CHAPTER 14
PROBABILITY

❖ *Some terms associated with the probability of an event*

- ***Random or non-deterministic experiment***

  An experiment, whose result cannot be uniquely predicted even if the previous results of the same experiment conducted under similar conditions are all known is called a random or a non-deterministic experiment.

- ***Sample Space & Event***

  The totality of all the possible outcomes of an experiment is called sample space of the experiment. Any component of the sample space is an event.

- ***Equally likely events***

  Events are said to be equally likely if there is no valid reason to say that one event has more chance to occur than the others.

- ***Mutually exclusive events***

  Events are said to be mutually exclusive if the happening of one forecloses the happening of all the others.

- ***Independent events***

  Events are said to be independent if the occurrence of one has no effect on the occurrence of the other or others.

- ***Elementary events***

  An elementary event is one which cannot be further subdivided.

- ***Exhaustive set of events***

  A set of events is said to be exhaustive if all the possible outcomes are included.

- ***Favourable outcomes***

  For every experiment, out of the set of exhaustive outcomes, those entailing the occurrence of a particular event are called the favourable ones for the event.

❖ *Classical or Mathematical or a Priori definition of Probability due to Laplace*

Out of n exhaustive, equally likely and mutually exclusive outcomes, if m are favourable to the event A, then the probability of the occurrence of the event A denoted by P(A) is the ratio m:n and we write

$$P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$$

Note

(i) $0 \le P(A) \le 1$
(ii) The probability of an impossible event is 0 (zero) and that of sure event is 1.
(iii) The symbol $P(\bar{A})$ denotes the probability of not happening of the event A.

❖ $P(A) + P(\bar{A}) = 1$

Proof: Out of n exhaustive, equally likely and mutually exclusive outcomes, if m are favourable to the event A, n − m are not favourable to the event A.

Then, $P(\bar{A}) = \frac{n-m}{n} = \frac{n}{n} - \frac{m}{n} = 1 - \frac{m}{n} = 1 - P(A)$

$\therefore P(A) + P(\bar{A}) = 1$

Note

(i) $P(\bar{A}) = 1 - P(A)$
(ii) If events A, B and C are mutually exclusive and exhaustive then $P(A) + P(B) + P(C) = 1$.
(iii) For independent events A and B, $P(AB) = P(A)P(B)$. Here the symbol $P(AB)$ denotes the happening of both the events A and B.

❖ *A note on playing cards*

A deck or pack of cards consists of 52 cards divided into two sets of 26 cards each. One set is red set and the other set is black set. In each colour set, there are two suits of 13 cards each. In a suit, there are 3 face cards called King, Queen and Jack. In each suit, there is one card called Ace and nine numerals starting from 2 and ending at 10. Ace, King, Queen and Jack of a suit are called four powers of the suit.

Demonstration of playing cards

Suit Name	Colour	Symbol	No. of Cards			
			Ace	Face	Numeral	Total
Heart	Red	♥	1	3	9	13
Diamond	Red	♦	1	3	9	13
Club	Black	♣	1	3	9	13
Spade	Black	♠	1	3	9	13

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SOLUTIONS

EXERCISE 14.1

1. Give some events in our day to day life that are mutually exclusive.

Solution:

The result of a lottery, the result of game, the gender of a person, the result of a student's examination, the result of the competition for a gold medal among the number of contenders etc.

2. If two events A and B are such that P(A)+P(B) =1, then, write P(B) in terms of $P(\bar{A})$.

Solution:

We know, $P(A) + P(\bar{A}) = 1$

But, $P(A) + P(B) = 1$ (given)

Then $P(A) + P(B) = P(A) + P(\bar{A})$

$\therefore P(B) = P(\bar{A})$

3. If A, B, C are three events which are equally likely but not forming an exhaustive system of events. Then, show that P(A) = P(B) = P(C)$\neq \frac{1}{3}$. Give one example of such a situation.

Solution:

We have, P(A) = P(B) = P(C) [$\because$ A, B, C are equally likely]

And P(A) + P(B) + P(C) $\neq$1 [$\because$ A, B, C are not forming an exhaustive system of events]

Then P(A) + P(A) + P(A) $\neq$1 [$\because$P(A) = P(B) = P(C)]

$\Rightarrow 3\times P(A) \neq 1$

$\Rightarrow P(A) \neq \frac{1}{3}$

$\therefore P(A) + P(B) + P(C) \neq \frac{1}{3}$

Example:

Let A, B and C be the events of happening 1, 2 and 3 in throwing a die.

Here A, B, C are equally likely as they have the chance of getting one another, but not exhaustive as other probabilities of getting 4, 5, 6 are not considered.

