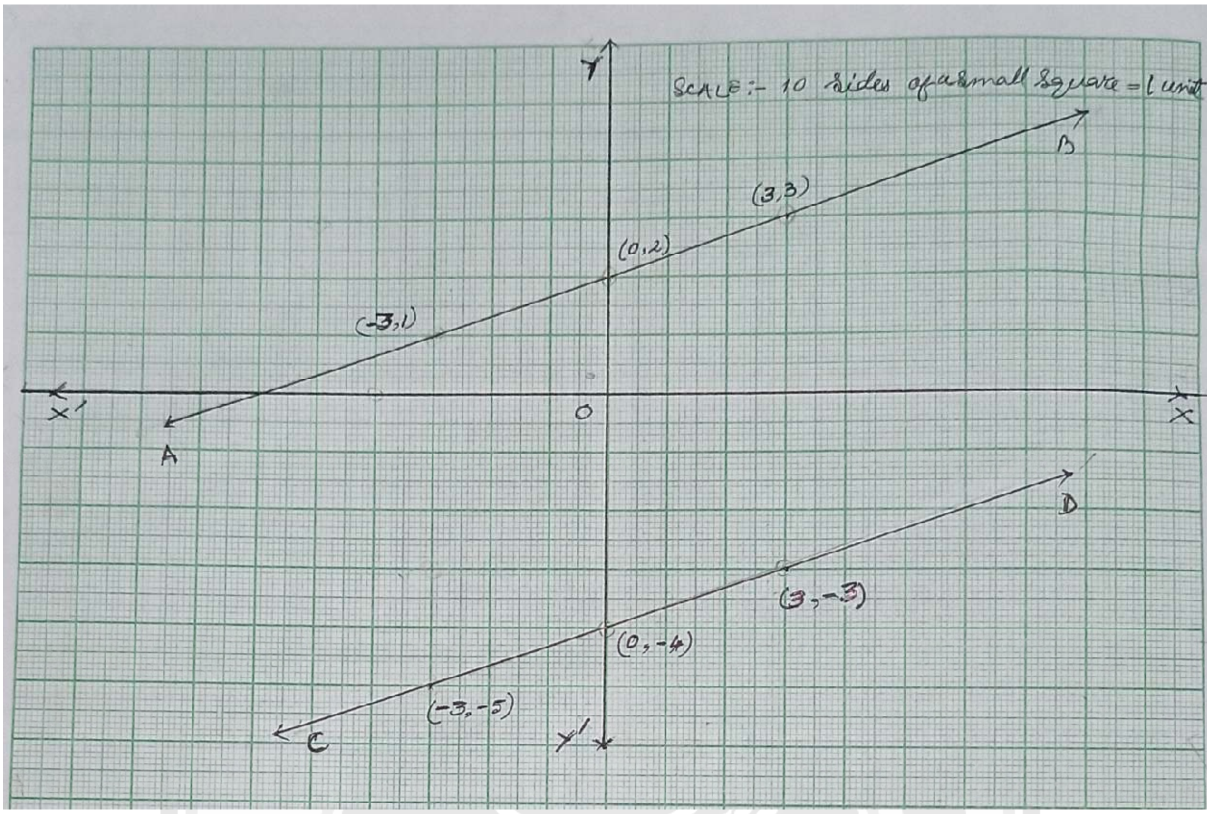
x	3	-3	0
y	3	1	2

Table (I)

Similarly, some of the values of x and y satisfying equation (2) are given in table (II).

x	-3	0	3
y	-5	-4	-3

Table (II)



The points given by table (I) are plotted in the Cartesian plane. Joining these points, we get the straight line AB. This is the graph of equation (1).

Again, in the same Cartesian plane and taking the same units the points given by table (II) are plotted. Joining these points we get the straight line CD. This is the graph of the equation (2).The two straight lines AB and CD are parallel.

Hence, the given pair of equations has no solution.

(ii) $\mathbf{2x - y = 2}$

$\mathbf{3x + 2y = 17}$

Solution: $2x - y = 2$ --------------- (1)

$3x + 2y = 17$ --------------- (2)

Some values of x and y satisfying equations (1) are given in table (I).

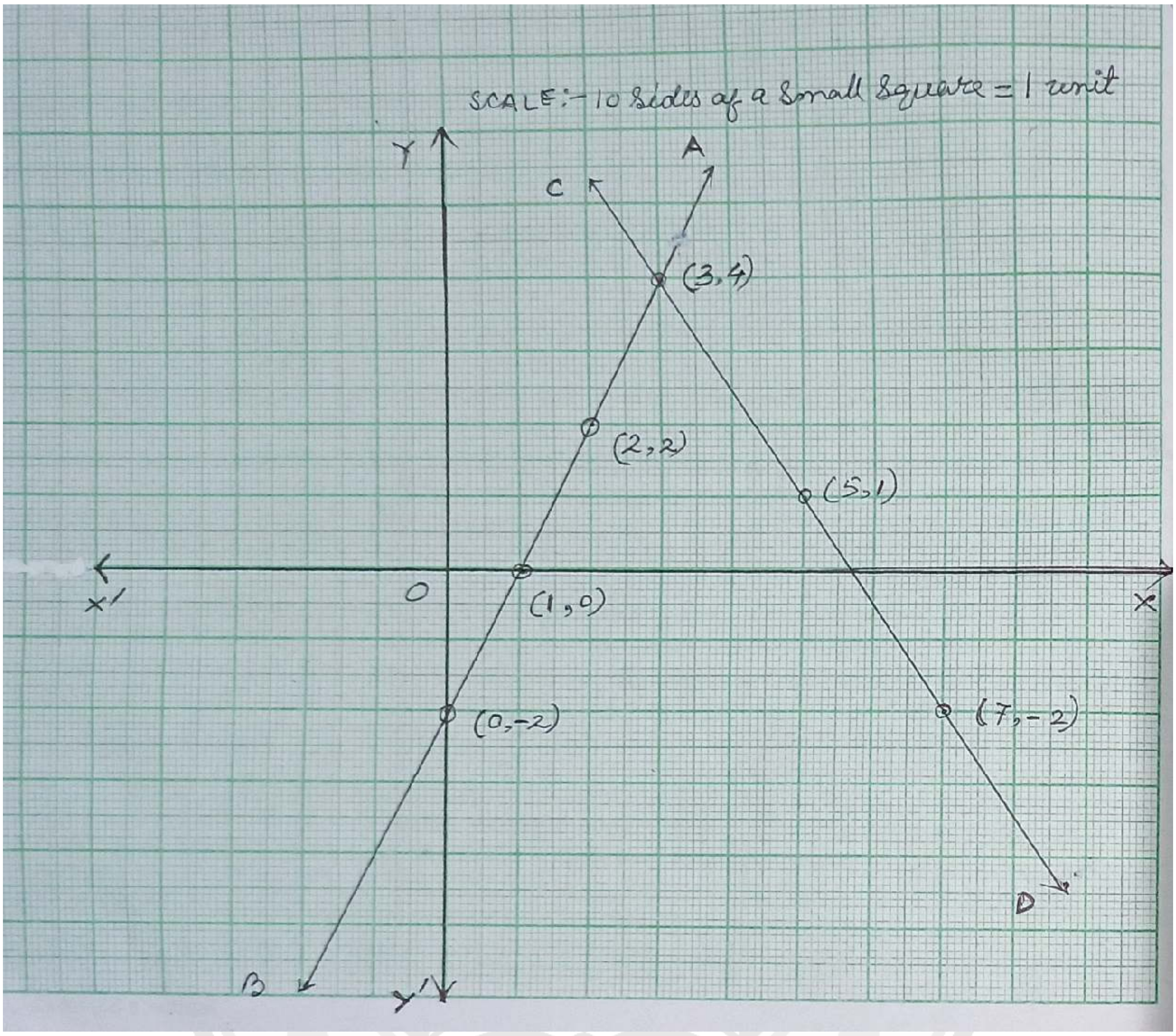
x	1	2	0
y	0	2	-2

Table (I)

Similarly, some of the values of x and y satisfying equation (2) are given in table (II).

x	3	5	7
y	4	1	-2

Table (II)



The lines (graphs) AB and CD of equations (1) and (2) respectively are drawn with the points given by tables (I) and (II). The lines AB and CD intersect at (3, 4). Hence, the solution of the given pair of equations is given by $x = 3$ and $y = 4$.

(iii) $\mathbf{2x + 3y = 5}$

$\mathbf{5x - 4y + 22 = 0}$

Solution: $2x + 3y = 5$ --------------- (1)

$5x - 4y + 22 = 0$ --------------- (2)

Some values of x and y satisfying equations (1) are given in table (I).

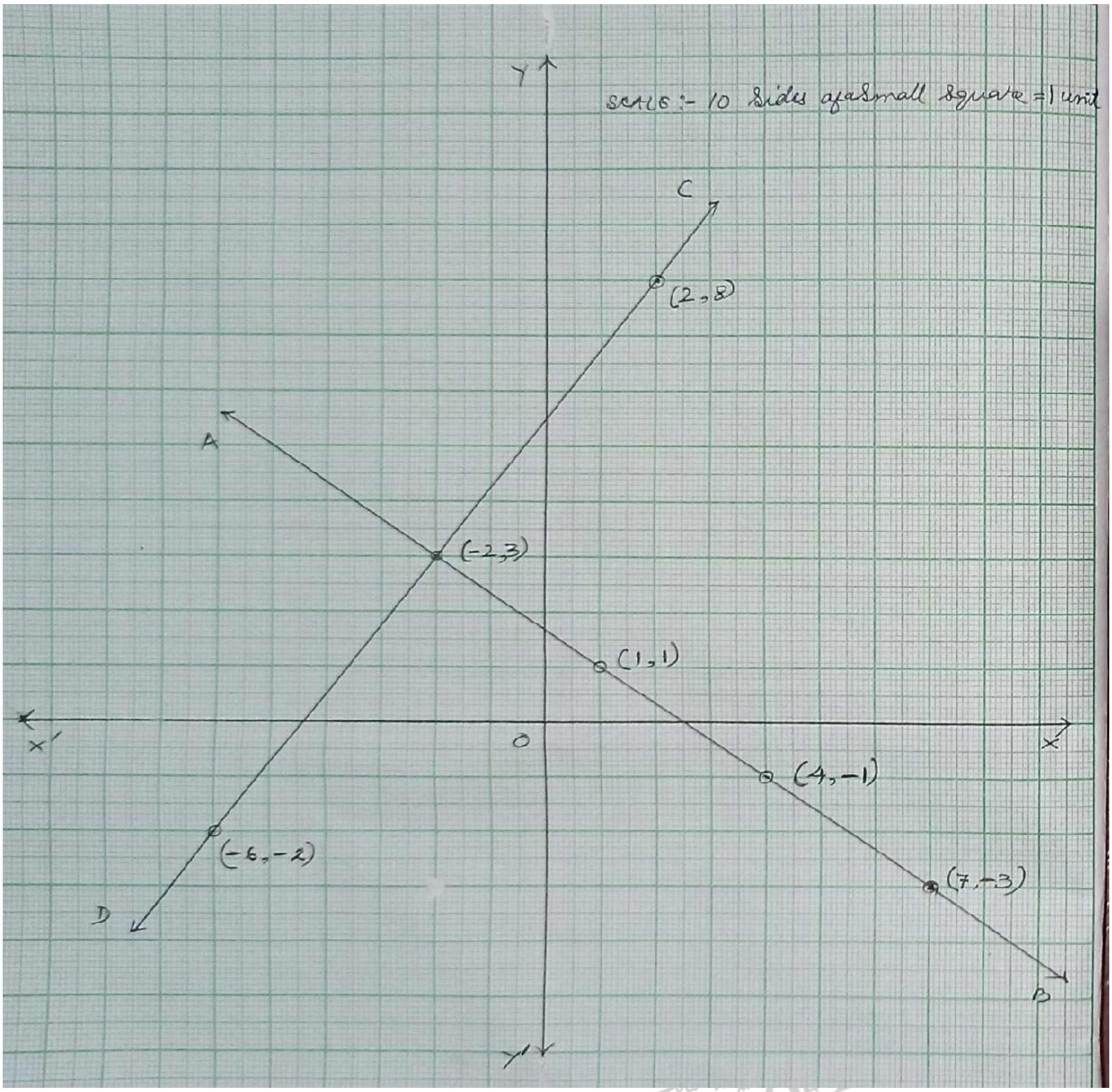
x	7	4	1
y	-3	-1	1

Table (I)

Similarly, some of the values of x and y satisfying equation (2) are given in table (II).

x	2	-2	-6
y	8	3	-2

Table (II)



When we draw the graph of equations (1) and (2) with the points given by tables (I) and (II) we get two intersecting straight lines AB and CD intersecting at $(-2, 3)$. Hence, the solution of the given pair of equations is given by $x = -2$ and $y = 3$.

(iv) $\boldsymbol{y = x}$

$\boldsymbol{5x - 2y = 9}$

Solution: $y = x$ --------------- (1)

$5x - 2y = 9$ --------------- (2)

Some values of x and y satisfying equations (1) are given in table (I).

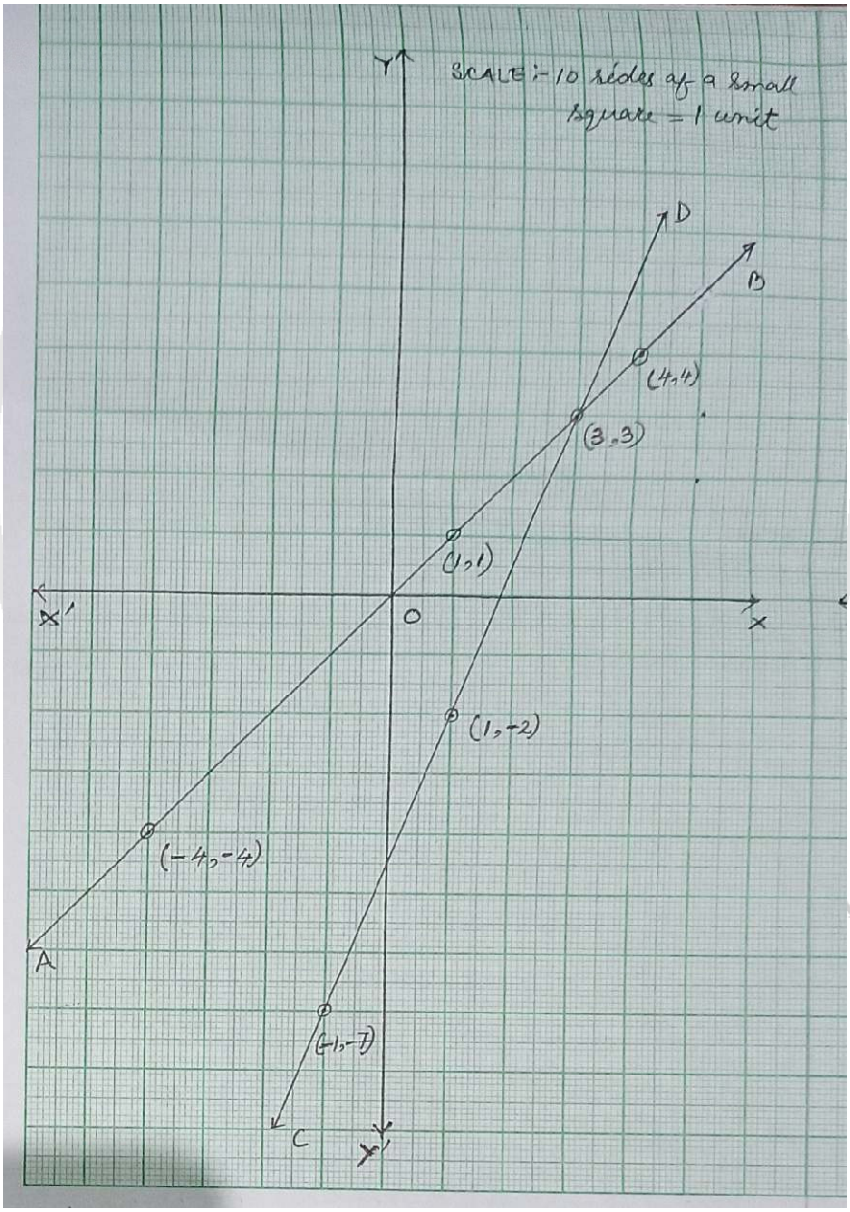
x	1	4	-4
y	1	4	-4

Table (I)

Similarly, some of the values of x and y satisfying equation (2) are given in table (II).

x	-1	1	3
y	-7	-2	3

Table (II)



When we draw the graph of equations (1) and (2) with the points given by tables (I) and (II) we get two intersecting straight lines AB and CD intersecting at (3, 3). Hence, the solution of the given pair of equations is given by $x = 3$ and $y = 3$.

(v) $\boldsymbol{x + 4y = 2}$

$\boldsymbol{3x + 12y = 6}$

Solution: $x + 4y = 2$ --------------- (1)

$3x + 12y = 6$ --------------- (2)

Some values of x and y satisfying equations (1) are given in table (I).

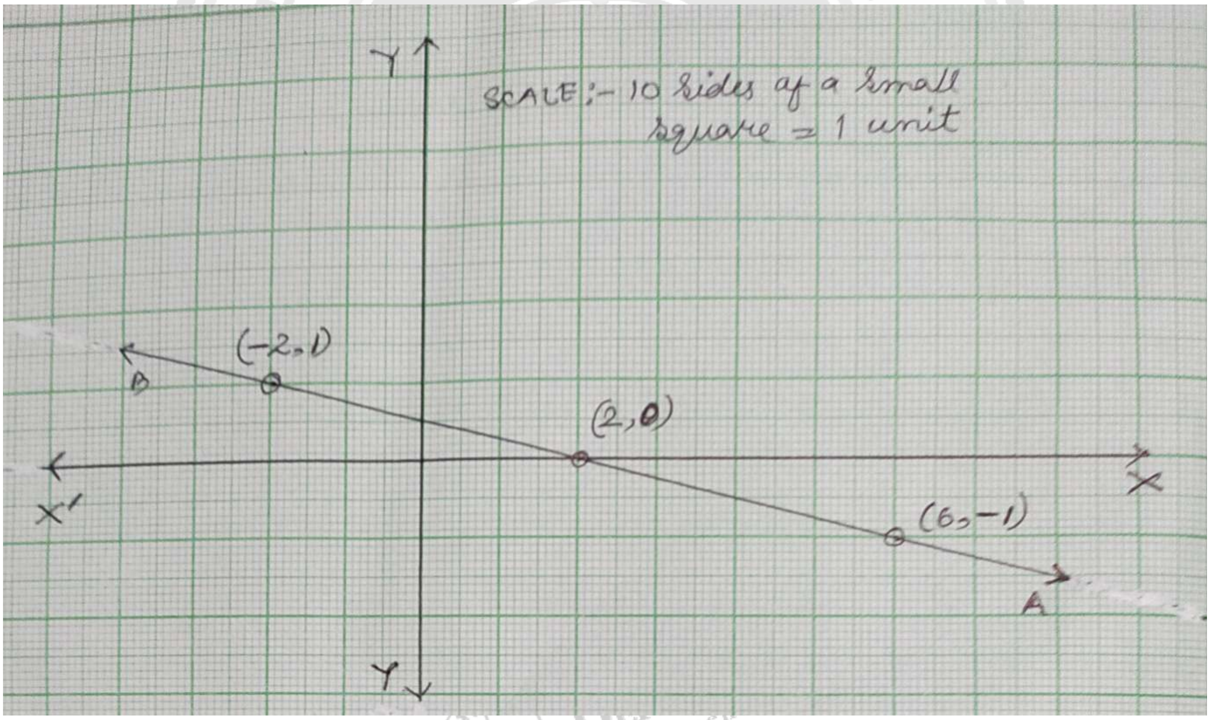
x	-2	2	6
y	1	0	-1

Table (I)

Similarly, some of the values of x and y satisfying equation (2) are given in table (II).

x	-2	2	6
y	1	0	-1

Table (II)



When we draw the graphs of equations (1) and (2) by plotting the points given by tables (I) and (II) in the same Cartesian plane, we get a single straight line AB.

