CHAPTER – 1
NUMBER SYSTEM

❖ **Euclid's Division Lemma (or Euclid's Division Algorithm)**

Let a and b be any two integers and b>0. Then there exists unique integers q and r such that $a = bq + r$ and $0 \leq r < b$.

- **Lemma**

  A lemma is a provable statement used in proving another statement.

- **Algorithm**

  An algorithm is a well defined sequence of steps forming a process of solving given problem.

❖ **Euclid's Algorithm for finding HCF of two given positive integers**

1. Find the quotient and remainder of the division of the greater number by the smaller.
2. If the remainder is zero, then the divisor is the HCF.
3. Else, taking the previous remainder as the new divisor and the previous divisor as the new dividend, find the quotient and remainder.
4. Continue the process till the remainder is zero.

   The last divisor is the required HCF.

❖ **Theorem:** If $a = bq + r$, then $(a, b) = (b, r)$.

Note: The symbol (a, b) denotes the HCF of two positive integers a and b.

SOLUTIONS

EXERCISE 1.1

1. Using Euclid's algorithm find the HCF of

(i) 1240 and 1984

Solution: We have

$1984 = 1240 \times 1 + 744$

$1240 = 744 \times 1 + 496$

$744 = 496 \times 1 + 248$

$496 = 248 \times 2 + 0$

$\therefore (1984, 1240) = 248$

(ii) **348 and 504**

Solution: We have

$504 = 348 \times 1 + 156$

$348 = 156 \times 2 + 36$

$156 = 36 \times 4 + 12$

$36 = 12 \times 3 + 0$

$\therefore (504,348) = 12$

(iii) **986 and 899**

Solution: We have

$986 = 899 \times 1 + 87$

$899 = 87 \times 10 + 29$

$87 = 29 \times 3 + 0$

$\therefore (986,899) = 29$

(iv) **4216 and 1240**

Solution: We have

$4216 = 1240 \times 3 + 496$

$1240 = 496 \times 2 + 248$

$496 = 248 \times 2 + 0$

$\therefore (4216,1240) = 248$

(v) **10605 and 5256**

Solution:-

We have

$10605 = 5256 \times 2 + 93$

$5256 = 93 \times 56 + 48$

$93 = 48 \times 1 + 45$

$48 = 45 \times 1 + 3$

$45 = 3 \times 15 + 0$

$\therefore (10605, 5256) = 3$

(vi) **10005 and 9269**

Solution:-

We have

$10005 = 9269 \times 1 + 736$

$9269 = 736 \times 12 + 437$

$736 = 437 \times 1 + 299$

$437 = 299 \times 1 + 138$

$299 = 138 \times 2 + 23$

$138 = 23 \times 6 + 0$

$\therefore$ (10005, 9269) =23

2. Show that the product of two consecutive integers is divisible by 2.

Solution:

Let a and $a+1$ be any two consecutive integers.

The integer a is of the form $2q$ or $2q+1$ for some integer q.

If $a = 2q$, $a(a+1) = 2q(2q+1)$ which is divisible by 2.

If $a = 2q+1$, $a(a+1) = (2q+1)(2q+1+1)$

$= (2q+1)(2q+2)$

$= (2q+1) \times 2(q+1)$

$= 2(2q+1)(q+1)$ which is divisible by 2.

Thus, the product of two consecutive integers is divisible by 2.

3. Show that the product of two consecutive even integers is divisible by 8.

Solution:

Let $2a$ and $2a+2$ be any two consecutive even integers.

The integer a is of the form $2q$ or $2q+1$.

If $a = 2q$, $2a(2a+2) = 2 \times 2q(2 \times 2q + 2)$

$= 4q(4q+2)$

$= 4q \times 2(2q+1)$

$= 8q(2q+1)$, which is divisible by 8.

If $a = 2q + 1$, $2a(2a + 2) = 2(2q + 1)\{2(2q + 1) + 2\}$

$= 2(2q + 1)(4q + 2 + 2)$

$= 2(2q + 1)(4q + 4)$

$= 2(2q + 1) \times 4(q + 1)$

$= 8(2q + 1)(q + 1)$, which is divisible by 8.

Thus, the product of two consecutive even integers is divisible by 8.

4. Show that every integer is of the form 4q, 4q+1, 4q+2 or 4q- 1.

Solution:

Let a be any integer.