$P(A) = \frac{1}{6}, P(B) = \frac{1}{6}, P(C) = \frac{1}{6}$

Here $P(A) = P(B) = P(C) \neq \frac{1}{3}$

4. Distinguish between subjective and objective probabilities giving examples in each case.

Solution: Suppose when A says that there is 90% chance of having rain tomorrow, B may say that there is hardly 50% chance of having rain tomorrow. In this case the probability of rain due to A is $\frac{9}{10}$. On the other hand probability of rain due to B is $\frac{1}{2}$. Such a probability whose value depends on the state of mind and opinion of the observer is subjective probability. By the objective probability of an event we mean the probability of happening of an event which is the same for all observers. Example:- The probability of a day of a week chosen at random to be a Sunday is $\frac{1}{7}$. In this case the probability is same for all observers.

5. When do you say that a die is fair?

Solution: A die is said to be fair when each of the six faces has equal probability of turning up in every toss.

6. When a fair coin is tossed three times, how many outcomes can be there? Are the outcomes equally likely?

Solution: When a fair coin is tossed 3 times,

Sample Space= {HHH, HHT, HTH, THH, TTT, TTH, THT, HTT}

So, the total number of possible outcomes = 8

Yes, all the possible outcomes are equally likely.

OR

Number of possible outcomes in each toss = 2 (viz. Head and Tail)

∴ number of possible outcomes when a fair coin is tossed three times $= 2 \times 2 \times 2 = 8$

Yes, all the possible outcomes are equally likely.

7. From an urn containing 4 white and 5 red balls, two balls are drawn at random. Find the probability that at least one is white.

Solution: Number of white balls =4

Number of red balls =5

Total number of balls = 4+5=9

The probability of getting a red ball in the first draw $= \frac{5}{9}$

The probability of getting a red ball in the second draw $= \frac{4}{8}$

∴ Probability that both are red $= \frac{5}{9} \times \frac{4}{8} = \frac{5}{18}$

∴ Probability that at least one is white = Probability that both are not red

$= 1 - \frac{5}{18} = \frac{13}{18}$ [$\because P(\bar{A}) = 1 - P(A)$]

8. *From a pack of cards, two cards are drawn at random after a thorough suffle. Find the probability that both are kings.*

Solution:

Total number of cards= 52

Number of kings= 4

Then, Probability of getting a king in the first draw $= \frac{4}{52} = \frac{1}{13}$

And probability of getting a king in the second draw $= \frac{3}{51} = \frac{1}{17}$

$\therefore$ probability that both are kings $= \frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$

9. *Two fair dice are rolled. Find the probability that sum of the points is 7. Also, find the sum of the points which is the most probable by showing all the possible sum of the points in a chart or table.*

Solution: Sample Space ={(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)

(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)

(3,1), (3,2), (2,3), (3,4), (3,5), (3,6)

(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)

(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)

(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

Total number of sample points = 6× 6 = 36

Number of sample points whose sum is 7 = 6 [viz. (1,5), (2,5), (3,4), (4,3), (5,2) and (6,1)]

$\therefore$ probability that sum of the points is 7 $= \frac{6}{36} = \frac{1}{6}$

Table showing all the possible sums:

Sum of the points	Favourable outcomes	No. of favourable outcomes	Probability
2	(1,1)	1	$\frac{1}{36}$
3	(1,2), (2,1)	2	$\frac{2}{36}=\frac{1}{18}$
4	(1,3), (2,2), (3,1)	3	$\frac{3}{36}=\frac{1}{12}$
5	(1,4), (2,3), (3,2), (4,1)	4	$\frac{4}{36}=\frac{1}{9}$
6	(1,5), (2,4), (3,4), (4,2), (5,1)	5	$\frac{5}{36}$
7	(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)	6	$\frac{6}{36}=\frac{1}{6}$
8	(2,6), (3,5), (4,4), (5,3), (6,2)	5	$\frac{5}{36}$
9	(3,6), (4,5), (5,4), (6,3)	4	$\frac{4}{36}=\frac{1}{9}$
10	(4,6), (5,5), (6,4)	3	$\frac{3}{36}=\frac{1}{12}$
11	(5,6), (6,5)	2	$\frac{2}{36}=\frac{1}{18}$
12	(6,6)	1	$\frac{1}{36}$
	Total	36	

We see that probability for the sum of the points is 7 is the greatest.

∴ required the sum of the points which is the most probable is 7.

10. Given that p is the probability that a person aged x years will die in a year, find the probability that none of the four persons all aged x years will die in a year.