Therefore, there is an infinite number of points common to both the graphs.

Hence, there are infinitely many solutions of the given pair of equations.

(vi) $\mathbf{4x + 6y = 18}$

$\mathbf{9 - 2x - 3y = 0}$

Solution: $4x + 6y = 18$ --------------- (1)

$9 - 2x - 3y = 0$ --------------- (2)

Some values of x and y satisfying equations (1) are given in table (I).

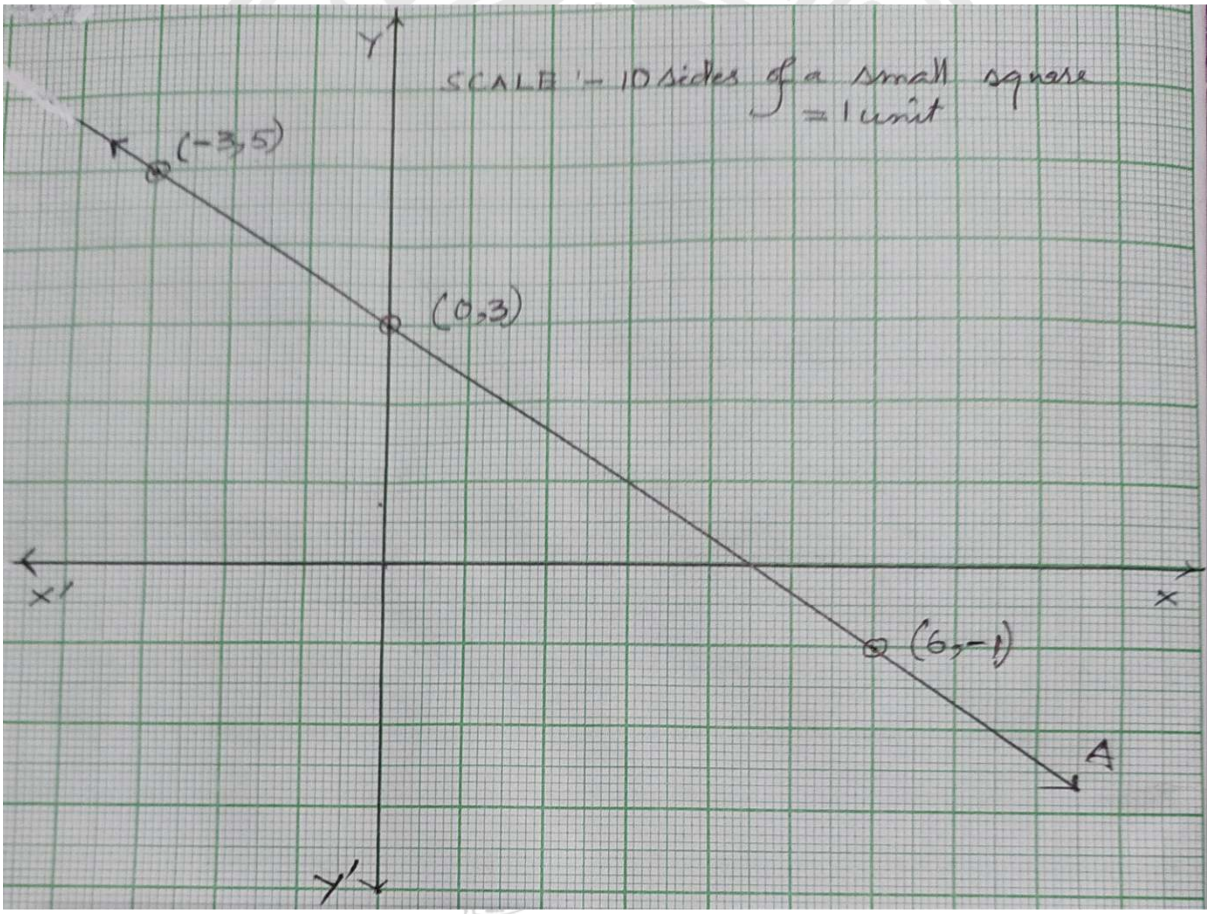
x	-3	0	6
y	5	3	-1

Table (I)

Similarly, some of the values of x and y satisfying equation (2) are given in table (II).

x	-3	0	6
y	5	3	-1

Table (II)



When we draw the graphs of equations (1) and (2) by plotting the points given by tables (I) and (II) in the same Cartesian plane, we get a single straight line AB.
Therefore, there is an infinite number of points common to both the graphs.
Hence, there are infinite number of solutions of the given pair of equations.

(vii) $\mathbf{2x + y = 7}$

$\mathbf{3x + 2y = 11}$

Solution: $2x + y = 7$ --------------- (1)

$3x + 2y = 11$ --------------- (2)

Some values of x and y satisfying equations (1) are given in table (I).

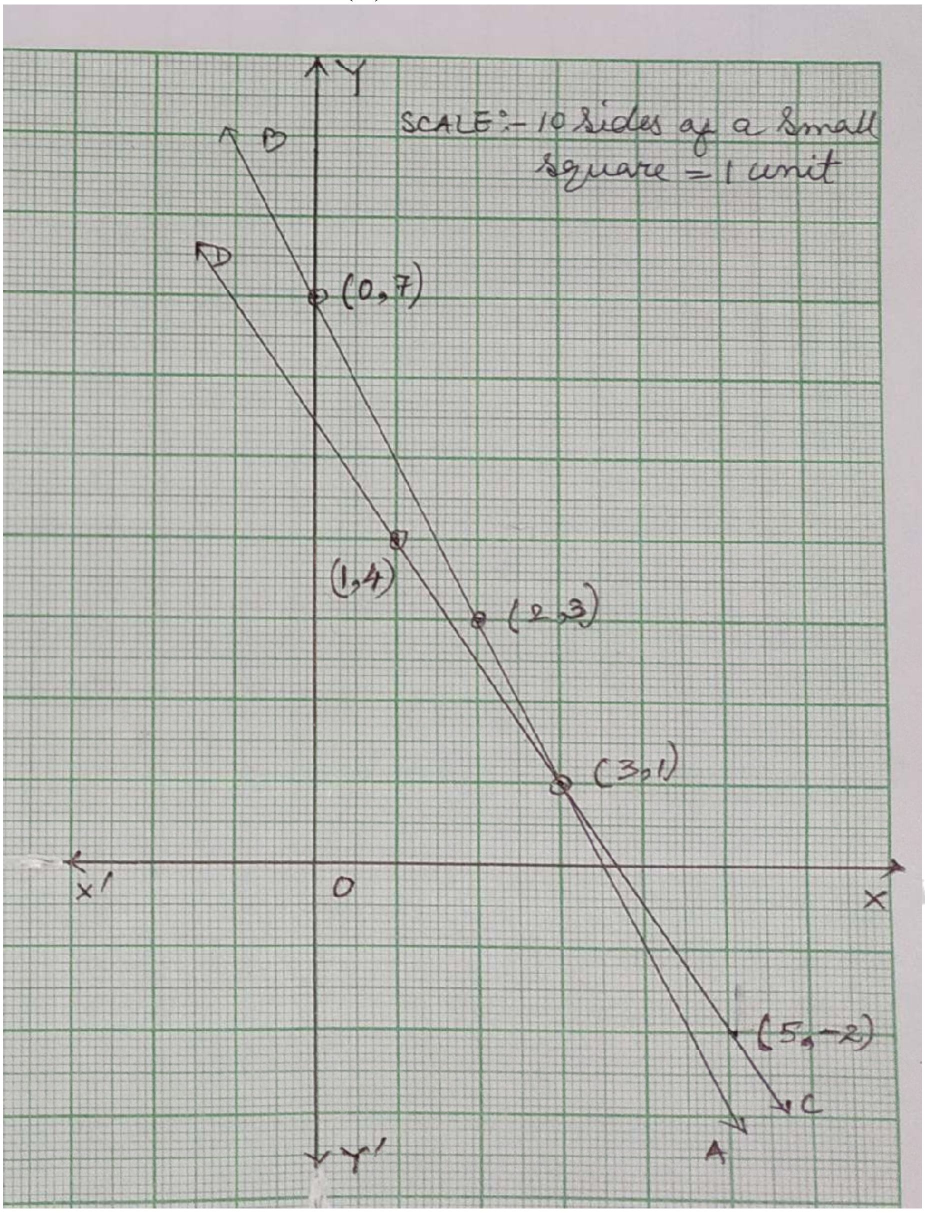
x	3	0	2
y	1	7	3

Table (I)

Similarly, some of the values of x and y satisfying equation (2) are given in table (II).

x	5	1	3
y	-2	4	1

Table (II)



When we draw the graph of equations (1) and (2) with the points given by tables (I) and (II) we get two intersecting straight lines AB and CD intersecting at (3, 1). Hence, the solution of the given pair of equations is given by $x = 3$ and $y = 1$.

(viii) $x + 2y = 8$
$3x - y = 3$

Solution: $x + 2y = 8$ --------------- (1)

$3x - y = 3$ --------------- (2)

Some values of x and y satisfying equations (1) are given in table (I).

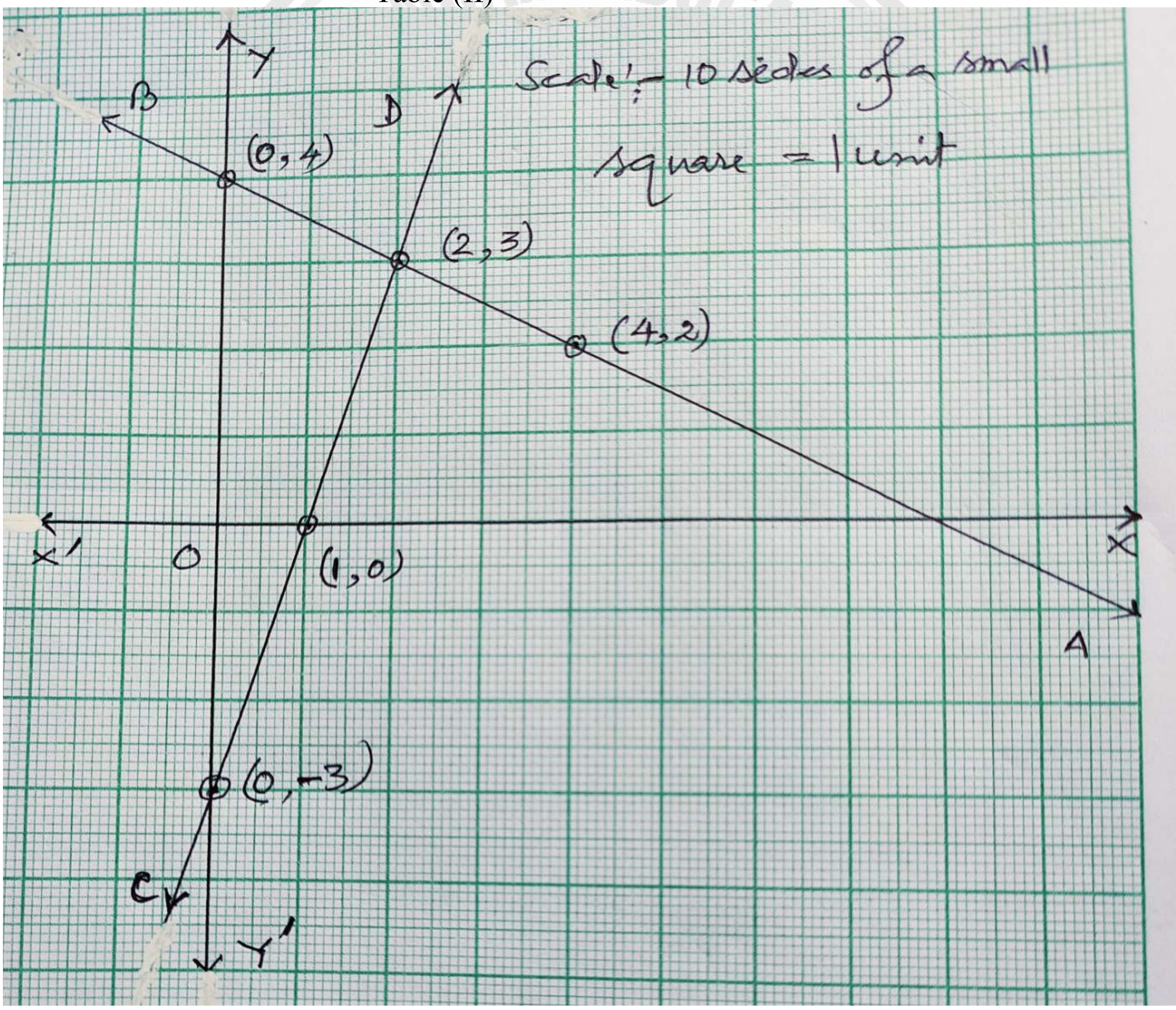
x	4	0	2
y	2	4	3

Table (I)

Similarly, some of the values of x and y satisfying equation (2) are given in table (II).

x	1	0	2
y	0	−3	3

Table (II)



When we draw the graph of equations (1) and (2) with the points given by tables (I) and (II) we get two intersecting straight lines AB and CD intersecting at (2, 3). Hence, the solution of the given pair of equations is given by $x = 2$ and $y = 3$.

(ix) $\mathbf{3x + y = 9}$

$\mathbf{2x - 3y + 16 = 0}$

Solution: $3x + y = 9$ --------------- (1)

$2x - 3y + 16 = 0$ --------------- (2)

Some values of x and y satisfying equations (1) are given in table (I).

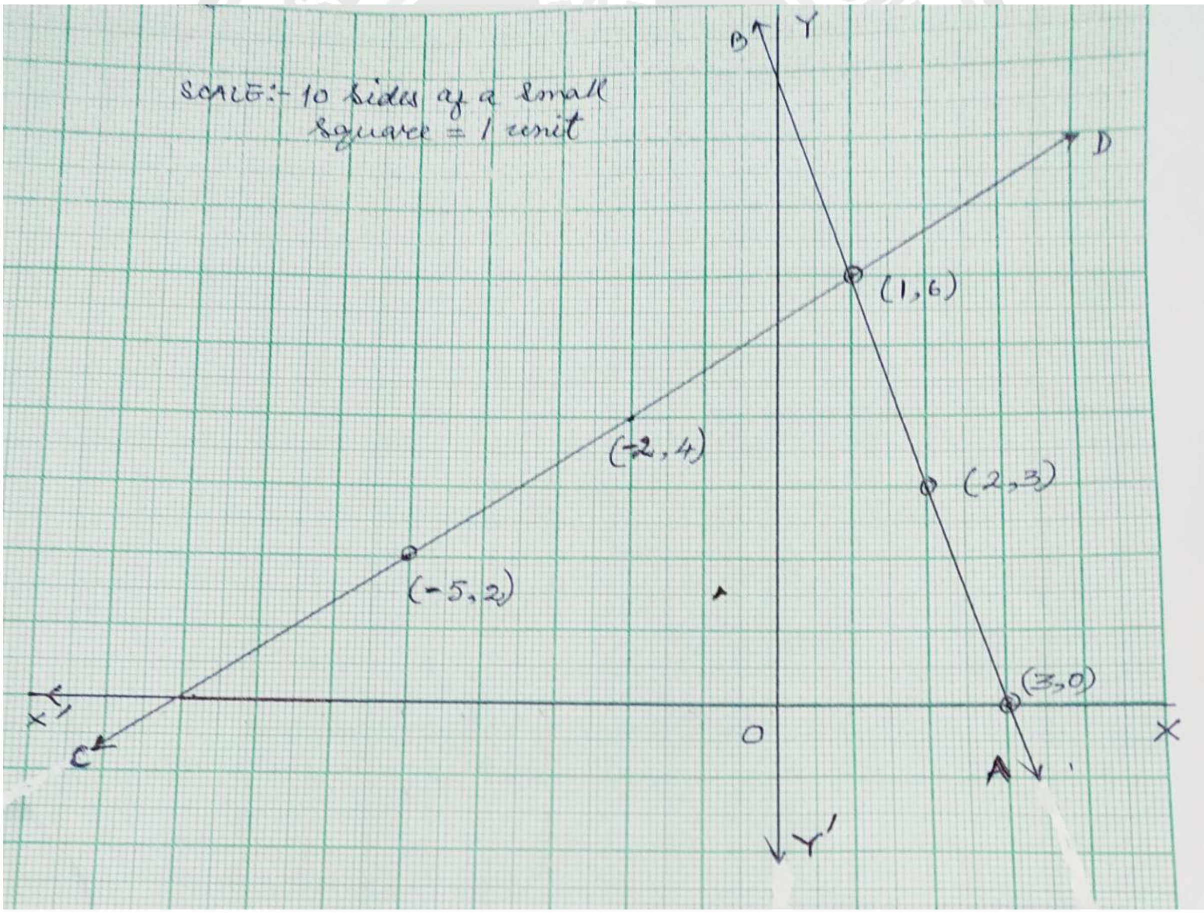
x	1	3	2
y	6	0	3

Table (I)

Similarly, some of the values of x and y satisfying equation (2) are given in table (II).

x	-2	-5	1
y	4	2	6

Table (II)



When we draw the graph of equations (1) and (2) with the points given by tables (I) and (II) we get two intersecting straight lines AB and CD intersecting at (1,6). Hence, the solution of the given pair of equations is given by $x = 1$ and $y = 6$.

(x) $\frac{x}{2}+\frac{y}{5}=\mathbf{1}$

$\mathbf{5x+2y=10}$

Solution: $\frac{x}{2}+\frac{y}{5}=1$ --------------- (1)

$5x+2y=10$ --------------- (2)

Some values of x and y satisfying equations (1) are given in table (I).