Taking $b = 4$ and applying Euclid's division lemma we get

$a = 4q + r$, where $r = 0$, 1, 2 or 3

If $r = 0, a = 4q + 0 = 4q$

If $r = 1, a = 4q + 1$

If $r = 2, a = 4q + 2$

If $r = 3, a = 4q + 3$

$= 4q + 4 - 1$

$= 4(q + 1) - 1$

$= 4k - 1$, where $k = q + 1$ is an integer.

Hence, every integer is of the form $4q$, $4q + 1$, $4q + 2$ or $4q - 1$.

5. Show that the product of three consecutive integers is divisible by 6.

Solution:-

Let $a - 1$, a and $a + 1$ be three consecutive integers.

The integer a is of the form $6q, 6q + 1, 6q + 2, 6q + 3, 6q - 2$ or $6q - 1$.

If $a = 6q$, $(a - 1)a(a + 1) = (6q - 1) \times 6q(6q + 1)$

$= 6q(6q - 1)(6q + 1)$ which is divisible by 6.

If $a = 6q + 1, (a - 1)a(a + 1) = (6q + 1 - 1)(6q + 1)(6q + 1 + 1)$

$= 6q(6q + 1)(6q + 2)$ which is divisible by 6.

If $a = 6q + 2, (a - 1)a(a + 1) = (6q + 2 - 1)(6q + 2)(6q + 2 + 1)$

$= (6q + 1)(6q + 2)(6q + 3)$

$= (6q + 1) \times 2(3q + 1) \times 3(2q + 1)$

$= 6(6q + 1)(3q + 1)(2q + 1)$ which is divisible by 6.

If $a = 6q + 3$, $(a-1)a(a+1) = (6q + 3 - 1)(6q + 3)(6q + 3 + 1)$

$= (6q + 2)(6q + 3)(6q + 4)$

$= 2(3q + 1) \times 3(2q + 1) \times 2(3q + 2)$

$= 6 \times 2(3q + 1)(2q + 1)(3q + 2)$ which is divisible by 6.

If $a = 6q - 2$, $(a-1)a(a+1) = (6q - 2 - 1)(6q - 2)(6q - 2 + 1)$

$= (6q - 3)(6q - 2)(6q - 1)$

$= 3(2q - 1) \times 2(3q - 1)(6q - 1)$

$= 6 \times (2q - 1)(3q - 1)(6q - 1)$ which is divisible by 6.

If $a = 6q - 1$, $(a-1)a(a+1) = (6q - 1 - 1)(6q - 1)(6q - 1 + 1)$

$= (6q - 2)(6q - 1) \times 6q$

$= 6q(6q - 2)(6q - 1)$ which is divisible by 6.

Thus, the product of three consecutive integers is divisible by 6.

6. Show that the square of an odd integer is of the form 8k + 1.

Solution:-

Let a be any odd integer.

Then a is of the form $4q \pm 1$.

Now, $a^2 = (4q \pm 1)^2$

$= (4q)^2 \pm 2 \times 4q \times 1 + 1^2$

$= 16q^2 \pm 8q + 1$

$= 8(2q^2 \pm q) + 1$

$= 8k + 1$, where $k = 2q^2 \pm q$ is an integer.

Thus, the square of an odd integer is of the form 8k + 1.

7. If a is divisible by neither 2 nor 3, show that $a^2 - 1$ is divisible by 24.

Solution:-

As a is divisible by neither 2 nor 3, a is of the form $12q \pm 1$ or $12q \pm 5$.

If $a = 12q \pm 1$, $a^2 - 1 = (12q \pm 1)^2 - 1$

$= (12q)^2 \pm 2 \times 12q \times 1 + 1^2 - 1$

$= 144q^2 \pm 24q + 1 - 1$

$= 24(6q^2 \pm q)$, which is divisible by 24.

If $a = 12q \pm 5, a^2 - 1 = (12q \pm 5)^2 - 1$

$= (12q)^2 \pm 2 \times 12q \times 5 + 5^2 - 1$

$= 144q^2 \pm 120q + 24$

$= 24(6q^2 \pm 5q + 1)$, which is divisible by 24.

Thus if a is divisible by neither 2 nor 3, then $a^2 - 1$ is divisible by 24.

8. Show that any square number cannot be put in the form 4k + 2.

Solution:-

We have,

4k+2=2(2k+1).