Solution: The probability that a person aged x years will die in a year $= p$

∴ The probability that a person aged x years will not die in a year $= 1 - p$ [∵P($\bar{A}$) =1− P(A)]

∴ The probability that none of the four persons all aged x years will die in a year

$$= (1-p)\times(1-p)\times(1-p)\times(1-p)$$
$$= (1-p)^4$$

11. In the above exercise if Mr. A is one of the four persons, find the probability that at least one of them will die in a year and Mr. A is the first person to die.

Solution:

The two events L, none of the four persons dies in a year; M, at least one of them dies in a year are mutually exclusive and they form an exhaustive set.

Then, $P(L) + P(M) = 1$

i.e. $P(M) = 1 - P(L)$

But $P(L) = (1-p)^4$

Also, probability that Mr. A will die first out of the four persons $= \frac{1}{4}$

∴ The probability that at least one of them will die in a year and Mr. A is the first person to die

$$= \frac{1}{4} \times \{1 - P(L)\} = \frac{1}{4} \times \{1 - (1-p)^4\} = \frac{1-(1-p)^4}{4}$$

12. From a well suffled pack of cards, two cards are drawn at random. Find the probability that both the cards are diamonds.

Solution:

Total number of cards= 52

Number of diamonds = 13

Then, probability of getting a diamonds in the first draw $= \frac{13}{52} = \frac{1}{4}$

Total no. of remaining cards= 51

And total no. of remaining diamonds = 12

And probability of getting a diamonds in the second draw $= \frac{12}{51} = \frac{4}{17}$

∴ Probability of getting both the cards are diamonds $= \frac{13}{52} \times \frac{12}{51} = \frac{1}{17}$

13. A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that the letter is (i) a vowel (ii) a consonant.

Solution:

Total number of letters = 13

Number of vowels = 6

Number of consonants = 7

(i) The probability that the letter chosen at random is a vowel $= \frac{6}{13}$

(ii) The probability that the letter chosen at random is a consonant $= \frac{7}{13}$

OR

(i) Total number of letters $= 13$

Number of vowels $= 6$

$\therefore$ The probability that the letter chosen at random is a vowel $= \frac{6}{13}$

(ii) The probability that the letter chosen at random is a consonant

=The probability that the letter chosen at random is not a vowel

$= 1 - \frac{6}{13}$ $[\because P(\bar{A}) = 1 - P(A)]$

$= \frac{7}{13}$

14. ***In a 20-20 cricket match, a batsman hits a boundary 5 times out of 24 balls he faced. Find the probability that he did not hit a boundary in a ball he faced.***

Solution:

Number of boundary balls out of 24 balls he faced $= 5$

So, the number of balls the batsman did not hit a boundary $= 24 - 5 = 19$

$\therefore$ Probability of hitting not a boundary $= \frac{19}{24}$

OR

Total number of balls he faced $= 24$

Number of balls hitting a boundary $= 5$

$\therefore$ Probability of hitting a boundary $= \frac{5}{24}$

$\therefore$ Probability of hitting not a boundary $= 1 - \frac{5}{24}$ $[\because P(\bar{A}) = 1 - P(A)]$

$$= \frac{19}{24}$$

15. A die is thrown. Find the probability of the following events.

(a) A prime number will appear.

(b) A number less than 6 will appear.

(c) A number more than 6 will appear.

(d) A number less than or equal to 3 will appear.

Solution:

Sample Space = {1, 2, 3, 4, 5, 6}

∴Number of sample points = 6

(a) Number of primes = 3 [viz. 2, 3, 5]

∴ Probability that a prime number will appear = $\frac{3}{6} = \frac{1}{2}$

(b) Number of sample points less than 6 = 5 [viz. 1, 2, 3, 4, 5]

∴ Probability that a number less than 6 will appear = $\frac{5}{6}$

(c) There is no any number more than 6.

∴Number of sample points more than 6 = 0

∴ Probability that a number more than 6 will appear= $\frac{0}{6}$ =0

(d) Number of sample points less than or equal to 3 = 3 [viz. 1, 2, 3]

∴ Probability that a number less than or equal to 3 will appear= $\frac{3}{6} = \frac{1}{2}$

16. There are four men and three ladies in a council. If two council members are selected at random for a committee, how likely is that both are ladies?

Solution:

Number of men =4

Number of ladies =3

Total number of members $= 4 + 3 = 7$

∴ Probability that the first selected member is a lady $= \frac{3}{7}$

No. of remaining ladies =2

No. of remaining members =6

∴ Probability that the second selected member is a lady $= \frac{2}{6} = \frac{1}{3}$

∴ Probability that both the members who are selected are ladies $= \frac{3}{7} \times \frac{1}{3} = \frac{1}{7}$

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