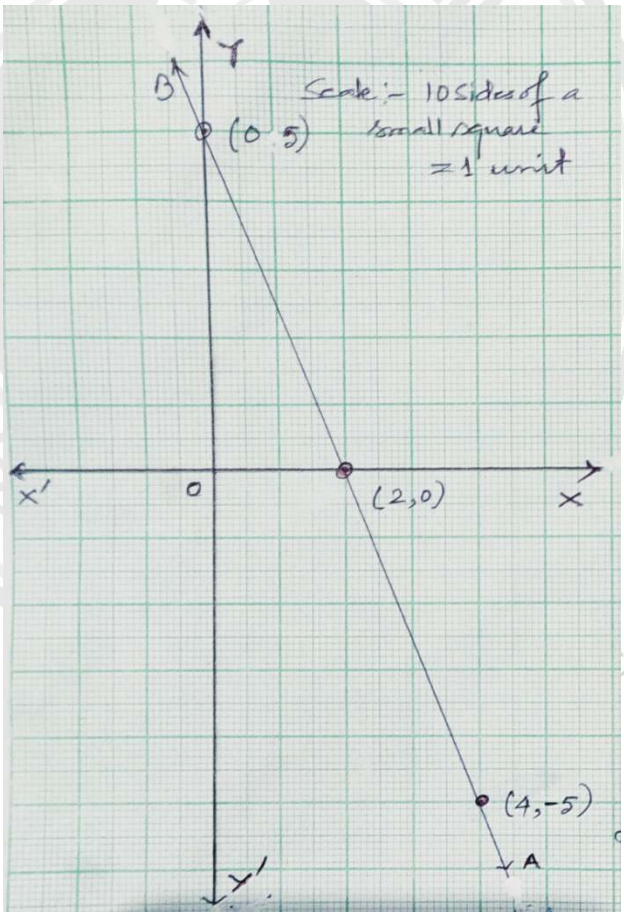
x	0	2	4
y	5	0	−5

Table (I)

Similarly, some of the values of x and y satisfying equation (2) are given in table (II).

x	0	2	4
y	5	0	−5

Table (II)



When we draw the graphs of equations (1) and (2) by plotting the points given by tables (I) and (II) in the same Cartesian plane, we get a single straight line AB. Therefore, there is an infinite number of points common to both the graphs.

Hence, there are infinite number of solutions of the given pair of equations.

(xi) $2x + 3y = 12$

$2x = 3y$

Solution: $2x + 3y = 12$ --------------- (1)

$2x = 3y$ --------------- (2)

Some values of x and y satisfying equations (1) are given in table (I).

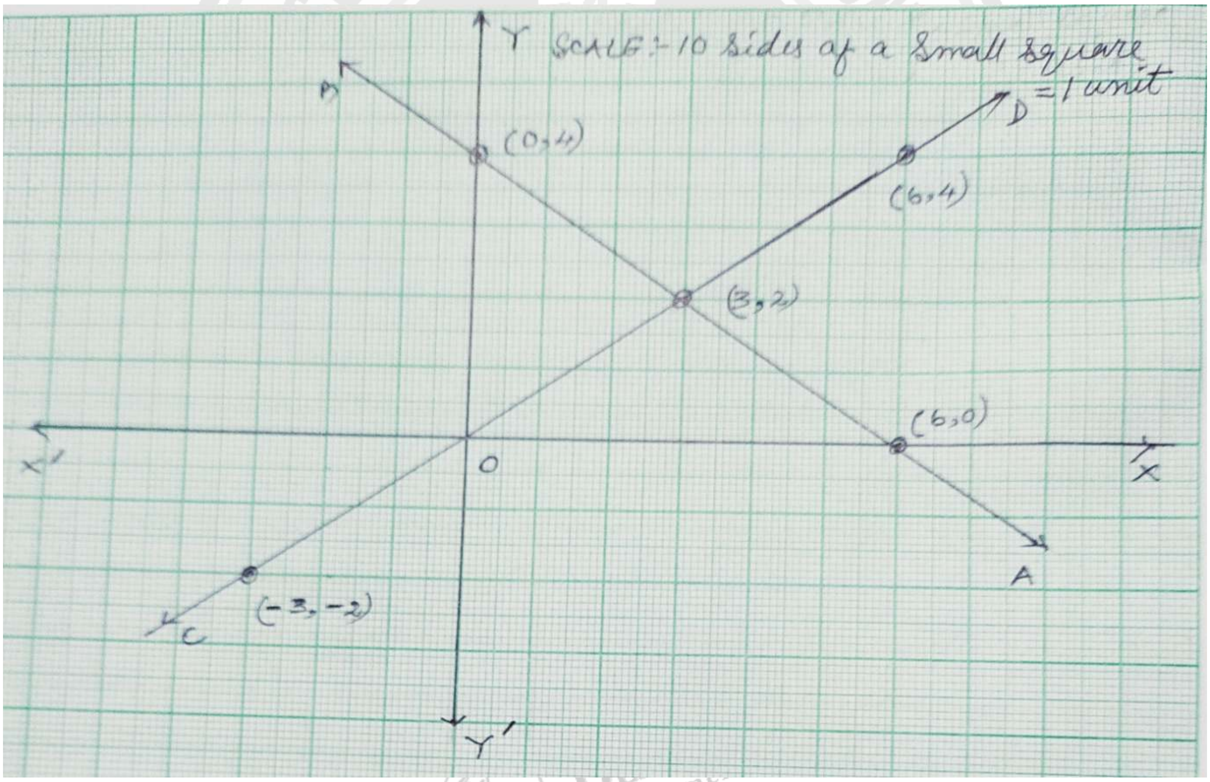
x	3	6	0
y	2	0	4

Table (I)

Similarly, some of the values of x and y satisfying equation (2) are given in table (II).

x	3	6	-3
y	2	4	-2

Table (II)



When we draw the graph of equations (1) and (2) with the points given by tables (I) and (II) we get two intersecting straight lines AB and CD intersecting at (3, 2). Hence, the solution of the given pair of equations is given by $x = 3$ and $y = 2$.

(xii) $3x + 2y = 4$

$6x + 4y = 13$

Solution: $3x + 2y = 4$ --------------- (1)

$6x + 4y = 13$ --------------- (2)

Some values of x and y satisfying equations (1) are given in table (I).

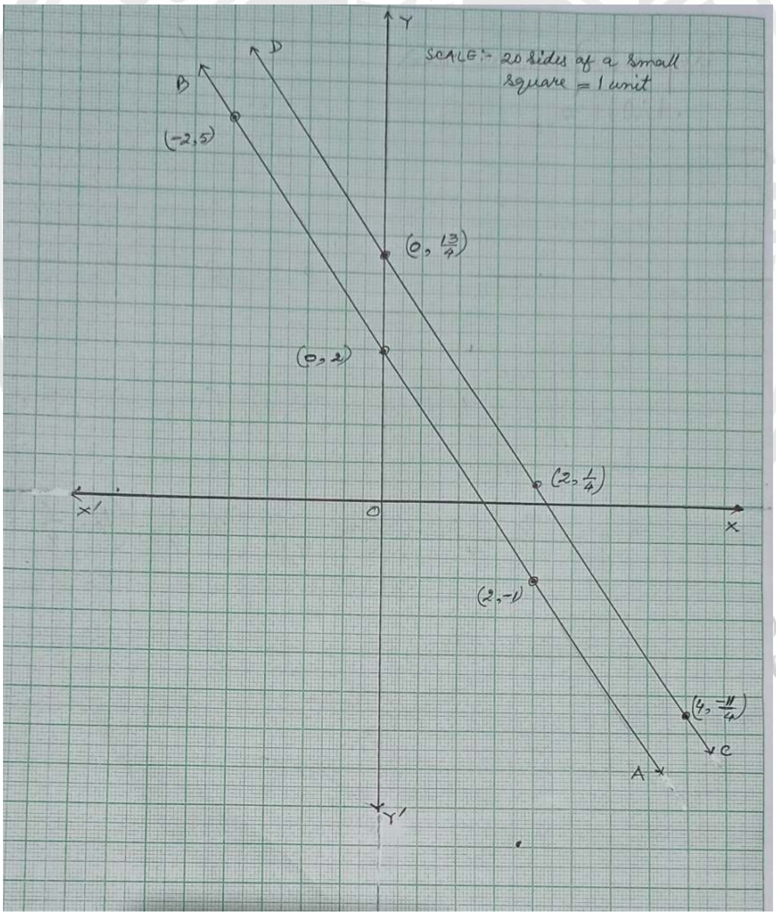
x	-4	2	0
y	5	-1	2

Table (I)

Similarly, some of the values of x and y satisfying equation (2) are given in table (II).

x	0	2	4
y	$\frac{13}{4}$	$\frac{1}{4}$	$-\frac{11}{4}$

Table (II)



When we draw the graph of equations (1) and (2) with the points given by tables (I) and (II) we get two parallel straight lines AB and CD.

Hence, the given pair of equations has no solution.

(xiii) $2x + y = 6$

$x - 2y = 8$

Solution: $2x + y = 6$ --------------- (1)

$x - 2y = 8$ --------------- (2)

Some values of x and y satisfying equations (1) are given in table (I).

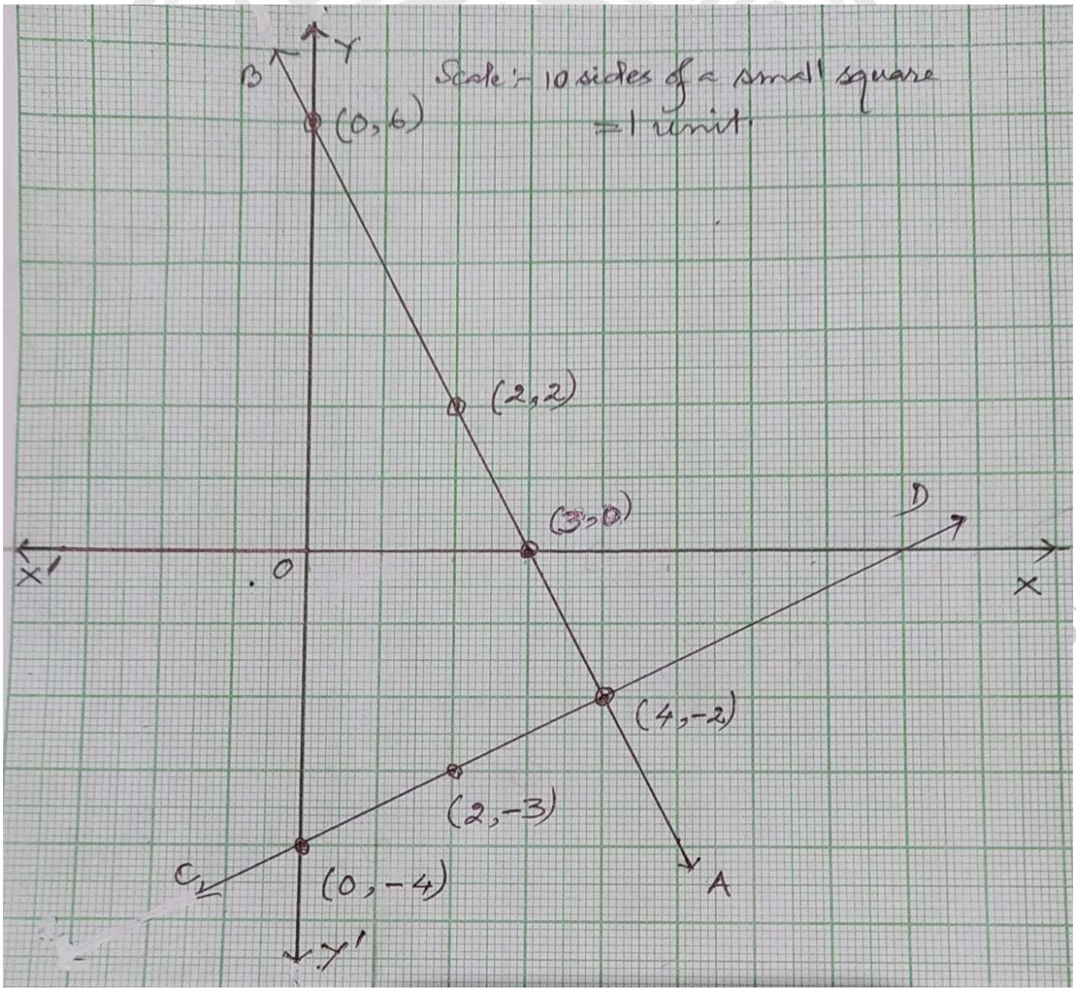
x	3	0	2
y	0	6	2

Table (I)

Similarly, some of the values of x and y satisfying equation (2) are given in table (II).

x	0	4	2
y	-4	-2	-3

Table (II)



When we draw the graphs of equations (1) and (2) with the points given by tables (I) and (II) we get two intersecting straight lines AB and CD intersecting at (4, −2). Hence, the solution of the given pair of equations is given by $x = 4$ and $y = -2$.

(xiv) $\mathbf{2x + y - 8 = 0}$

$\mathbf{x - y - 1 = 0}$

Solution: $2x + y - 8 = 0$ --------------- (1)

$x - y - 1 = 0$ --------------- (2)

Some values of x and y satisfying equation (1) are given in table (I).

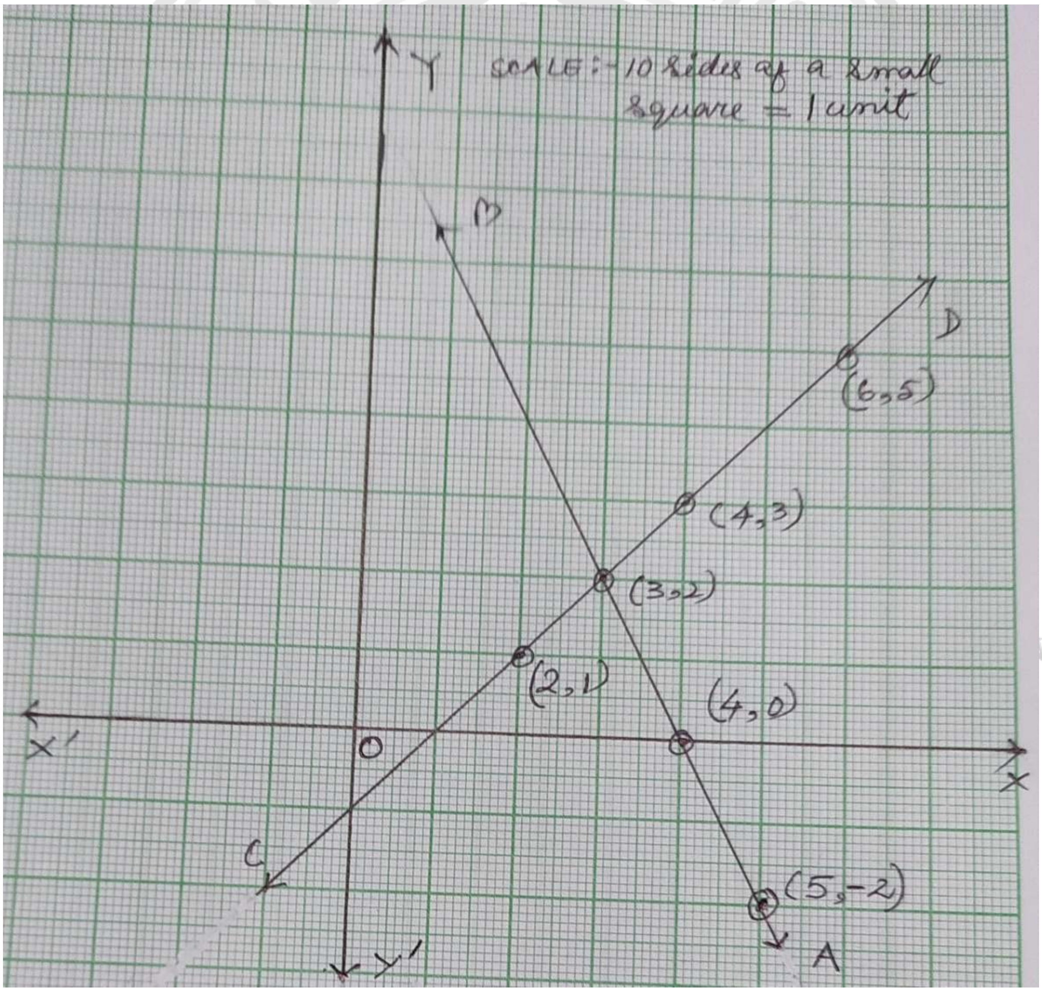
x	3	4	5
y	2	0	-2

Table (I)

Similarly, some of the values of x and y satisfying equation (2) are given in table (II).

x	2	4	6
y	1	3	5

Table (II)



When we draw the graphs of equations (1) and (2) with the points given by tables (I) and (II) we get two straight lines AB and CD intersecting at (3,2). Hence, the solution of the given pair of equations is given by $x = 3$ and $y = 2$.

(xv) $\frac{x}{3}+\frac{y}{4}=1$

$5x-3y=15$

Solution: $\frac{x}{3}+\frac{y}{4}=1$ --------------- (1)

$5x-3y=15$ --------------- (2)

Some values of x and y satisfying equations (1) are given in table (I).

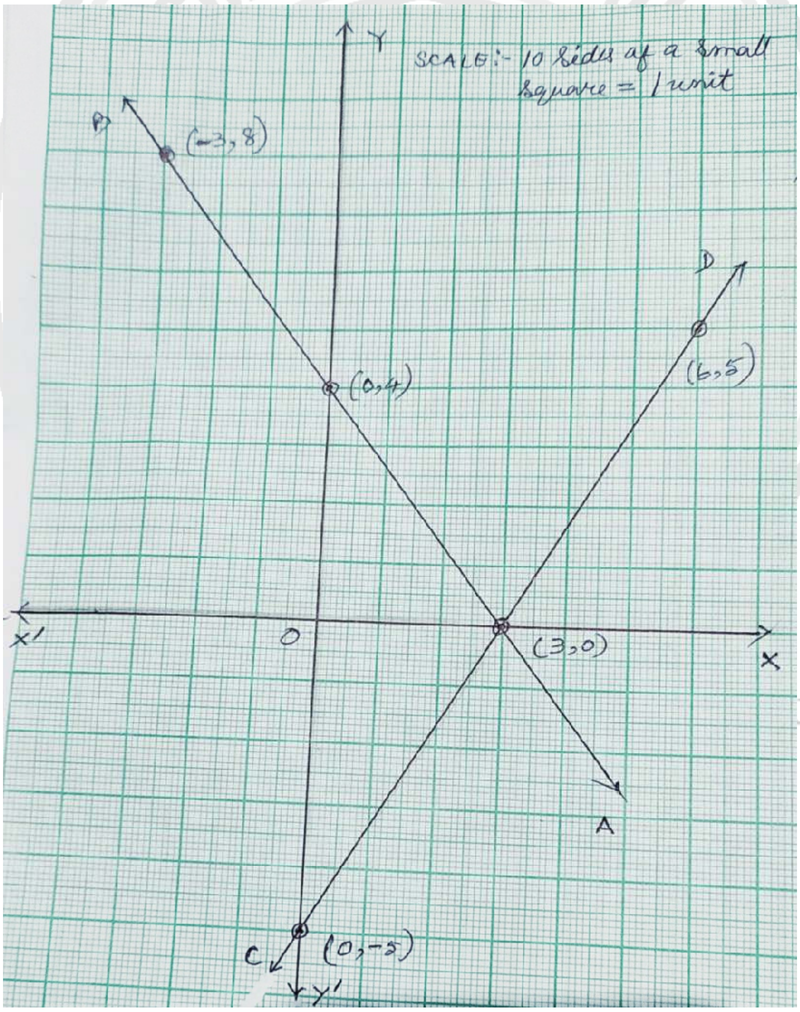
x	-3	0	3
y	8	4	0

Table (I)

Similarly, some of the values of x and y satisfying equation (2) are given in table (II).

x	0	6	3
y	-5	5	0

Table (II)



When we draw the graphs of equations (1) and (2) with the points given by tables (I) and (II) we get two intersecting straight lines AB and CD intersecting at (3,0). Hence, the solution of the given pair of equations is given by $x=3$ and $y=0$.