For this product to be a perfect square, the number 2k+1 must have 2 as its prime factor, which is impossible as the number 2k+1 is odd. Therefore, the number 4k+2 cannot be a perfect square.

Hence, any square number cannot be put in the form 4k+2.

9. Show that any square number is of the form 3n or 3n+1.

Solution:-

Let a be any integer. Then a is of the form $3q$, $3q+1$ or $3q-1$

If $a = 3q$, $\boldsymbol{a^2} = (3q)^2$

$= 3 \times 3q^2$

$= 3n$ where $n = 3q^2$ is an integer.

If $a = 3q + 1$, $\boldsymbol{a^2} = (3q + 1)^2$

$= (3q)^2 + 2 \times 3q \times 1 + 1^2$

$= 9q^2 + 6q + 1$

$= 3(3q^2 + 2q) + 1$

$= 3n + 1$, where $n = 3q^2 + 2q$ is an integer.

If $a = 3q - 1$, $\boldsymbol{a^2} = (3q - 1)^2$

$= (3q)^2 - 2 \times 3q \times 1 + 1^2$

$= 9q^2 - 6q + 1$

$= 3(3q^2 - 2q) + 1$

$= 3n + 1$, where $n = 3q^2 - 2q$ is an integer.

Thus, any square integer is of the form $3n$ or $3n + 1$.

10. Show that one of three consecutive odd integers is a multiple of 3.

Solution:-

Let 2a+1, 2a+3 and 2a+5 be any three consecutive odd integer.

Then a is of the form $3q, 3q+1$ or $3q+2$.

If $a = 3q,\ 2a+3 = 2\times 3q+3$

$= 6q+3$

$= 3(2q+1)$ which is a multiple of 3.

If $a = 3q+1, 2a+1 = 2(3q+1)+1$

$= 6q+2+1$

$= 6q+3$

$= 3(2q+1)$ which is a multiple of 3.

If $a = 3q+2, 2a+5 = 2(3q+2)+5$

$= 6q+4+5$

$= 6q+9$

$= 3(2q+3)$ which is a multiple of 3

Thus, one of three consecutive odd integers is a multiple of 3.

11. Show that the product of any three consecutive even integers is divisible by 48.

Solution:-

Let $a-2, a$ and $a+2$ be any three consecutive even integers.

Then a is of the form $2q$.

Now, $(a-2)a(a+2) = (2q-2)\times 2q\times(2q+2)$

$= 2(q-1)\times 2q\times 2(q+1)$

$= 8(q-1)q(q+1)$

$= 8\times 6k$ where $6k = (q-1)q(q+1) \in Z$ for the product of three consecutive integers is divisible by 6.

$= 48k$ which is divisible by 48.

Thus, the product of any three consecutive even integers is divisible by 48.

Extra Question

Show that one of three consecutive even integers is a multiple of 3.

Solution:-

Let $2a, 2a+2$ and $2a+4$ be any three consecutive even integers.
The integer a is of the form $3q, 3q+1$ or $3q+2$ for some integer q.

If $a = 3q, 2a = 2 \times 3q$

$= 3 \times 2q$ which is a multiple of 3.

If $a = 3q + 1, 2a + 4 = 2(3q + 1) + 4$

$= 6q + 2 + 4$

$= 6q + 6$

$= 6(q + 1)$

$= 3 \times 2(q + 1)$ which is a multiple of 3.

If $a = 3q + 2, 2a + 2 = 2(3q + 2) + 2$

$= 6q + 4 + 2$

$= 6q + 6$

$= 6(q + 1)$

$= 3 \times 2(q + 1)$ which is a multiple of 3.

Thus, one of three consecutive even integers is a multiple of 3.

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- **Fundamental Theorem of Arithmetic or Unique Factorisation Theorem:**

  Every composite number can be expressed as a product of primes uniquely except for the order of the factors.

  OR

  Every integer n > 1 can be expressed uniquely in the form

  $$n = p_1^{a_1} p_2^{a_2} p_3^{a_3}, \dots \dots \dots \dots \dots . p_k^{a_k}$$

  where $p_1, p_2, p_3, \dots \dots \dots \dots, p_k$ are primes such that $p_1 < p_2 < p_3 < \dots \dots \dots \dots < p_k$ and $a_1, a_2, a_3, \dots \dots \dots \dots, a_k$ are all positive integers.