4. **Find the values of p for which the following pair of equations has a unique solution:**

(i) $\mathbf{4x + py + 5 = 0}$

$\mathbf{2x + 3y + 7 = 0}$

Solution: Here $a_1 = 4$, $b_1 = p$, $a_2 = 2$, $b_2 = 3$

For the given pair of equations to have a unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \frac{4}{2} \neq \frac{p}{3}$$

$$\Rightarrow 2 \neq \frac{p}{3}$$

$$\Rightarrow p \neq 6$$

(ii) $\mathbf{px + 2y = 5}$

$\mathbf{3x + 4y = 1}$

Solution: We have

$$px + 2y = 5 \Rightarrow px + 2y - 5 = 0$$

$$3x + 4y = 1 \Rightarrow 3x + 4y - 1 = 0$$

Here $a_1 = p$, $b_1 = 2$, $a_2 = 3$, $b_2 = 4$

For the given pair of equations to have a unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \frac{p}{3} \neq \frac{2}{4}$$

$$\Rightarrow \frac{p}{3} \neq \frac{1}{2}$$

$$\Rightarrow p \neq \frac{3}{2}$$

(iii) $\mathbf{7x - 5y - 4 = 0}$

$\mathbf{14x + py + 4 = 0}$

Solution: Here $a_1 = 7$, $b_1 = -5$, $a_2 = 14$, $b_2 = p$

For the given pair of equations to have a unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \frac{7}{14} \neq \frac{-5}{p}$$

$$\Rightarrow \frac{1}{2} \neq \frac{-5}{p}$$

$$\Rightarrow p \neq -10$$

5. For what values of a does the pair of equations

$\mathbf{2x + 3y = 7}$

and $\mathbf{(a-1)x + (a+1)y = 3a - 1}$

have infinitely many solutions?

Solution: We have

$$2x + 3y = 7 \Rightarrow 2x + 3y - 7 = 0$$

$$(a-1)x + (a+1)y = 3a - 1 \Rightarrow (a-1)x + (a+1)y - (3a-1) = 0$$

Here , $\frac{a_1}{a_2} = \frac{2}{a-1}$, $\frac{b_1}{b_2} = \frac{3}{a+1}$, $\frac{c_1}{c_2} = \frac{-7}{-(3a-1)}$

For the given pair of equations to have infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \frac{2}{a-1} = \frac{3}{a+1} = \frac{-7}{-(3a-1)} \text{ ---------------- (1)}$$

$$\Rightarrow \frac{2}{a-1} = \frac{3}{a+1} \qquad \text{[Taking the first two ratios]}$$

$$\Rightarrow 3(a-1) = 2(a+1)$$

$$\Rightarrow 3a - 3 = 2a + 2$$

$$\Rightarrow 3a - 2a = 2 + 3$$

$$\Rightarrow a = 5$$

Putting $a = 5$ in the last two ratios of (1), we have

$\frac{3}{5+1} = \frac{-7}{-(3\times5-1)} \Rightarrow \frac{3}{6} = \frac{7}{14} \Rightarrow \frac{1}{2} = \frac{1}{2}$, this is also true.

Hence the given pair of equations has infinitely many solutions when $a = 5$.

6. Find the value of k for which the pair of equations

$$3x + y = 1$$

and $$(2k-1)x + (k-1)y = 2k+1$$

has no solution.

Solution: We have

$$3x + y = 1 \Rightarrow 3x + y - 1 = 0$$

$$(2k-1)x + (k-1)y = 2k+1 \Rightarrow (2k-1)x + (k-1)y - (2k+1) = 0$$

Here, $\frac{a_1}{a_2} = \frac{3}{2k-1}$, $\frac{b_1}{b_2} = \frac{1}{k-1}$, $\frac{c_1}{c_2} = \frac{-1}{-(2k+1)} = \frac{1}{2k+1}$

For the given pair of equations to have no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\Rightarrow \frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$ ---------------- (1)

$\Rightarrow \frac{3}{2k-1} = \frac{1}{k-1}$ [Taking the first two ratios]

$\Rightarrow 3(k-1) = 2k-1$

$\Rightarrow 3k - 3 = 2k - 1$

$\Rightarrow k = 2$

Putting $a = 5$ in the last two ratios of (1), we have

$\frac{1}{2-1} \neq \frac{1}{2\times2+1} \Rightarrow \frac{1}{1} \neq \frac{1}{5}$, this is also true.

Hence the given pair of equations has no solutions when $k = 2$.

➢ Algebraic Method of Solving a Pair of Linear Equations

- **Substitution Method**

Steps to find the solution of a pair of equations in two variables by substitution method:

1. From either equation, whichever is convenient, find the value of one variable, say y in terms of the other variable i.e. x.
2. Substitute the value of y thus obtained in Step 1 in the other equation and reduce it to an equation in only one variable x, which can be solved; and hence obtain the value of x.

   Sometimes after substitution you may get an equality relation with no variable. If this relation is true, you can conclude that the pair of linear equations has infinitely many solutions. If the relation is false, then the pair of linear equations is inconsistent.
3. Substitute the value of x obtained in Step 2 in the equations of Step 1 and obtain the value of y.

- **Elimination method**

Steps to solve of a pair of equations in two variables by elimination method:

1. Multiply or divide both the equations by suitable non-zero constant so that the coefficients of one variable (either x or y) become numerically equal.
2. Then add one equation to the other or subtract one from the other so that one variable gets eliminated. If we get an equation in one variable, go to Step 3.

   If we obtain a true equality relation involving no variable, then the original pair of equations has infinitely many solutions.

   If we obtain a false relation involving no variable, then the original pair of equations has no solution.
3. Solve the equation obtained in Step 2 and obtain the value of the variable which is not eliminated.
4. Substitute the value of the variable obtained in Step 3 in any of the given equations to get the value of the other variable.

- **Cross-Multiplication Method**

The pair of linear equations

$$a_1x + b_1y + c_1 = 0$$

and $a_2x + b_2y + c_2 = 0$ can be combined as

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}, \; a_1b_2 - a_2b_1 \neq 0.$$

Steps to find the solution of a pair of linear equations in two variables by Cross-Multiplication Method:

1. Write the given equations in the general form.
2. Write the pair of equations as $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$.
3. From the equations obtained in Step 2, find the values of x and y, provided $a_1b_2 - a_2b_1 \neq 0$

SOLUTIONS

EXERCISE 4.2

1. Solve the following pair of linear equations by the substitution method:

(i) $\boldsymbol{x + y = 5}$

$\boldsymbol{x - y = 1}$

Solution: The given equations are

$x + y = 5$ --------------- (1)

and $x - y = 1$ --------------- (2)

From equation (1), we get

$y = 5 - x$ --------------- (3)

Substituting this value of y in equation (2), we get

$$x - (5 - x) = 1$$

$$\Rightarrow x - 5 + x = 1$$

$$\Rightarrow 2x = 6$$

$$\Rightarrow x = \frac{6}{2}$$

$$\Rightarrow x = 3$$

Substituting this value of $x = 3$ in equation (3), we get

$$y = 5 - 3$$

$$\Rightarrow y = 2$$

Hence the solution is $x = 3, y = 2$.

(ii) $\boldsymbol{x + y = 7}$
$\boldsymbol{2x + 3y = 18}$

Solution: The given equations are

$x + y = 7$ --------------- (1)

and $2x + 3y = 18$ --------------- (2)

From equation (1), we get

$x = 7 - y$ --------------- (3)

Substituting this value of x in equation (2), we get

$2 \times (7 - y) + 3y = 18$

$\Rightarrow 14 - 2y + 3y = 18$

$\Rightarrow y = 18 - 14$

$\Rightarrow y = 4$

Substituting this value of $y = 4$ in equation (3), we get

$x = 7 - 4 = 3$

Hence the solution is $x = 3, y = 4$.

(iii) $\boldsymbol{3x + 5y = 4}$
$\boldsymbol{4x - 3y = 15}$

Solution: The given equations are

$3x + 5y = 4$ --------------- (1)

and $4x - 3y = 15$ --------------- (2)

From equation (1), we get

$3x = 4 - 5y$

$\Rightarrow x = \frac{4-5y}{3}$ --------------- (3)

Substituting this value of x in equation (2), we get

$4 \times \left(\frac{4-5y}{3}\right) - 3y = 15$

$\Rightarrow \frac{16-20y}{3} - 3y = 15$

$\Rightarrow \frac{16-20y-9y}{3} = 15$

$\Rightarrow \frac{16-29y}{3} = 15$

$\Rightarrow 45 = 16 - 29y$

$\Rightarrow 29y = 16 - 45$

$\Rightarrow 29y = -29$

$\Rightarrow y = \frac{-29}{29} = -1$

Substituting this value of $y = -1$ in equation (3), we get

$$x = \frac{4-5\times(-1)}{3}$$

$$\Rightarrow x = \frac{4+5}{3}$$

$$\Rightarrow x = \frac{9}{3}$$

$$\Rightarrow x = 3$$

Hence the solution is $x = 3, y = -1$.

(iv) $\mathbf{3x - 4y - 11 = 0}$

$\mathbf{5x + 3y + 1 = 0}$

Solution: The given equations are

$3x - 4y - 11 = 0$ ---------------- (1)

and $5x + 3y + 1 = 0$ ---------------- (2)

From equation (1), we get

$$3x = 4y + 11$$

$\Rightarrow x = \frac{4y+11}{3}$ ---------------- (3)

Substituting this value of x in equation (2), we get

$$5 \times \left(\frac{4y+11}{3}\right) + 3y + 1 = 0$$

$$\Rightarrow \frac{20y+55}{3} + 3y = -1$$

$$\Rightarrow \frac{20y+55+9y}{3} = -1$$

$$\Rightarrow \frac{29y+55}{3} = -1$$

$$\Rightarrow 29y + 55 = -3$$

$$\Rightarrow 29y = -58$$

$$\Rightarrow y = \frac{-58}{29}$$

$$\Rightarrow y = -2$$

Substituting this value of $y = -2$ in equation (3), we get

$$x = \frac{4\times(-2)+11}{3}$$

$$\Rightarrow x = \frac{-8+11}{3}$$

$$\Rightarrow x = \frac{3}{3}$$

$$\Rightarrow x = 1$$

Hence the solution is $x = 1, y = -2$.

(v) $\mathbf{2x + 3y = 0}$
$\mathbf{3x - 2y = 13}$

Solution: The given equations are

$2x + 3y = 0$ --------------- (1)

and $3x - 2y = 13$ --------------- (2)

From equation (1), we get

$2x = -3y$

$\Rightarrow x = \frac{-3y}{2}$ --------------- (3)

Substituting this value of x in equation (2), we get

$3 \times \left(\frac{-3y}{2}\right) - 2y = 13$

$\Rightarrow \frac{-9y}{2} - 2y = 13$

$\Rightarrow \frac{-9y-4y}{2} = 13$

$\Rightarrow \frac{-13y}{2} = 13$

$\Rightarrow y = 13 \times \left(-\frac{2}{13}\right)$

$\Rightarrow y = -2$

Substituting this value of $y = -2$ in equation (3), we get

$x = \frac{-3\times(-2)}{2}$

$\Rightarrow x = \frac{6}{2}$

$\Rightarrow x = 3$

Hence the solution is $x = 3, y = -2$.

(vi) $\mathbf{x + 2y = 3}$
$\mathbf{2x + y = 0}$

Solution: The given equations are

$x + 2y = 3$ --------------- (1)

and $2x + y = 0$--------------- (2)

From equation (2), we get

$y = -2x$ --------------- (3)

Substituting this value of y in equation (1), we get

$x + 2 \times (-2x) = 3$

$\Rightarrow x - 4x = 3$

$\Rightarrow -3x = 3$

$\Rightarrow x = -\frac{3}{3} = -1$

Substituting this value of $x = -1$ in equation (3), we get

$$y = -2 \times (-1) = 2$$

Hence the solution is $x = -1, y = 2$.

(vii) $\boldsymbol{x - y = 3}$
$\boldsymbol{\frac{x}{3} + \frac{y}{2} = 6}$

Solution: The given equations are

$x - y = 3$ --------------- (1)

and $\frac{x}{3} + \frac{y}{2} = 6$ --------------- (2)

From equation (1), we get

$x = y + 3$ --------------- (3)

Substituting this value of y in equation (2), we get

$$\frac{y+3}{3} + \frac{y}{2} = 6$$

$$\Rightarrow \frac{2y+6+3y}{6} = 6$$

$$\Rightarrow \frac{5y+6}{6} = 6$$

$$\Rightarrow 5y + 6 = 36$$

$$\Rightarrow 5y = 30$$

$$\Rightarrow y = 6$$

Substituting this value of $y = 6$ in equation (3), we get

$$x = 6 + 3$$

$$\Rightarrow x = 9$$

Hence the solution is $x = 9, y = 6$.

(viii) $\boldsymbol{3x - y = 3}$
$\boldsymbol{6x - 2y = 6}$

Solution: The given equations are

$3x - y = 3$ --------------- (1)

and $6x - 2y = 6$ --------------- (2)

From equation (1), we get

$\Rightarrow y = 3x - 3$ --------------- (3)

Substituting this value of y in equation (2), we get

$$6x - 2(3x - 3) = 6$$

$$\Rightarrow 6x - 6x + 6 = 6$$

$\Rightarrow 6 = 6$, which is true.

Hence the given pair of equations has infinitely many solutions.

(ix) $\boldsymbol{x + 2y = 5}$

$\boldsymbol{3x + 6y = 7}$

Solution: The given equations are

$x + 2y = 5$ --------------- (1)

and $3x + 6y = 7$ --------------- (2)

From equation (1), we get

$x = 5 - 2y$ --------------- (3)

Substituting this value of x in equation (2), we get

$3x + 6y = 7$

$\Rightarrow 3(5 - 2y) + 6y = 7$

$\Rightarrow 15 - 6y + 6y = 7$

$\Rightarrow 15 = 7$, which is false.

Hence the given pair of equations has no solution.

(x) $\boldsymbol{\sqrt{2}x + \sqrt{3}y = 0}$

$\boldsymbol{\sqrt{3}x - 2\sqrt{2}y = 0}$

Solution: The given equations are

$\sqrt{2}x + \sqrt{3}y = 0$ --------------- (1)

and $\sqrt{3}x - 2\sqrt{2}y = 0$ --------------- (2)

From equation (1), we get

$\sqrt{3}y = -\sqrt{2}x$

$\Rightarrow y = \frac{-\sqrt{2}x}{\sqrt{3}}$ --------------- (3)

Substituting this value of y in equation (2), we get

$\sqrt{3}x - 2\sqrt{2} \times \left(\frac{-\sqrt{2}x}{\sqrt{3}}\right) = 0$

$\Rightarrow \sqrt{3}x + \frac{4x}{\sqrt{3}} = 0$

$\Rightarrow 3x + 4x = 0$ [Multiplying both sides by $\sqrt{3}$]

$\Rightarrow 7x = 0$

$\Rightarrow x = 0$

Substituting this value of $x = 0$ in equation (3), we get

$y = \frac{-\sqrt{2}}{\sqrt{3}} \times 0$

$\Rightarrow y = 0$

Hence the solution is $x = 0, y = 0$.

(xi) $\mathbf{17x + 12y = 27}$

$\mathbf{12x + 17y = 2}$

Solution: The given equations are

$17x + 12y = 27$ --------------- (1)

and $12x + 17y = 2$ --------------- (2)

From equation (1), we get

$12y = 27 - 17x$

$\Rightarrow y = \frac{27-17x}{12}$ --------------- (3)

Substituting this value of y in equation (2), we get

$12x + 17\left(\frac{27-17x}{12}\right) = 2$

$\Rightarrow 144x + 17(27 - 17x) = 24$ [Multiplying both sides by 12]

$\Rightarrow 144x + 459 - 289x = 24$

$\Rightarrow -145x = 24 - 459$

$\Rightarrow -145x = -435$

$\Rightarrow x = \frac{-435}{-145}$

$\Rightarrow x = 3$

Substituting this value of $x = 3$ in equation (3), we get

$y = \frac{27-17x}{12}$

$\Rightarrow y = \frac{27-17\times3}{12}$

$\Rightarrow y = \frac{27-51}{12}$

$\Rightarrow y = \frac{-24}{12}$

$\Rightarrow y = -2$

Hence the solution is $x = 3, y = -2$.

(xii) $\mathbf{2x - 3y = 11}$

$\mathbf{3x + 4y = 8}$

Solution: The given equations are

$2x - 3y = 11$--------------- (1)

and $3x + 4y = 8$ --------------- (2)

From equation (1), we get

$2x = 11 + 3y$

$\Rightarrow x = \frac{11+3y}{2}$ --------------- (3)

Substituting this value of x in equation (2), we get

$$3\left(\frac{11+3y}{2}\right)+4y=8$$

$$\Rightarrow 3(11+3y)+8y=16$$

$$\Rightarrow 33+9y+8y=16$$

$$\Rightarrow 17y=16-33$$

$$\Rightarrow 17y=-17$$

$$\Rightarrow y=\frac{-17}{17}$$

$$\Rightarrow y=-1$$

Substituting this value of $y=-1$ in equation (3), we get

$$x=\frac{11+3\times(-1)}{2}$$

$$\Rightarrow x=\frac{11-3}{2}$$

$$\Rightarrow x=\frac{8}{2}$$

$$\Rightarrow x=4$$

Hence the solution is $x=4, y=-1$.

2. Solve the following pair of linear equations by the elimination method:

(i) $\boldsymbol{x+y=14}$

$\boldsymbol{x-y=4}$

Solution: The given equations are

$x+y=14$--------------- (1)

and $x-y=4$ --------------- (2)

Adding equations (1) and (2), we get

$$2x=18$$

$$\Rightarrow x=9$$

Substituting the value of x in equation (1), we get

$$9+y=14$$

$$\Rightarrow y=5$$

Hence the solution is $x=9, y=5$.

(ii) $\boldsymbol{x + y = 3}$

$\boldsymbol{7x - 3y = 41}$

Solution: The given equations are

$x + y = 3$--------------- (1)

and $7x - 3y = 41$ --------------- (2)

Multiplying equation (1) by 3, we get

$3x + 3y = 9$ --------------- (3)

Adding equations (2) and (3), we get

$10x = 50$

$\Rightarrow x = 5$

Substituting the value of x in equation (1), we get

$5 + y = 3$

$\Rightarrow y = 3 - 5$

$\Rightarrow y = -2$

Hence the solution is $x = 5, y = -2$.

(iii) $\boldsymbol{x + 2y = 0}$

$\boldsymbol{3x - y = 7}$

Solution: The given equations are

$x + 2y = 0$--------------- (1)

and $3x - y = 7$ --------------- (2)

Multiplying equation (2) by 2, we get

$6x - 2y = 14$ --------------- (3)

Adding equations (1) and (3), we get

$7x = 14$

$\Rightarrow x = 2$

Substituting the value of x in equation (1), we get

$2 + 2y = 0$

$\Rightarrow 2y = -2$

$\Rightarrow y = -1$

Hence the solution is $x = 2, y = -1$.

(iv) $\mathbf{8u - 9v = 20}$
$\mathbf{7u - 10v = 9}$

Solution: The given equations are

$8u - 9v = 20$ --------------- (1)

and $7u - 10v = 9$ --------------- (2)

Multiplying equation (1) by 10, we get

$80u - 90v = 200$ --------------- (3)

Multiplying equation (2) by 9, we get

$63u - 90v = 81$ --------------- (4)

Subtracting equations (4) from equation (3), we get

$(80u - 90v) - (63u - 90v) = 200 - 81$

$\Rightarrow 80u - 90v - 63u + 90v = 119$

$\Rightarrow 17u = 119$

$\Rightarrow u = \frac{119}{17}$

$\Rightarrow u = 7$

Substituting the value of u in equation (2), we get

$7 \times 7 - 10v = 9$

$\Rightarrow 49 - 10v = 9$

$\Rightarrow 49 - 9 = 10v$

$\Rightarrow 10v = 40$

$\Rightarrow v = 4$

Hence the solution is $u = 7, v = 4$.

(v) $\mathbf{5x + 2y = 4}$

$\mathbf{7x + y = 5}$

Solution: The given equations are

$5x + 2y = 4$ --------------- (1)

and $7x + y = 5$ --------------- (2)

Multiplying equation (2) by 2, we get

$14x + 2y = 10$ --------------- (3)

Subtracting equation (1) from equation (3), we get

$9x = 6$

$\Rightarrow x = \frac{6}{9}$

$\Rightarrow x = \frac{2}{3}$

Substituting the value of x in equation (1), we get

$$5 \times \frac{2}{3} + 2y = 4$$

$$\Rightarrow \frac{10}{3} + 2y = 4$$

$$\Rightarrow 2y = 4 - \frac{10}{3}$$

$$\Rightarrow 2y = \frac{12-10}{3}$$

$$\Rightarrow 2y = \frac{2}{3}$$

$$\Rightarrow y = \frac{1}{3}$$

Hence the solution is $x = \frac{2}{3}, y = \frac{1}{3}$.

(vi) $\mathbf{4u + 7v = 21}$

$\mathbf{21u - 13v = 160}$

Solution: The given equations are

$4u + 7v = 21$ ---------------- (1)

and $21u - 13v = 160$ ---------------- (2)

Multiplying equation (1) by 13, we get

$52u + 91v = 273$ ---------------- (3)

Multiplying equation (2) by 7, we get

$147u - 91v = 1120$ ---------------- (4)

Adding equations (3) and (4), we get

$$52u + 91v + 147u - 91v = 273 + 1120$$

$$\Rightarrow 199u = 1393$$

$$\Rightarrow u = \frac{1393}{199}$$

$$\Rightarrow u = 7$$

Substituting the value of u in equation (1), we get

$$4 \times 7 + 7v = 21$$

$$\Rightarrow 28 + 7v = 21$$

$$\Rightarrow 7v = 21 - 28$$

$$\Rightarrow 7v = -7$$

$$\Rightarrow v = \frac{-7}{7}$$

$$\Rightarrow v = -1$$

Hence the solution is $u = 7, v = -1$.

(vii) $\mathbf{3x + 5y = 7}$

$\mathbf{12x - 13y = -5}$

Solution: The given equations are

$3x + 5y = 7$ --------------- (1)

and $12x - 13y = -5$ --------------- (2)

Multiplying equation (1) by 4, we get

$12x + 20y = 28$ --------------- (3)

Subtracting equation (2) from equation (3), we get

$(12x + 20y) - (12x - 13y) = 28 - (-5)$

$\Rightarrow 12x + 20y - 12x + 13y = 28 + 5$

$\Rightarrow 33y = 33$

$\Rightarrow y = 1$

Substituting the value of y in equation (1), we get

$3x + 5 \times 1 = 7$

$\Rightarrow 3x + 5 = 7$

$\Rightarrow 3x = 2$

$\Rightarrow x = \frac{2}{3}$

Hence the solution is $x = \frac{2}{3}, y = 1$.

(viii) $\mathbf{4x - 3y = 10}$

$\mathbf{-5x + 4y = -13}$

Solution: The given equations are

$4x - 3y = 10$ --------------- (1)

and $-5x + 4y = -13$ --------------- (2)

Multiplying equation (1) by 4, we get

$16x - 12y = 40$ --------------- (3)

Multiplying equation (2) by 3, we get

$-15x + 12y = -39$ --------------- (4)

Adding equations (3) and (4), we get

$(16x - 12y) + (-15x + 12y) = 40 + (-39)$

$\Rightarrow 16x - 12y - 15x + 12y = 40 - 39$

$\Rightarrow x = 1$

Substituting the value of x in equation (1), we get

$$4 \times 1 - 3y = 10$$

$$\Rightarrow 4 - 3y = 10$$

$$\Rightarrow -3y = 6$$

$$\Rightarrow y = \frac{6}{-3}$$

$$\Rightarrow y = -2$$

Hence the solution is $x = 1, y = -2$.

(ix) $\boldsymbol{5x + 2y = 4}$

$\boldsymbol{10x + 4y = 8}$

Solution: The given equations are

$5x + 2y = 4$ --------------- (1)

and $10x + 4y = 8$ --------------- (2)

Multiplying equation (1) by 2, we get

$10x + 4y = 8$ --------------- (3)

Subtracting equation (2) from equation (3), we get

$$(10x + 4y) - (10x + 4y) = 8 - 8$$

$\Rightarrow 0 = 0$, which is true.

Hence the given pair of equations has infinitely many solutions.

(x) $\boldsymbol{2x - 5y = 6}$

$\boldsymbol{4x - 10y = 9}$

Solution: The given equations are

$2x - 5y = 6$ --------------- (1)

and $4x - 10y = 9$ --------------- (2)

Multiplying equation (1) by 2, we get

$4x - 10y = 12$ --------------- (3)

Subtracting equation (2) from equation (3), we get

$$(4x - 10y) - (4x - 10y) = 12 - 9$$

$\Rightarrow 0 = 3$, which is false.

Hence the given pair of equations has no solution.

(xi) $\frac{x}{2}+\frac{2y}{3}=-1$

$\frac{x}{3}-\frac{y}{9}=1$

Solution: The given equations are

$\frac{x}{2}+\frac{2y}{3}=-1$

$\Rightarrow \frac{3x+4y}{6}=-1$

$\Rightarrow 3x+4y=-6$ --------------- (1)

and $\frac{x}{3}-\frac{y}{9}=1$

$\Rightarrow \frac{3x-y}{9}=1$

$\Rightarrow 3x-y=9$ --------------- (2)

Subtracting equation (2) from equation (1), we get

$(3x+4y)-(3x-y)=-6-9$

$\Rightarrow 3x+4y-3x+y=-15$

$\Rightarrow 5y=-15$

$\Rightarrow y=\frac{-15}{5}$

$\Rightarrow y=-3$

Substituting the value of y in equation (2), we get

$3x-(-3)=9$

$\Rightarrow 3x+3=9$

$\Rightarrow 3x=6$

$\Rightarrow x=2$

Hence the solution is $x=2, y=-3$.

(xii) $3x-5y+2=0$

$9x=2y+7$

Solution: The given equations are

$3x-5y+2=0$

$\Rightarrow 3x-5y=-2$ --------------- (1)

and $9x=2y+7$

$\Rightarrow 9x-2y=7$ --------------- (2)

Multiplying equation (1) by 3, we get

$9x-15y=-6$ --------------- (3)

Subtracting equation (3) from equation (2), we get

$$(9x - 2y) - (9x - 15y) = 7 - (-6)$$

$$\Rightarrow 9x - 2y - 9x + 15y = 7 + 6$$

$$\Rightarrow 13y = 13$$

$$\Rightarrow y = 1$$

Substituting the value of y in equation (1), we get

$$3x - 5y = -2$$

$$\Rightarrow 3x - 5 \times 1 = -2$$

$$\Rightarrow 3x - 5 = -2$$

$$\Rightarrow 3x = -2 + 5$$

$$\Rightarrow 3x = 3$$

$$\Rightarrow x = 1$$

Hence the solution is $x = 1, y = 1$.

3. Examine whether the following pairs of linear equations have a unique solution, no solution or infinitely many solutions. Not? In case there is a unique solution, find it by using cross-multiplication method:

(i) $\boldsymbol{2x - 3y - 4 = 0}$

$\boldsymbol{4x - 6y + 5 = 0}$

Solution: Comparing the given pair of equations with the general form, we get

$a_1 = 2,\quad b_1 = -3,\quad c_1 = -4$

and $a_2 = 4,\quad b_2 = -6,\quad c_2 = 5$

$$\therefore \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2},\quad \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2},\quad \frac{c_1}{c_2} = \frac{-4}{5}$$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence the given pair of equations has no solution.

(ii) $\boldsymbol{4x - 3y = 5}$

$\boldsymbol{3x - 5y = 1}$

Solution: The given equations can be written as

$$4x - 3y - 5 = 0$$

$$3x - 5y - 1 = 0$$

Here $a_1 = 4,\quad b_1 = -3,\quad c_1 = -5$

and $a_2 = 3,\quad b_2 = -5,\quad c_2 = -1$

$$\therefore \frac{a_1}{a_2} = \frac{4}{3},\ \frac{b_1}{b_2} = \frac{-3}{-5} = \frac{3}{5},\quad \frac{c_1}{c_2} = \frac{-5}{-1} = 5$$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

∴ the given pair of equations has a unique solution.

Then by cross-multiplication method, we get

$$\frac{x}{(-3)\times(-1)-(-5)\times(-5)} = \frac{y}{(-5)\times 3-(-1)\times 4} = \frac{1}{4\times(-5)-3\times(-3)}$$

$$\Rightarrow \frac{x}{3-25} = \frac{y}{-15+4} = \frac{1}{-20+9}$$

$$\Rightarrow \frac{x}{-22} = \frac{y}{-11} = \frac{1}{-11}$$

$$\Rightarrow x = \frac{-22}{-11} \text{ and } y = \frac{-11}{-11}$$

$$\therefore x = 2 \text{ and } y = 1$$

(iii) $\boldsymbol{2x + y - 5 = 0}$

$\boldsymbol{3x + 2y - 8 = 0}$

Solution: Comparing the given pair of equations with the general form, we get

$a_1 = 2,\quad b_1 = 1,\quad c_1 = -5$

and $a_2 = 3,\quad b_2 = 2,\quad c_2 = -8$

$\therefore \frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-5}{-8} = \frac{5}{8}$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence the given pair of equations has a unique solution.

Then by cross-multiplication method, we get

$$\frac{x}{1\times(-8)-2\times(-5)} = \frac{y}{(-5)\times 3-(-8)\times 2} = \frac{1}{2\times 2-3\times 1}$$

$$\Rightarrow \frac{x}{-8+10} = \frac{y}{-15+16} = \frac{1}{4-3}$$

$$\Rightarrow \frac{x}{2} = y = 1$$

$$\therefore x = 2 \text{ and } y = 1$$

(iv) $\boldsymbol{x - 2y + 3 = 0}$

$\boldsymbol{3x - 6y + 9 = 0}$

Solution: Comparing the given pair of equations with the general form, we get

$a_1 = 1,\quad b_1 = -2,\quad c_1 = 3$

and $a_2 = 3,\quad b_2 = -6,\quad c_2 = 9$

$\therefore \frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-2}{-6} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{3}{9} = \frac{1}{3}$

We see that $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Hence the given pair of equations has infinitely many solutions.

(v) $\boldsymbol{x - 3y = 7}$
$\boldsymbol{x - y = 5}$

Solution: The given equations can be written as

$$x - 3y - 7 = 0$$
$$x - y - 5 = 0$$

Here $a_1 = 1,\quad b_1 = -3,\quad c_1 = -7$

and $a_2 = 1,\quad b_2 = -1,\quad c_2 = -5$

$\therefore \frac{a_1}{a_2} = \frac{1}{1} = 1,\quad \frac{b_1}{b_2} = \frac{-3}{-1} = 3,\quad \frac{c_1}{c_2} = \frac{-7}{-5} = \frac{7}{5}$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ the given pair of equations has a unique solution.

Then by cross-multiplication method, we get

$$\frac{x}{(-3)\times(-5)-(-1)\times(-7)} = \frac{y}{(-7)\times1-(-5)\times1} = \frac{1}{1\times(-1)-1\times(-3)}$$

$$\Rightarrow \frac{x}{15-7} = \frac{y}{-7+5} = \frac{1}{-1+3}$$

$$\Rightarrow \frac{x}{8} = \frac{y}{-2} = \frac{1}{2}$$

$$\Rightarrow x = \frac{8}{2} \text{ and } y = \frac{-2}{2}$$

$$\therefore x = 4 \text{ and } y = -1$$

(vi) $\boldsymbol{ax + by = c^2}$
$\boldsymbol{\frac{x+a}{b} - \frac{y+b}{a} = 0}$

Solution: The given equations are

$$ax + by = c^2$$

$$\Rightarrow ax + by - c^2 = 0 \text{ --------------- (1)}$$

and $\frac{x+a}{b} - \frac{y+b}{a} = 0$

$$\Rightarrow \frac{a(x+a)-b(y+b)}{ab} = 0$$

$$\Rightarrow a(x+a) - b(y+b) = 0$$

$$\Rightarrow ax + a^2 - by - b^2 = 0$$

$$\Rightarrow ax - by + a^2 - b^2 = 0 \text{ --------------- (2)}$$

Comparing the pair of equations (1) and (2) with the general form, we get

$a_1 = a,\quad b_1 = b,\quad c_1 = -c^2$

and $a_2 = a,\quad b_2 = -b,\quad c_2 = a^2 - b^2$

$\therefore \frac{a_1}{a_2} = \frac{a}{a} = 1,\quad \frac{b_1}{b_2} = \frac{b}{-b} = -1,\quad \frac{c_1}{c_2} = \frac{-c^2}{a^2-b^2}$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ the given pair of equations has a unique solution.

Then from equations (1) and (2), by cross-multiplication method, we get

$$\frac{x}{b(a^2-b^2)-(-b)\times(-c^2)} = \frac{y}{(-c^2)a-(a^2-b^2)a} = \frac{1}{a(-b)-ab}$$

$$\Rightarrow \frac{x}{b(a^2-b^2)-(-b)\times(-c^2)} = \frac{y}{(-c^2)a-(a^2-b^2)a} = \frac{1}{a(-b)-ab}$$

$$\Rightarrow \frac{x}{b(a^2-b^2)-bc^2} = \frac{y}{-c^2a-a(a^2-b^2)} = \frac{1}{-ab-ab}$$

$$\Rightarrow \frac{x}{b(a^2-b^2-c^2)} = \frac{y}{-a(c^2-a^2+b^2)} = \frac{1}{-2ab}$$

$$\Rightarrow \frac{x}{b(a^2-b^2-c^2)} = \frac{1}{-2ab} \text{ and } \frac{y}{-a(c^2-a^2+b^2)} = \frac{1}{-2ab}$$

$$\Rightarrow x = \frac{b(a^2-b^2-c^2)}{-2ab} \text{ and } y = \frac{-a(c^2-a^2+b^2)}{-2ab}$$

$$\Rightarrow x = \frac{-(a^2-b^2-c^2)}{2a} \text{ and } y = \frac{c^2-a^2+b^2}{2b}$$

$$\therefore x = \frac{b^2+c^2-a^2}{2a} \text{ and } y = \frac{c^2-a^2+b^2}{2b}$$

(vii) $\boldsymbol{7x - 5y = 11}$

$\boldsymbol{3x + 2y = 13}$

Solution: The given equations can be written as

$7x - 5y - 11 = 0$

$3x + 2y - 13 = 0$

Here $a_1 = 7$, $b_1 = -5$, $c_1 = -11$

and $a_2 = 3$, $b_2 = 2$, $c_2 = -13$

$\therefore \frac{a_1}{a_2} = \frac{7}{3}$, $\frac{b_1}{b_2} = \frac{-3}{2}$, $\frac{c_1}{c_2} = \frac{-11}{-13} = \frac{11}{13}$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ the given pair of equations has a unique solution.

Then by cross-multiplication method, we get

$$\frac{x}{(-5)\times(-13)-2\times(-11)} = \frac{y}{(-11)\times3-(-13)\times7} = \frac{1}{7\times2-3\times(-5)}$$

$$\Rightarrow \frac{x}{65+22} = \frac{y}{-33+91} = \frac{1}{14+15}$$

$$\Rightarrow \frac{x}{87} = \frac{y}{58} = \frac{1}{29}$$

$$\Rightarrow x = \frac{87}{29} \text{ and } y = \frac{58}{29}$$

$$\therefore x = 3 \text{ and } y = 2$$

(viii) $\boldsymbol{2x + 3y - 8 = 0}$

$\boldsymbol{3x - 4y + 5 = 0}$

Solution: Comparing the given pair of equations with the general form, we get

$a_1 = 2, \qquad b_1 = 3, \qquad c_1 = -8$

and $a_2 = 3, \qquad b_2 = -4, \qquad c_2 = 5$

$\therefore \frac{a_1}{a_2} = \frac{2}{3}, \qquad \frac{b_1}{b_2} = \frac{3}{-4} = -\frac{3}{4}, \qquad \frac{c_1}{c_2} = \frac{-8}{5}$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ the given pair of equations has a unique solution.

Then by cross-multiplication method, we get

$$\frac{x}{3\times5-(-4)\times(-8)} = \frac{y}{(-8)\times3-5\times2} = \frac{1}{2\times(-4)-3\times3}$$

$$\Rightarrow \frac{x}{15-32} = \frac{y}{-24-10} = \frac{1}{-8-9}$$

$$\Rightarrow \frac{x}{-17} = \frac{y}{-34} = \frac{1}{-17}$$

$$\Rightarrow x = \frac{-17}{-17} \text{ and } y = \frac{-34}{-17}$$

$$\therefore x = 1 \text{ and } y = 2$$

(ix) $\boldsymbol{\frac{x}{a} + \frac{y}{b} = 2}$

$\boldsymbol{ax - by = a^2 - b^2}$

Solution: The given equations are

$$\frac{x}{a} + \frac{y}{b} = 2$$

$$\Rightarrow \frac{bx+ay}{ab} = 2$$

$$\Rightarrow bx + ay = 2ab$$

$\Rightarrow bx + ay - 2ab = 0$ --------------- (1)

and $ax - by = a^2 - b^2$

$\Rightarrow ax - by - (a^2 - b^2) = 0$ --------------- (2)

Comparing the pair of equations (1) and (2) with the general form, we get

$a_1 = b, \qquad b_1 = a, \qquad c_1 = -2ab$

and $a_2 = a, \qquad b_2 = -b, \qquad c_2 = -(a^2 - b^2)$

$\therefore \frac{a_1}{a_2} = \frac{b}{a}, \qquad \frac{b_1}{b_2} = \frac{a}{-b} = -\frac{a}{b}, \qquad \frac{c_1}{c_2} = \frac{-2ab}{-(a^2-b^2)} = \frac{2ab}{a^2-b^2}$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ the given pair of equations has a unique solution.

Then from equations (1) and (2), by cross-multiplication method, we get

$$\frac{x}{a\{-(a^2-b^2)\}-(-b)\times(-2ab)} = \frac{y}{-2aba-\{-(a^2-b^2)\}b} = \frac{1}{b(-b)-a.a}$$

$$\Rightarrow \frac{x}{-a(a^2-b^2)-2ab^2} = \frac{y}{-2a^2b+b(a^2-b^2)} = \frac{1}{-b^2-a^2}$$

$$\Rightarrow \frac{x}{-a\{a^2-b^2+2b^2\}} = \frac{y}{b(a^2-b^2-2a^2)} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \frac{x}{-a(a^2+b^2)} = \frac{y}{-b(a^2+b^2)} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \frac{x}{-a(a^2+b^2)} = \frac{1}{-(a^2+b^2)} \text{ and } \frac{y}{-b(a^2+b^2)} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow x = \frac{-a(a^2+b^2)}{-(a^2+b^2)} \text{ and } y = \frac{-b(a^2+b^2)}{-(a^2+b^2)}$$

$\therefore x = a$ and $y = b$

(x) $\boldsymbol{ax + by = a - b}$
$\boldsymbol{bx - ay = a + b}$

Solution: The given equations are

$ax + by = a - b$

$\Rightarrow ax + by - (a - b) = 0$ --------------- (1)

and $bx - ay = a + b$

$\Rightarrow bx - ay - (a + b) = 0$ --------------- (2)

Comparing the pair of equations (1) and (2) with the general form, we get

$a_1 = a,\quad b_1 = b,\quad c_1 = -(a - b)$

and $a_2 = b,\quad b_2 = -a,\quad c_2 = -(a + b)$

$\therefore \frac{a_1}{a_2} = \frac{a}{b},\quad \frac{b_1}{b_2} = \frac{b}{-a} = -\frac{b}{a},\quad \frac{c_1}{c_2} = \frac{-(a-b)}{-(a+b))} = \frac{a-b}{a+b}$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ the given pair of equations has a unique solution.

Then from equations (1) and (2), by cross-multiplication method, we get

$$\frac{x}{b\{-(a+b)\}-(-a)\times\{-(a-b)\}} = \frac{y}{-(a-b)b-\{-(a+b)\}a} = \frac{1}{a(-a)-b.b}$$

$$\Rightarrow \frac{x}{-b(a+b)-a(a-b)} = \frac{y}{-b(a-b)+a(a+b)} = \frac{1}{-a^2-b^2}$$

$$\Rightarrow \frac{x}{-ab-b^2-a^2+ab} = \frac{y}{-ab+b^2+a^2+ab} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \frac{x}{-(a^2+b^2)} = \frac{y}{a^2+b^2} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow \frac{x}{-(a^2+b^2)} = \frac{1}{-(a^2+b^2)} \text{ and } \frac{y}{a^2+b^2} = \frac{1}{-(a^2+b^2)}$$

$$\Rightarrow x = \frac{-(a^2+b^2)}{-(a^2+b^2)} \text{ and } y = \frac{a^2+b^2}{-(a^2+b^2)}$$

$\therefore x = 1$ and $y = -1$

(xi) $\frac{x}{a} + \frac{y}{b} = a + b$

$\frac{x}{a^2} + \frac{y}{b^2} = 2$

Solution: The given equations are

$$\frac{x}{a} + \frac{y}{b} = a + b$$

$$\Rightarrow \frac{bx+ay}{ab} = a + b$$

$$\Rightarrow bx + ay = ab(a + b)$$

$$\Rightarrow bx + ay - ab(a + b) = 0 \text{ --------------- (1)}$$

and $\frac{x}{a^2} + \frac{y}{b^2} = 2$

$$\Rightarrow \frac{b^2x+a^2y}{a^2b^2} = 2$$

$$\Rightarrow b^2x + a^2y = 2a^2b^2$$

$$\Rightarrow b^2x + a^2y - 2a^2b^2 = 0 \text{ --------------- (2)}$$

Comparing the pair of equations (1) and (2) with the general form, we get

$a_1 = b$, $b_1 = a$, $c_1 = -ab(a + b)$

and $a_2 = b^2$, $b_2 = a^2$, $c_2 = -2a^2b^2$

$\therefore \frac{a_1}{a_2} = \frac{b}{b^2} = \frac{1}{b}$, $\frac{b_1}{b_2} = \frac{a}{a^2} = \frac{1}{a}$, $\frac{c_1}{c_2} = \frac{-ab(a+b)}{-2a^2b^2} = \frac{a+b}{2ab}$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ the given pair of equations has a unique solution.

Then from equations (1) and (2), by cross-multiplication method, we get

$$\frac{x}{a(-2a^2b^2)-a^2\{-ab(a+b)\}} = \frac{y}{-ab(a+b)b^2-(-2a^2b^2)b} = \frac{1}{b.a^2-b^2.a}$$

$$\Rightarrow \frac{x}{-2a^3b^2+a^3b(a+b)} = \frac{y}{-ab^3(a+b)+2a^2b^3} = \frac{1}{ab(a-b)}$$

$$\Rightarrow \frac{x}{a^3b(-2b+a+b)} = \frac{y}{-ab^3(a+b-2a)} = \frac{1}{ab(a-b)}$$

$$\Rightarrow \frac{x}{a^3b(a-b)} = \frac{y}{-ab^3(-a+b)} = \frac{1}{ab(a-b)}$$

$$\Rightarrow \frac{x}{a^3b(a-b)} = \frac{y}{ab^3(a-b)} = \frac{1}{ab(a-b)}$$

$$\Rightarrow \frac{x}{a^3b(a-b)} = \frac{1}{ab(a-b)} \text{ and } \frac{y}{ab^3(a-b)} = \frac{1}{ab(a-b)}$$

$$\Rightarrow x = \frac{a^3b(a-b)}{ab(a-b)} \text{ and } y = \frac{ab^3(a-b)}{ab(a-b)}$$

$$\therefore x = a^2 \text{ and } y = b^2$$

(xii) $\frac{x}{a} - \frac{y}{b} = 0$

$ax + by = a^2 + b^2$

Solution: The given equations are

$$\frac{x}{a} - \frac{y}{b} = 0$$

$$\Rightarrow \frac{bx-ay}{ab} = 0$$

$$\Rightarrow bx - ay = 0$$

$$\Rightarrow bx - ay + 0 = 0 \text{ --------------- (1)}$$

and $ax + by = a^2 + b^2$

$$\Rightarrow ax + by - (a^2 + b^2) = 0 \text{ --------------- (2)}$$

Comparing the pair of equations (1) and (2) with the general form, we get

$a_1 = b,\quad b_1 = -a,\quad c_1 = 0$

and $a_2 = a,\quad b_2 = b,\quad c_2 = -(a^2 + b^2)$

$\therefore \frac{a_1}{a_2} = \frac{b}{a},\quad \frac{b_1}{b_2} = \frac{-a}{b},\quad \frac{c_1}{c_2} = \frac{0}{-(a^2+b^2)} = 0$

We see that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\therefore$ the given pair of equations has a unique solution.

Then from equations (1) and (2), by cross-multiplication method, we get

$$\frac{x}{-a\{-(a^2+b^2)\}-b.0} = \frac{y}{0.a-\{-(a^2+b^2)\}\text{b}} = \frac{1}{b.b-a.(-a)}$$

$$\Rightarrow \frac{x}{a(a^2+b^2)} = \frac{y}{b(a^2+b^2)} = \frac{1}{b^2+a^2}$$

$$\Rightarrow \frac{x}{a(a^2+b^2)} = \frac{1}{a^2+b^2} = \frac{y}{b(a^2+b^2)} = \frac{1}{a^2+b^2}$$

$$\Rightarrow x = \frac{a(a^2+b^2)}{a^2+b^2} = y = \frac{b(a^2+b^2)}{a^2+b^2}$$

$$\therefore x = a \text{ and } y = b$$

4. Solve the following pair of equations by reducing them to a pair of linear equations (by any algebraic method):

(i) $\frac{6}{x} + \frac{8}{y} = 5$

$\frac{8}{x} + \frac{12}{y} = 7$

Solution: Writing $u = \frac{1}{x}$ and $v = \frac{1}{y}$, the given equations can be written as

$$6u + 8v - 5 = 0$$

$$8u + 12v - 7 = 0$$

Then by cross-multiplication, we get

$$\frac{u}{8\times(-7)-12\times(-5)} = \frac{v}{(-5)\times8-(-7)\times6} = \frac{1}{6\times12-8\times8}$$

$$\Rightarrow \frac{u}{-56+60} = \frac{v}{-40+42} = \frac{1}{72-64}$$

$$\Rightarrow \frac{u}{4} = \frac{v}{2} = \frac{1}{8}$$

$$\Rightarrow \frac{u}{4} = \frac{1}{8} \text{ and } \frac{v}{2} = \frac{1}{8}$$

$$\Rightarrow u = \frac{4}{8} \text{ and } v = \frac{2}{8}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{2} \text{ and } \frac{1}{y} = \frac{1}{4}$$

$$\therefore x = 2 \text{ and } y = 4$$

(ii) $\frac{6}{x} + \frac{10}{y} = 7$

$\frac{2}{x} + \frac{3}{y} = \frac{13}{6}$

Solution: Writing $u = \frac{1}{x}$ and $v = \frac{1}{y}$, the given equations can be written as

$6u + 10v = 7$ ---------------- (1)

$2u + 3v = \frac{13}{6}$ ---------------- (2)

Multiplying equation (2) by 3 and subtracting from equation (1), we get

$$(6u + 10v) - (6u + 9v) = 7 - \frac{13}{2}$$

$$\Rightarrow 6u + 10v - 6u - 9v = \frac{14-13}{2}$$

$$\Rightarrow v = \frac{1}{2}$$

$$\Rightarrow \frac{1}{y} = \frac{1}{2}$$

$$\therefore y = 2$$

Substituting the value of $v = \frac{1}{2}$ in equation (1), we get

$$6u + 10 \times \frac{1}{2} = 7$$

$$\Rightarrow 6u + 5 = 7$$

$$\Rightarrow 6u = 2$$

$$\Rightarrow u = \frac{2}{6}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{3}$$

$$\therefore x = 3$$

(iii) $\frac{3}{y} - \frac{1}{x} = 1$

$\frac{2}{5x} + \frac{5}{2y} = 7$

Solution: Writing $u = \frac{1}{x}$ and $v = \frac{1}{y}$, the given equations can be written as

$3v - u = 1$

$\Rightarrow u - 3v + 1 = 0$ --------------- (1)

and $\frac{2}{5}u + \frac{5}{2}v = 7$

$\Rightarrow \frac{4u+25v}{10} = 7$

$\Rightarrow 4u + 25v = 70$

$\Rightarrow 4u + 25v - 70 = 0$ --------------- (2)

Then from equations (1) and (2), by cross-multiplication method, we get

$\frac{u}{(-3)\times(-70)-25\times1} = \frac{v}{1\times4-(-70)\times1} = \frac{1}{1\times25-4\times(-3)}$

$\Rightarrow \frac{u}{210-25} = \frac{v}{4+70} = \frac{1}{25+12}$

$\Rightarrow \frac{u}{185} = \frac{v}{74} = \frac{1}{37}$

$\Rightarrow \frac{u}{185} = \frac{1}{37}$ and $\frac{v}{74} = \frac{1}{37}$

$\Rightarrow u = \frac{185}{37}$ and $v = \frac{74}{37}$

$\Rightarrow \frac{1}{x} = 5$ and $\frac{1}{y} = 2$

$\Rightarrow 5x = 1$ and $2y = 1$

$\therefore x = \frac{1}{5}$ and $y = \frac{1}{2}$

(iv) $\frac{a}{x} + \frac{b}{y} = p$

$\frac{b}{x} + \frac{a}{y} = q$, where $\frac{a}{b} \neq \frac{p}{q} \neq \frac{q}{p}$

Solution: Writing $u = \frac{1}{x}$ and $v = \frac{1}{y}$, the given equations can be written as

$au + bv = p$ --------------- (1)

$bu + av = q$ --------------- (2)

Multiplying equation (1) by a; equation (2) by$b$ and subtracting each other, we get

$(a^2u + abv) - (b^2u + abv) = ap - bq$

$\Rightarrow a^2u + abv - b^2u - abv = ap - bq$

$\Rightarrow a^2u - b^2u = ap - bq$

$\Rightarrow (a^2 - b^2)u = ap - bq$

$\Rightarrow u = \frac{ap-bq}{a^2-b^2}$

$\Rightarrow \frac{1}{x} = \frac{ap-bq}{a^2-b^2}$

$\therefore x = \frac{a^2-b^2}{ap-bq}$

Multiplying equation (1) by b; equation (2) by$a$ and subtracting each other, we get

$(abu + b^2v) - (abv + a^2v) = bp - aq$

$\Rightarrow abu + b^2v - abv - a^2v = bp - aq$

$\Rightarrow b^2v - a^2v = bp - aq$

$\Rightarrow a^2v - b^2v = aq - bp$

$\Rightarrow (a^2 - b^2)v = aq - bp$

$\Rightarrow v = \frac{aq-bp}{a^2-b^2}$

$\Rightarrow \frac{1}{y} = \frac{aq-bp}{a^2-b^2}$

$\therefore y = \frac{a^2-b^2}{aq-bp}$

(v) $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$

$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

Solution: Writing $u = \frac{1}{\sqrt{x}}$ and $v = \frac{1}{\sqrt{y}}$, the given equations can be written as

$2u + 3v - 2 = 0$

$4u - 9v + 1 = 0$

Then by cross-multiplication, we get

$\frac{u}{3\times1-(-9)\times(-2)} = \frac{v}{(-2)\times4-2\times1} = \frac{1}{2\times(-9)-4\times3}$

$\Rightarrow \frac{u}{3-18} = \frac{v}{-8-2} = \frac{1}{-18-12}$

$\Rightarrow \frac{u}{-15} = \frac{v}{-10} = \frac{1}{-30}$

$\Rightarrow \frac{u}{-15} = \frac{1}{-30}$ and $\frac{v}{-10} = \frac{1}{-30}$

$\Rightarrow u = \frac{-15}{-30}$ and $v = \frac{-10}{-30}$

$\Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2}$ and $\frac{1}{\sqrt{y}} = \frac{1}{3}$

$\Rightarrow \frac{1}{x} = \frac{1}{4}$ and $\frac{1}{y} = \frac{1}{9}$ [squaring both sides]

$\therefore x = 4$ and $y = 9$

(vi) $\frac{7x-2y}{xy} = 5$

$\frac{8x+7y}{xy} = 15$

Solution: The given equations are

$$\frac{7x-2y}{xy} = 15$$

$$\Rightarrow \frac{7x}{xy} - \frac{2y}{xy} = 5$$

$$\Rightarrow \frac{7}{y} - \frac{2}{x} = 5$$

$$\Rightarrow \frac{2}{x} - \frac{7}{y} + 5 = 0 \text{ ---------------- (1)}$$

and $\frac{8x+7y}{xy} = 15$

$$\Rightarrow \frac{8x}{xy} + \frac{7y}{xy} = 15$$

$$\Rightarrow \frac{8}{y} + \frac{7}{x} = 15$$

$$\Rightarrow \frac{7}{x} + \frac{8}{y} - 15 = 0 \text{ ---------------- (2)}$$

Writing $u = \frac{1}{x}$ and $v = \frac{1}{y}$, the equations (1) and (2) can be written as

$$2u - 7v + 5 = 0$$

$$7u + 8v - 15 = 0$$

Then by cross-multiplication, we get

$$\frac{u}{(-7)\times(-15)-8\times5} = \frac{v}{5\times7-(-15)\times2} = \frac{1}{2\times8-7\times(-7)}$$

$$\Rightarrow \frac{u}{105-40} = \frac{v}{35+30} = \frac{1}{16+49}$$

$$\Rightarrow \frac{u}{65} = \frac{v}{65} = \frac{1}{65}$$

$$\Rightarrow u = 1 \text{ and } v = 1$$

$$\Rightarrow \frac{1}{x} = 1 \text{ and } \frac{1}{y} = 1$$

$$\therefore x = 1 \text{ and } y = 1$$

(vii) $\frac{5}{x-1} + \frac{1}{y-2} = 2$

$\frac{6}{x-1} - \frac{3}{y-2} = 1$

Solution: Writing $u = \frac{1}{x-1}$ and $v = \frac{1}{y-2}$, the given equations can be written as

$$5u + v - 2 = 0$$

$$6u - 3v - 1 = 0$$

Then by cross-multiplication, we get

$$\frac{u}{1\times(-1)-(-3)\times(-2)} = \frac{v}{(-2)\times 6-(-1)\times 5} = \frac{1}{5\times(-3)-6\times 1}$$

$$\Rightarrow \frac{u}{-1-6} = \frac{v}{-12+5} = \frac{1}{-15-6}$$

$$\Rightarrow \frac{u}{-7} = \frac{v}{-7} = \frac{1}{-21}$$

$$\Rightarrow u = v = \frac{1}{3}$$

$$\Rightarrow u = \frac{1}{3} \text{ and } v = \frac{1}{3}$$

$$\Rightarrow \frac{1}{x-1} = \frac{1}{3} \text{ and } \frac{1}{y-2} = \frac{1}{3}$$

$$\Rightarrow x - 1 = 3 \text{ and } y - 2 = 3$$

$$\therefore x = 4 \text{ and } y = 5$$

(viii) $\boldsymbol{2x + y = 2xy}$

$\boldsymbol{2x + 4y = 5xy}$

Solution: The given equations are

$$2x + y = 2xy$$

$$\Rightarrow \frac{2x+y}{xy} = 2$$

$$\Rightarrow \frac{2x}{xy} + \frac{y}{xy} = 2$$

$$\Rightarrow \frac{2}{y} + \frac{1}{x} = 2$$

$\Rightarrow \frac{1}{x} + \frac{2}{y} = 2$ --------------- (1)

and $2x + 4y = 5xy$

$$\Rightarrow \frac{2x+4y}{xy} = 5$$

$$\Rightarrow \frac{2x}{xy} + \frac{4y}{xy} = 15$$

$$\Rightarrow \frac{2}{y} + \frac{4}{x} = 5$$

$\Rightarrow \frac{4}{x} + \frac{2}{y} = 5$ --------------- (2)

Writing $u = \frac{1}{x}$ and $v = \frac{1}{y}$, the equations (1) and (2) can be written as

$u + 2v = 2$ --------------- (3)

$4u + 2v = 5$ --------------- (4)

Subtracting equation (3) from equation (4), we get

$(4u + 2v) - (u + 2v) = 5 - 3$

$\Rightarrow 4u + 2v - u - 2v = 3$

$\Rightarrow 3u = 3$

$\Rightarrow u = 1$

$\Rightarrow \frac{1}{x} = 1$

$\therefore x = 1$

Substituting the value of $u = 1$ in equation (3), we get

$1 + 2v = 2$

$\Rightarrow 2v = 1$

$\Rightarrow v = \frac{1}{2}$

$\Rightarrow \frac{1}{y} = \frac{1}{2}$

$\therefore y = 2$

(ix) $\frac{5}{x+y} + \frac{1}{x-y} = 2$

$\frac{15}{x+y} - \frac{5}{x-y} = -2$

Solution: Writing $u = \frac{1}{x+y}$ and $v = \frac{1}{x-y}$, the given equations can be written as

$5u + v - 2 = 0$
$15u - 5v + 2 = 0$

Then by cross-multiplication, we get

$$\frac{u}{1\times2-(-5)\times(-2)} = \frac{v}{(-2)\times15-2\times5} = \frac{1}{5\times(-5)-15\times1}$$

$\Rightarrow \frac{u}{2-10} = \frac{v}{-30-10} = \frac{1}{-25-15}$

$\Rightarrow \frac{u}{-8} = \frac{v}{-40} = \frac{1}{-40}$

$\Rightarrow \frac{u}{-8} = \frac{1}{-40}$ and $\frac{v}{-40} = \frac{1}{-40}$

$\Rightarrow u = \frac{-8}{-40}$ and $v = \frac{-40}{-40}$

$\Rightarrow \frac{1}{x+y} = \frac{1}{5}$ and $\frac{1}{x-y} = 1$

$\Rightarrow x + y = 5$ --------------- (1)

and $x - y = 1$ --------------- (2)

Adding equations (1) and (2), we get

$2x = 6$

$\therefore x = 3$

Substituting $x = 3$ in equation (1), we get

$$3 + y = 5$$

$$\therefore y = 2$$

(x) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$

$\frac{1}{3x+y} - \frac{1}{3x-y} = \frac{-1}{4}$

Solution: Writing $u = \frac{1}{3x+y}$ and $v = \frac{1}{3x-y}$, the given equations can be written as

$u + v = \frac{3}{4}$ --------------- (1)

$u - v = \frac{-1}{4}$ --------------- (2)

Adding the above relations, we get

$$2u = \frac{2}{4}$$

$$\Rightarrow u = \frac{1}{4}$$

$$\Rightarrow \frac{1}{3x+y} = \frac{1}{4}$$

$\Rightarrow 3x + y = 4$ --------------- (3)

Substituting $u = \frac{1}{4}$ in equation (1), we get

$$\frac{1}{4} + v = \frac{3}{4}$$

$$\Rightarrow v = \frac{3}{4} - \frac{1}{4}$$

$$\Rightarrow v = \frac{3-1}{4}$$

$$\Rightarrow v = \frac{2}{4}$$

$$\Rightarrow \frac{1}{3x-y} = \frac{1}{2}$$

$\Rightarrow 3x - y = 2$ --------------- (4)

Adding equations (3) and (4), we get

$$6x = 6$$

$$\therefore x = 1$$

Substituting $x = 1$ in equation (3), we get

$$3 \times 1 + y = 4$$

$$\Rightarrow 3 + y = 4$$

$$\therefore y = 1$$

(xi) $\frac{8}{x+y}+\frac{4}{x-y}=1$

$\frac{4}{x+y}+\frac{8}{x-y}=\frac{5}{4}$

Solution: Writing $u=\frac{1}{x+y}$ and $v=\frac{1}{x-y}$, the given equations can be written as

$8u+4v=1$ --------------- (1)

$4u+8v=\frac{5}{4}$ --------------- (2)

Multiplying equation (2) by $\frac{1}{2}$ and subtracting equation (1), we get

$(2u+4v)-(8u+4v)=\frac{5}{8}-1$

$\Rightarrow 2u+4v-8u-4v=\frac{5-8}{8}$

$\Rightarrow -6u=\frac{-3}{8}$

$\Rightarrow u=\frac{3}{8\times 6}$

$\Rightarrow u=\frac{1}{16}$

$\Rightarrow \frac{1}{x+y}=\frac{1}{16}$

$\Rightarrow x+y=16$ --------------- (3)

Substituting $u=\frac{1}{16}$ in equation (1), we get

$8\times\frac{1}{16}+4v=1$

$\Rightarrow \frac{1}{2}+4v=1$

$\Rightarrow 4v=\frac{1}{2}$

$\Rightarrow v=\frac{1}{8}$

$\Rightarrow \frac{1}{x-y}=\frac{1}{8}$

$\Rightarrow x-y=8$ --------------- (4)

Adding equations (1) and (2), we get

$2x=24$

$\therefore x=12$

Substituting $x=12$ in equation (3), we get

$12+y=16$

$\therefore y=4$

(xii) $\frac{6}{x+y}+\frac{4}{x-y}=3$

$\frac{9}{x+y}-\frac{4}{x-y}=\frac{-1}{2}$

Solution: Writing $u=\frac{1}{x+y}$ and $v=\frac{1}{x-y}$, the given equations can be written as

$6u+4v=3$ --------------- (1)

$9u-4v=\frac{-1}{2}$ --------------- (2)

Adding equations (1) and (2), we get

$15u=2\frac{1}{2}$

$\Rightarrow 15u=\frac{5}{2}$

$\Rightarrow 3u=\frac{1}{2}$

$\Rightarrow u=\frac{1}{6}$

$\Rightarrow \frac{1}{x+y}=\frac{1}{6}$

$\Rightarrow x+y=6$ --------------- (3)

Substituting $u=\frac{1}{6}$ in equation (1), we get

$6\times\frac{1}{6}+4v=3$

$\Rightarrow 1+4v=3$

$\Rightarrow 4v=2$

$\Rightarrow v=\frac{2}{4}$

$\Rightarrow \frac{1}{x-y}=\frac{1}{2}$

$\Rightarrow x-y=2$ --------------- (4)

Adding equations (1) and (2), we get

$2x=8$

$\therefore x=4$

Substituting $x=4$ in equation (3), we get

$4+y=6$

$\therefore y=2$

SOLUTIONS

EXERCISE 4.3

1. Divide 250 into two parts so that 3 times the first part and 5 times the second part together make 950.

Solution: Let x and y respectively be the first and the second parts.

From the given conditions of the problem, we get

$x + y = 250$ --------------- (1)

and $3x + 5y = 950$ --------------- (2)

Multiplying equation (1) by 5, we get

$5x + 5y = 1250$ --------------- (3)

Subtracting equation (2) from equation (3), we get

$(5x + 5y) - (3x + 5y) = 1250 - 950$

$\Rightarrow 5x + 5y - 3x - 5y = 300$

$\Rightarrow 2x = 300$

$\Rightarrow x = \frac{300}{2}$

$\Rightarrow x = 150$

Substituting this value of x in equation (1), we get

$150 + y = 250$

$\Rightarrow y = 250 - 150$

$\Rightarrow y = 100$

Hence, the required first and the second parts are 150 and 100 respectively.

2. There are two numbers. when 1 is added to each of the numbers their ratio becomes 1:2 and, when 5 is subtracted from each their ratio becomes 5:11. Find the numbers.

Solution: Let x and y be the two numbers.

From the given conditions of the problem, we get

$\frac{x+1}{y+1} = \frac{1}{2}$

$\Rightarrow 2(x + 1) = y + 1$

$\Rightarrow 2x + 2 = y + 1$

$\Rightarrow 2x - y = -1$ --------------- (1)

and $\frac{x-5}{y-5} = \frac{5}{11}$

$\Rightarrow 11(x-5) = 5(y-5)$

$\Rightarrow 11x - 55 = 5y - 25$

$\Rightarrow 11x - 5y = -25 + 55$

$\Rightarrow 11x - 5y = 30$ --------------- (2)

Multiplying equation (1) by 5, we get

$10x - 5y = -5$ --------------- (3)

Subtracting equation (3) from equation (2), we get

$(11x - 5y) - (10x - 5y) = 30 - (-5)$

$\Rightarrow 11x - 5y - 10x + 5y = 30 + 5$

$\Rightarrow x = 35$

Substituting this value of x in equation (1), we get

$2 \times 35 - y = -1$

$\Rightarrow 70 + 1 = y$

$\Rightarrow y = 71$

Hence, the required numbers are 35 and 71.

3. The sum of the digits of a two digit number is 12. If the digits interchanged, the number is increased by 18. Find the number.

Solution: Let x and y respectively be the digits in the unit's and ten's places.

Then the number $= x + 10y$ --------------- (1)

The number formed by interchanging the digits $= 10x + y$

According to the given condition, we have

$x + y = 12$ --------------- (2)

and $10x + y = x + 10y + 18$

$\Rightarrow 10x + y - x - 10y = 18$

$\Rightarrow 9x - 9y = 18$

$\Rightarrow x - y = 2$ --------------- (3) [dividing both sides by 9]

Adding equations (2) and (3), we get

$(x + y) + (x - y) = 12 + 2$

$\Rightarrow x + y + x - y = 14$

$\Rightarrow 2x = 14$

$\Rightarrow x = \frac{14}{2} = 7$

Substituting the value of x in equation (2), we get

$7 + y = 12$

$\Rightarrow y = 12 - 7$

$\Rightarrow y = 5$

So, substituting the values of x and y in equation (1), we get

the required number= $7 + 10 \times 5 = 7 + 50 = 57$

4. The sum of the two digits of a two digit number is 9. When 9 is added to the number, the digits are reversed, find the number.

Solution: Let x and y respectively be the digits in the unit's and ten's places.

Then the number = $x + 10y$ --------------- (1)

The number formed by reversing the digits = $10x + y$

According to the given condition, we have

$x + y = 9$ --------------- (2)

and $x + 10y + 9 = 10x + y$

$\Rightarrow 10x + y = x + 10y + 9$

$\Rightarrow 10x + y - x - 10y = 9$

$\Rightarrow 9x - 9y = 9$

$\Rightarrow x - y = 1$ --------------- (3) [dividing both sides by 9]

Adding equations (2) and (3), we get

$(x + y) + (x - y) = 9 + 1$

$\Rightarrow x + y + x - y = 10$

$\Rightarrow 2x = 10$

$\Rightarrow x = \frac{10}{2} = 5$

Substituting the value of x in equation (2), we get

$5 + y = 9$

$\Rightarrow y = 9 - 5$

$\Rightarrow y = 4$

So, substituting the values of x and y in equation (1), we get

the required number= $5 + 10 \times 4 = 5 + 40 = 45$

5. In a given fraction, the denominator is greater than the numerator by 2. If 7 is added to the numerator, the resulting fraction becomes greater than the given fraction by 1. Find the fraction.

Solution: Let $\frac{x}{y}$ be the fraction.

By the question, we have

$y - x = 2$ --------------- (1)

and $\frac{x+7}{y} = \frac{x}{y} + 1$

$\Rightarrow \frac{x+7}{y} - \frac{x}{y} = 1$

$\Rightarrow \frac{x+7-x}{y} = 1$

$\Rightarrow \frac{7}{y} = 1$

$\Rightarrow y = 7$

Substituting this value of y in equation (1), we have

$7 - x = 2$

$\Rightarrow 7 - 2 = x$

$\Rightarrow x = 5$

Hence the required fraction is $\frac{5}{7}$.

6. Four years ago a father was nine times as old as his son, and 8 years hence the father's age will be three times the son's age. Find their present ages.

Solution: Let x and y (in years) respectively be the present ages of the father and the son.

From the given conditions of the problem, we get

$x - 4 = 9(y - 4)$

$\Rightarrow x - 4 = 9y - 36$

$\Rightarrow x - 9y = -36 + 4$

$\Rightarrow x - 9y = -32$ --------------- (1)

and $x + 8 = 3(y + 8)$

$\Rightarrow x + 8 = 3y + 24$

$\Rightarrow x - 3y = 24 - 8$

$\Rightarrow x - 3y = 16$ --------------- (2)

Subtracting equation (1) from equation (2), we get

$$(x-3y)-(x-9y)=16-(-32)$$

$$\Rightarrow x-3y-x+9y=16+32$$

$$\Rightarrow 6y=48$$

$$\Rightarrow y=\frac{48}{6}=8$$

Substituting this value of y in equation (2), we get

$$x-3\times 8=16$$

$$\Rightarrow x-24=16$$

$$\Rightarrow x=16+24$$

$$\Rightarrow x=40$$

Hence, the present age of the father is 40 years and that of the son is 8 years.

7. The present age of a father exceeds that of his son by 20 years. Twenty years ago the age of the father was five times that of his son. Find their present ages.

Solution: Let x and y (in years) respectively be the present ages of the father and the son.

From the given conditions of the problem, we get

$x=y+20$ --------------- (1)

and $x-20=5(y-20)$

$$\Rightarrow x-20=5y-100$$

$$\Rightarrow x=5y-100+20$$

$\Rightarrow x=5y-80$ --------------- (2)

From equations (1) and (2), we get

$$5y-80=y+20$$

$$\Rightarrow 5y-y=20+80$$

$$\Rightarrow 4y=100$$

$$\Rightarrow y=\frac{100}{4}=25$$

$$\Rightarrow y=25$$

Substituting this value of y in equation (1), we get

$$x=25+20$$

$$\Rightarrow x=45$$

Hence, the present age of the father is 45 years and that of the son is 25 years.

8. A chair and a table cost Rs. 1,200. By selling the chair at a profit of 20% and the table at a loss of 5%, there is a profit of 4% on the whole. Find the cost price of the table and the chair.

Solution: Let x and y (in Rs.) respectively be the cost prices of a chair and a table.

Then, $x + y = 1200$ --------------- (1)

And $\frac{20}{100}x - \frac{5}{100}y = \frac{4}{100} \times 1200$

[$\because$ gain on chair− loss on the table= gain on the whole transaction]

$\Rightarrow 20x - 5y = 4 \times 1200$

$\Rightarrow 4x - y = 4 \times 240$ [Dividing both sides by 5]

$\Rightarrow 4x - y = 960$ --------------- (2)

Adding (1)and (2), we get

$5x = 2160$

$\Rightarrow x = \frac{2160}{5} = 432$

Substituting $x = 432$ in equation (i), we have

$432 + y = 1200$

$\therefore y = 1200 - 432 = 768$

Hence the required cost price of the chair is Rs. 432 and that of the table is Rs. 768.

9. Two tables and three chairs cost Rs. 3,500, and three tables and two chairs cost Rs. 4,000. What is the cost of a table and that of a chair?

Solution: Let x and y (in Rs.) be the cost prices of a table and a chair respectively.

By the given conditions of the problem, we get

$2x + 3y = 3500$

$\Rightarrow 2x + 3y - 3500 = 0$ --------------- (1)

and $3x + 2y = 4000$

$\Rightarrow 3x + 2y - 4000 = 0$ --------------- (2)

From equations (1) and (2), by cross-multiplication method, we have

$$\frac{x}{3\times(-4000)-2\times(-3500)} = \frac{y}{3\times(-3500)-2\times(-4000)} = \frac{1}{2\times2-3\times3}$$

$$\Rightarrow \frac{x}{-12000+7000} = \frac{y}{-10500+8000} = \frac{1}{4-9}$$

$$\Rightarrow \frac{x}{-5000} = \frac{y}{-2500} = \frac{1}{-5}$$

$$\Rightarrow x = \frac{-5000}{-5} \text{ and } y = \frac{-2500}{-5}$$

$$\therefore x = 1000 \text{ and } y = 500$$

Hence the required cost price of the table is Rs. 1000 and that of the chair is Rs. 500.

10. A farmer sold a cow and a calf for Rs.12750 thereby making a profit of 25% on the cow and 10% on the calf. By selling them for Rs. 11925, he would have realized a profit of 10% on the cow and 25% on the calf. Find the cost prices of the cow and the calf.

Solution: Let *x* and *y* (in Rs.) respectively be the cost prices of the cow and the calf.

Then, by the given conditions of the problem, we get

$$\left(x+\frac{25}{100}x\right)+\left(y+\frac{10}{100}y\right)=12750$$

$$\Rightarrow \left(x+\frac{x}{4}\right)+\left(y+\frac{y}{10}\right)=12750$$

$$\Rightarrow \left(\frac{4x+x}{4}\right)+\left(\frac{10y+y}{10}\right)=12750$$

$$\Rightarrow \frac{5x}{4}+\frac{11y}{10}=12750$$

$$\Rightarrow \frac{25x+22y}{20}=12750$$

$$\Rightarrow 25x+22y=255000 \text{ --------------- (1)}$$

and $\left(x+\frac{10}{100}x\right)+\left(y+\frac{25}{100}y\right)=11925$

$$\Rightarrow \left(x+\frac{x}{10}\right)+\left(y+\frac{y}{4}\right)=11925$$

$$\Rightarrow \left(\frac{10x+x}{10}\right)+\left(\frac{4y+y}{4}\right)=11925$$

$$\Rightarrow \frac{11x}{10}+\frac{5y}{4}=11925$$

$$\Rightarrow \frac{22x+25y}{20}=11925$$

$$\Rightarrow 22x+25y=238500 \text{ --------------- (2)}$$

Multiplying equation (1) by 22, we get

$$550x+484y=5610000 \text{ --------------- (3)}$$

Multiplying equation (2) by 25, we get

$$550x+625y=5962500 \text{ --------------- (4)}$$

Subtracting equation (3) from equation (4), we get

$$(550x+625y)-(550x+484y)=5962500-5610000$$

$$\Rightarrow 550x+625y-550x-484y=352500$$

$$\Rightarrow 141y=352500$$

$$\Rightarrow y=\frac{352500}{141}$$

$$\Rightarrow y=2500$$

Substituting this value of y in equation (1), we get

$$25x + 22 \times 2500 = 255000$$
$$\Rightarrow 25x + 55000 = 255000$$
$$\Rightarrow 25x = 255000 - 55000$$
$$\Rightarrow 25x = 200000$$
$$\Rightarrow x = \frac{200000}{25}$$
$$\Rightarrow x = 8000$$

∴the required cost prices of the cow and the calf are Rs. 8000 and Rs. 2500 respectively.

11. If we buy 3 tickets for Imphal to Dimapur and 2 tickets for Imphal to Mao, the total cost is Rs. 633, but if we buy 2 tickets for Imphal to Dimapur and 5 tickets for Imphal to Mao, the total cost is Rs. 642. Find the bus fares from Imphal to Dimapur and Mao.

Solution: Let Rs. x and Rs. y be the bus fare from Imphal to Dimapur and Mao respectively.

Then, by the given conditions of the problem, we get

$$3x + 2y = 633$$
$$\Rightarrow 3x + 2y - 633 = 0 \text{ --------------- (1)}$$
and $2x + 5y = 642$
$$\Rightarrow 2x + 5y - 642 = 0 \text{ --------------- (2)}$$

From equations (1) and (2), by cross-multiplication method, we have

$$\frac{x}{2\times(-642)-5\times(-633)} = \frac{y}{2\times(-633)-3\times(-642)} = \frac{1}{3\times5-2\times2}$$
$$\Rightarrow \frac{x}{-1284+3165} = \frac{y}{-1266+1926} = \frac{1}{15-4}$$
$$\Rightarrow \frac{x}{1881} = \frac{y}{660} = \frac{1}{11}$$
$$\Rightarrow x = \frac{1881}{11} \text{ and } y = \frac{660}{11}$$
$$\therefore x = 171 \text{ and } y = 60$$

Hence, the required the bus fare from Imphal to Dimapur is Rs. 171 and that from Imphal to Mao is Rs. 60.

12. Two stations A and B on a highway are 90 km. apart. A car starts from A another car starts from B at the same time. If they travel in the same direction they meet in 9 hours, but if they travel towards each other they meet in 1 hour after start. Find the speeds of the two cars, the car from A moving faster.

Solution: Let x and y (in km/hr) respectively be the speeds of the cars at A and B respectively.

If they travel in the same direction, we get

Distance travelled by the car at A− distance travelled by the car at B

= Distance between the stations

$\Rightarrow 9x - 9y = 90$ [∵ Distance = speed× time]

$\Rightarrow x - y = 10$ --------------- (1)

If they travel in opposite direction, we get

And distance travelled by the car at A+ distance travelled by the car at B

= Distance between the stations

$\Rightarrow x + y = 90$ --------------- (2) [∵ Distance = speed× time]

Adding equations (1) and (2), we get

$(x - y) + (x + y) = 10 + 90$

$\Rightarrow x - y + x + y = 100$

$\Rightarrow 2x = 100$

$\Rightarrow x = \frac{100}{2}$

$\Rightarrow x = 50$

Substituting this value of x in equation (1), we get

$50 + y = 90$

$\Rightarrow y = 90 - 50$

$\Rightarrow y = 40$

Hence, the speeds of the cars at A and B are 50 km/hr and 40 km/hr respectively.

13. 90% and 95% pure mustard oils are mixed to obtained 20 litres of 92% pure mustard oil. How many litres of each kind of mustard oil are needed?

Solution: Let x and y (in litres)be the quantities of 90% amd 95% pure mustard oils.

By the given conditions of the problem, we get

$x + y = 20$ --------------- (1)

and 90% of x + 95% of y = 92% of 20

$\Rightarrow \frac{90}{100}x + \frac{95}{100}y = \frac{92}{100} \times 20$

$\Rightarrow 90x + 95y = 92 \times 20$

$\Rightarrow 18x + 19y = 92 \times 4$ [Dividing both sides by 5]

$\Rightarrow 18x + 19y = 368$ --------------- (2)

From equation (1), we get

$x = 20 - y$ --------------- (3)

Substituting this value of x in equation (2), we get

$$18(20 - y) + 19y = 368$$

$$\Rightarrow 360 - 18y + 19y = 368$$

$$\Rightarrow 360 + y = 368$$

$$\Rightarrow y = 368 - 360 = 8$$

Substituting this value of y in equation (3), we get

$$x = 20 - 8 = 12$$

Hence, the required quantity of 90% pure mustard oil is 12 litres and that of 95% pure mustard oil is 8 litres.

14. A steamer goes 50 km. downstream and 45 km. upstream in 5 hours. In 5 hours 8 minutes it can go 50 km. upstream and 45 km. downstream. Find the speed of the stream and that of the steamer in still water.

Solution: Let x and y (in Km/hr) respectively be the speeds of the steamer in still water and the stream.

Then speed of the steamer in downstream $= x + y$

and speed of the steamer in upstream$= x - y$

By the given conditions of the problem, we get

$$\frac{50}{x+y} + \frac{45}{x-y} = 5 \qquad [\because \text{time} = \frac{\text{distance}}{\text{speed}}]$$

$$\Rightarrow 5\left(\frac{10}{x+y} + \frac{9}{x-y}\right) = 5$$

$$\Rightarrow \frac{10}{x+y} + \frac{9}{x-y} = 1 \text{ --------------- (1)}$$

and $$\frac{45}{x+y} + \frac{50}{x-y} = 5 + \frac{8}{60} \qquad [\because \text{time} = \frac{\text{distance}}{\text{speed}}]$$

$$\Rightarrow \frac{45}{x+y} + \frac{50}{x-y} = 5 + \frac{2}{15}$$

$$\Rightarrow \frac{45}{x+y} + \frac{50}{x-y} = \frac{77}{15}$$

$$\Rightarrow 15\left(\frac{45}{x+y} + \frac{50}{x-y}\right) = 77$$

$$\Rightarrow \frac{675}{x+y} + \frac{750}{x-y} = 77 \text{ --------------- (2)}$$

Writing $u = \frac{1}{x+y}$ and $v = \frac{1}{x-y}$, equations (1) and (2) respectively become

$$10u + 9v = 1$$

$$\Rightarrow 10u + 9v - 1 = 0 \text{ --------------- (3)}$$

and $675u + 750v = 77$

$$\Rightarrow 675u + 750v - 77 = 0 \text{ --------------- (4)}$$

From equations (3) and (4), by cross-multiplication method, we get

$$\frac{u}{9\times(-77)-750\times(-1)}=\frac{v}{675\times(-1)-10\times(-77)}=\frac{1}{10\times750-675\times9}$$

$$\Rightarrow \frac{u}{-693+750}=\frac{v}{-675+770}=\frac{1}{7500-6075}$$

$$\Rightarrow \frac{u}{57}=\frac{v}{95}=\frac{1}{1425}$$

$$\therefore \frac{u}{57}=\frac{1}{1425}$$

$$\Rightarrow u=\frac{57}{1425}$$

$$\Rightarrow u=\frac{1}{25}$$

$$\Rightarrow \frac{1}{x+y}=\frac{1}{25}$$

$\Rightarrow x+y=25$ --------------- (5)

and $\frac{v}{95}=\frac{1}{1425}$

$$\Rightarrow v=\frac{95}{1425}$$

$$\Rightarrow \frac{1}{x-y}=\frac{1}{15}$$

$\Rightarrow x-y=15$ --------------- (6)

Adding equations (5) and (6), we get

$$2x=40$$

$$\Rightarrow x=20$$

Substituting this value of x in equation (5), we get

$$20+y=25$$

$$\Rightarrow y=5$$

Hence, the speed of the steamer in still water is 20 km/hr and the speed of the stream is 5km/hr.

15. In a rectangle, if the length is reduced by 5cm. and the breadth is increased by 2cm, the area is reduced by 40 sq.cm. If however, the length is increased by 2cm. and the breadth by 4cm, the area is increased by 92 sq.cm. Find the length and the breadth of the rectangle.

Solution: Let x and y (in cm) respectively be the length and the breadth of the rectangle.

Then, Area of the rectangle$=xy$

By the given conditions of the problem, we get

$$(x-5)(y+2)=xy-40$$

$\Rightarrow xy + 2x - 5y - 10 = xy - 40$

$\Rightarrow 2x - 5y = -40 + 10$

$\Rightarrow 2x - 5y = -30$ --------------- (1)

and $(x + 2)(y + 4) = xy + 92$

$\Rightarrow xy + 4x + 2y + 8 = xy + 92$

$\Rightarrow 4x + 2y = 92 - 8$

$\Rightarrow 2(2x + y) = 84$

$\Rightarrow 2x + y = 42$ --------------- (2)

Subtracting equation (1) from equation (2), we get

$(2x + y) - (2x - 5y) = 42 - (-30)$

$\Rightarrow 2x + y - 2x + 5y = 42 + 30$

$\Rightarrow 6y = 72$

$\Rightarrow y = \frac{72}{6}$

$\Rightarrow y = 12$

Substituting this value of y in equation (2), we get

$2x + 12 = 42$

$\Rightarrow 2x = 30$

$\Rightarrow x = \frac{30}{2}$

$\Rightarrow x = 15$

Hence, the length and the breadth of the rectangle are 15 cm and 12 cm respectively.

16. The area of a rectangular garden is increased by 55 sq.m. if its length is reduced by 2 m. and the breadth is increased by 5 m. If the length is increased by 3m. and the breadth by 2m. the area is increased by 70 sq.m. Find the area of the rectangular garden.

Solution: Let x and y (in metres) respectively be the length and the breadth of the rectangular garden.

Then, Area of the rectangular garden $= xy$

By the given conditions of the problem, we get

$(x - 2)(y + 5) = xy + 55$

$\Rightarrow xy + 5x - 2y - 10 = xy + 55$

$\Rightarrow 5x - 2y = 55 + 10$

$\Rightarrow 5x - 2y = 65$

$\Rightarrow 5x - 2y - 65 = 0$ --------------- (1)

and $(x+3)(y+2) = xy + 70$

$\Rightarrow xy + 2x + 3y + 6 = xy + 70$

$\Rightarrow 2x + 3y = 70 - 6$

$\Rightarrow 2x + 3y = 64$

$\Rightarrow 2x + 3y - 64 = 0$ --------------- (2)

From equations (1) and (2), by cross-multiplication method, we get

$$\frac{x}{(-2)\times(-64)-3\times(-65)} = \frac{y}{2\times(-65)-5\times(-64)} = \frac{1}{5\times3-2\times(-2)}$$

$\Rightarrow \frac{x}{128+195} = \frac{y}{-130+320} = \frac{1}{15+4}$

$\Rightarrow \frac{x}{323} = \frac{y}{190} = \frac{1}{19}$

$\Rightarrow \frac{x}{323} = \frac{1}{19}$ and $\frac{y}{190} = \frac{1}{19}$

$\Rightarrow x = \frac{323}{19}$ and $y = \frac{190}{19}$

$\Rightarrow x = 17$ and $y = 10$

So, area of the rectangular garden $= 17 \times 10$ sq.m. $= 170$ sq.m.

17. A rectangle is of the same area as another which is 6 m longer and 4 m narrower. It is also of the same area as a third rectangle which is 8 m longer and 5 m. narrower. Find the area of the rectangle.

Solution: Let x and y (in metres) respectively be the length and the breadth of the rectangle.

Then, Area of the rectangle$= xy$

By the given conditions of the problem, we get

$(x+6)(y-4) = xy$

$\Rightarrow xy - 4x + 6y - 24 = xy$

$\Rightarrow -4x + 6y - 24 = 0$

$\Rightarrow -2x + 3y - 12 = 0$ --------------- (1) [Dividing both sides by 2]

and $(x+8)(y-5) = xy$

$\Rightarrow xy - 5x + 8y - 40 = xy$

$\Rightarrow -5x + 8y - 40 = 0$ --------------- (2)

From equations (1) and (2), by cross-multiplication method, we get

$$\frac{x}{3\times(-40)-8\times(-12)} = \frac{y}{(-5)\times(-12)-(-2)\times(-40)} = \frac{1}{(-2)\times8-3\times(-5)}$$

$\Rightarrow \frac{x}{-120+96} = \frac{y}{60-80} = \frac{1}{-16+15}$

$\Rightarrow \frac{x}{-24} = \frac{y}{-20} = \frac{1}{-1}$

$\Rightarrow \frac{x}{24} = \frac{y}{20} = 1$

$\Rightarrow x = 24$ and $y = 20$

So, area of the rectangle $= 24 \times 20$ sq.m. $= 480$ sq.m.

18. An annual income of Rs. 1,200 is derived from two sums invested, one at 4% and the other at 6% per annum simple interest. If the rates of interest are changed to 5% and 7% per annum simple interest respectively, the annual income derived from the investments is Rs. 1450; find the sums invested.

Solution: Let x and y (in Rs.) respectively be the sums invested at 4% and 6% per annum.

Then, by the given conditions of the problem, we have

$\frac{4x}{100} + \frac{6y}{100} = 1200$

$\Rightarrow 4x + 6y = 120000$

$\Rightarrow 2(2x + 3y) = 120000$

$\Rightarrow 2x + 3y = 60000$ --------------- (1)

and $\frac{5x}{100} + \frac{7y}{100} = 1450$

$\Rightarrow 5x + 7y = 145000$ --------------- (2)

Multiplying equations (1) by 5 and (2) by 2 respectively become

$10x + 15y = 300000$ --------------- (3)

and $10x + 14y = 290000$ --------------- (4)

Subtracting equation (4) from equation (3), we get

$y = 10000$

Substituting this value of y in equation (1), we get

$2x + 3 \times 10000 = 60000$

$\Rightarrow 2x + 30000 = 60000$

$\Rightarrow 2x = 60000 - 30000 = 30000$

$\Rightarrow x = 15000$

Hence, the sums invested at 4% and 6% per annum are Rs. 15,000 and Rs. 10,000 respectively.

19. A man invested Rs. 36,000, a part of it at 12% and the rest at 15% per annum simple interest. If he received a total annual interest of Rs. 4,890, how much did he invest at each rate?

Solution: Let x and y (in Rs.) respectively be the sums invested at 12% and 15% per annum.

Then, by the given conditions of the problem, we have

$x + y = 36000$ --------------- (1)

and $\frac{12x}{100} + \frac{15y}{100} = 4890$

$\Rightarrow 3\left(\frac{4x}{100} + \frac{5y}{100}\right) = 4890$

$\Rightarrow \frac{4x}{100} + \frac{5y}{100} = 1630$

$\Rightarrow 4x + 5y = 163000$ ---------------- (2)

From equation (1), we get

$x = 36000 - y$ --------------- (3)

Substituting this value of x in equation (2), we get

$4(36000 - y) + 5y = 163000$

$\Rightarrow 144000 - 4y + 5y = 163000$

$\Rightarrow y = 163000 - 144000 = 19000$

Substituting this value of y in equation (3), we get

$x = 36000 - 19000$

$\Rightarrow x = 17000$

Hence, the sums invested at 12% and 15% per annum are Rs. 17,000 and Rs. 19,000 respectively.

20. In a classroom there are a number of benches. If 4 students sit on each bench, five benches are left vacant; and if 3 students sit on each bench, 4 students are left standing. Find the number of benches and students in the class room.

Solution: Let x and y respectively be the number of benches and students in the classroom.

Then, by the given conditions of the problem, we have

$y = 4(x - 5)$--------------- (1)

and $y = 3x + 4$ ---------------- (2)

From equations (1) and (2), we get

$4(x - 5) = 3x + 4$

$\Rightarrow 4x - 20 = 3x + 4$

$\Rightarrow 4x - 3x = 20 + 4$

$\therefore x = 24$

Substituting x =24 in equation (2), we have

$y = 3 \times 24 + 4$

$\Rightarrow y = 72 + 4$

$\therefore y$ =76

Hence, the number of benches & students in the classroom are 24 and76 respectively.

21. The sum of a two-digit number and the number obtained by reversing the digits is 110. If the difference of the digits of the number is 4, find the number. How many such numbers are there?

Solution: Let x and y respectively be the digits in ten's and unit's places.

Then the number $= 10x + y$

The number formed by reversing the digits $= 10y + x$

By the given conditions of the problem, we have

$(10x + y) + (10y + x) = 110$

$\Rightarrow 11x + 11y = 110$

$\Rightarrow 11(x + y) = 110$

$\Rightarrow x + y = 10$ --------------- (1)

And either $x - y = 4$ --------------- (2)

or $y - x = 4$ --------------- (3)

If $x - y = 4$, adding equations (1) and (2), we get

$2x = 14$

$\Rightarrow x = 7$

Substituting this value of x in equation (1), we get

$7 + y = 10$

$\Rightarrow y = 3$

In this case, we get the number $= 10 \times 7 + 3 = 73$

If $y - x = 4$, adding equations (1) and (3), we get

$2y = 14$

$\Rightarrow y = 7$

Substituting this value of y in equation (1), we get

$x + 7 = 10$

$\Rightarrow x = 3$

In this case, the number $= 10 \times 3 + 7 = 37$

Thus, there are two such numbers, 73 and 37.

22. The ratio of incomes of two persons is 9:7 and the ratio of their expenditures is 4:3. If each of them saves Rs. 2000 per month, find their monthly incomes.

Solution: Let x and y (in Rs.) respectively be the incomes of the two persons.

Then, by the given conditions of the problem, we get

$$\frac{x}{y} = \frac{9}{7}$$

$$\Rightarrow 9y = 7x$$

$$\Rightarrow y = \frac{7x}{9} \text{ --------------- (1)}$$

and $\frac{x-2000}{y-2000} = \frac{4}{3}$ [∵ income – saving = expenditure]

$$\Rightarrow 4y - 8000 = 3x - 6000$$

$$\Rightarrow y = \frac{3x+2000}{4} \text{ --------------- (2)}$$

From equations (1) and (2), we get

$$\frac{7x}{9} = \frac{3x+2000}{4}$$

$$\Rightarrow 28x = 27x + 18000$$

$$\Rightarrow 28x - 27x = 18000$$

$$\therefore x = 18000$$

Substituting x =18000 in equation (1), we have

$$y = \frac{7 \times 18000}{9}$$

$$\Rightarrow y = 7 \times 2000$$

$$\Rightarrow y = 14000$$

Hence, the incomes of the two persons are Rs.18000 and Rs.14000.

23. A two-digit number is obtained either by multiplying the sum of the digits by 8 and adding 1, or by multiplying the difference of the digits by 13 and adding 2. Find the number. How many such numbers are there?

Solution: Let x and y respectively be the digits in ten's and unit's places.

Then the number $= 10x + y$

By the given conditions of the problem, we have

$$10x + y = 8(x + y) + 1$$

$$\Rightarrow 10x + y = 8x + 8y + 1$$

$$\Rightarrow 2x - 7y - 1 = 0 \text{ --------------- (1)}$$

And either $10x + y = 13(x - y) + 2$

$\Rightarrow 10x + y = 13x - 13y + 2$

$\Rightarrow 10x - 13x + y + 13y - 2 = 0$

$\Rightarrow -3x + 14y - 2 = 0$

$\Rightarrow 3x - 14y + 2 = 0$ --------------- (2)

or $10x + y = 13(y - x) + 2$

$\Rightarrow 10x + y = 13y - 13x + 2$

$\Rightarrow 10x + 13x + y - 13y = 2$

$\Rightarrow 23x - 12y - 2 = 0$ --------------- (3)

Now from equations (1) and (2), by cross-multiplication method, we get

$$\frac{x}{(-7)\times 2-(-14)\times(-1)} = \frac{y}{3\times(-1)-2\times 2} = \frac{1}{2\times(-14)-(-7)\times 3}$$

$$\Rightarrow \frac{x}{-14-14} = \frac{y}{-3-4} = \frac{1}{-28+21}$$

$$\Rightarrow \frac{x}{-28} = \frac{y}{-7} = \frac{1}{-7}$$

$$\Rightarrow \frac{x}{-28} = \frac{1}{-7} \text{ and } \frac{y}{-7} = \frac{1}{-7}$$

$$\Rightarrow x = \frac{-28}{-7} \text{ and } y = \frac{-7}{-7}$$

$$\Rightarrow x = 4 \text{ and } y = 1$$

In this case, the number $= 10 \times 4 + 1 = 41$

Or from equations (1) and (3), by cross-multiplication method, we get

$$\frac{x}{(-7)\times(-2)-(-12)\times(-1)} = \frac{y}{23\times(-1)-2\times(-2)} = \frac{1}{2\times(-12)-(-7)\times 23}$$

$$\Rightarrow \frac{x}{14-12} = \frac{y}{-23+4} = \frac{1}{-24+161}$$

$$\Rightarrow \frac{x}{2} = \frac{y}{-19} = \frac{1}{137}$$

$$\Rightarrow \frac{x}{2} = \frac{1}{137} \text{ and } \frac{y}{-19} = \frac{1}{137}$$

$\Rightarrow x = \frac{2}{137}$ and $y = \frac{-19}{137}$ which is impossible as a digit of number cannot be fraction or negative.

Thus, there is only one such number i.e.41.

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