Note:

(i) For every integer $n > 1$, the expression $n = p_1^{a_1} p_2^{a_2} p_3^{a_3} \dots \dots \dots \dots \dots \dots . . p_k^{a_k}$, where $p_1, p_2, p_3, \dots \dots \dots p_k$ are primes such that $p_1 < p_2 < p_3 <, \dots \dots \dots < p_k$ and $a_1, a_2, a_3, \dots \dots \dots, a_k$ are all positive integers is called canonical decomposition of n.

(ii) (a,b) denotes the HCF of a and b and [a,b] denotes the LCM of a and b.

(iii) HCF= Product of the smallest power of each common factor in the numbers

(iv) LCM= Product of the greatest power of each prime factor involved in the numbers

(v) For two integers,

(a) HCF×LCM= Product of the integers.

(b) HCF= $\frac{\text{Product of the integers}}{\text{LCM}}$

(c) LCM= $\frac{\text{Product of the integers}}{\text{HCF}}$

(d) One of the numbers= $\frac{\text{HCF×LCM}}{\text{other number}}$

SOLUTIONS

EXERCISE 1.2

1. ***Find*** **the canonical decomposition of the numbers:**

(i) 1914 (ii) 2332 (iii) 4284 (iv) 190575 (v) 133848 (vi) 217350

Solution:-

(i) $1914 = 2 \times 3 \times 11 \times 29$

(ii) $2332 = 2^2 \times 11 \times 53$

(iii) $4284 = 2^2 \times 3^2 \times 7 \times 17$

(iv) $190575 = 3^2 \times 5^2 \times 7 \times 11^2$

(v) $133848 = 2^3 \times 3^2 \times 11 \times 13^2$

(vi) $217350 = 2 \times 3^3 \times 5^2 \times 7 \times 23$

2. Find (a,b), [a,b] and verify that (a,b)[a,b] = ab for each of the following pairs of integers:

(i) a =429, b = 715

Solution:-

$$a = 429 = 3 \times 11 \times 13$$
$$b = 715 = 5 \times 11 \times 13$$

$\therefore (a, b) = 11 \times 13 = 143$

and $[a, b] = 3 \times 5 \times 11 \times 13 = 2145$

Verification:

$$\begin{aligned}(a, b)[a, b] &= 143 \times 2145 \\ &= (11 \times 13) \times (3 \times 5 \times 11 \times 13) \\ &= (3 \times 11 \times 13) \times (5 \times 11 \times 13) \\ &= 429 \times 715 \\ &= ab\end{aligned}$$

Hence the result.

(ii) $\mathbf{a = 756, b = 1044}$

Solution:-

$a = 756 = 2^2 \times 3^3 \times 7$
$b = 1044 = 2^2 \times 3^2 \times 29$

$\therefore (a, b) = 2^2 \times 3^2 = 36$

and $[a, b] = 2^2 \times 3^3 \times 7 \times 29 = 21924$

Verification:

$(a, b)[a, b] = 36 \times 21924$

$= (2^2 \times 3^2) \times (2^2 \times 3^3 \times 7 \times 29)$

$= (2^2 \times 3^3 \times 7) \times (2^2 \times 3^2 \times 29)$

$= 756 \times 1044$

$= ab$

Hence the result.

(iii) $\mathbf{a = 576, b = 2520}$

Solution:-

$a = 576 = 2^6 \times 3^2$
$b = 2520 = 2^3 \times 3^2 \times 5 \times 7$

$\therefore (a, b) = 2^3 \times 3^2 = 72$

and $[a, b] = 2^6 \times 3^2 \times 5 \times 7 = 20160$

Verification:

$(a, b)[a, b] = 2 \times 20160$

$= (2^3 \times 3^2) \times (2^6 \times 3^2 \times 5 \times 7)$

$= (2^6 \times 3^2) \times (2^3 \times 3^2 \times 5 \times 7)$

$= 576 \times 2520$

$= ab$

Hence the result.

3. Find the HCF and LCM of the following integers by prime factorization method:

(i) 204, 1020, 1190

Solution:

204= $2^2 \times 3 \times 17$
1020= $2^2 \times 3 \times 5 \times 17$

1190= $2 \times 5 \times 7 \times 17$

$\therefore$ HCF= $2 \times 17 = 34$

and LCM= $2^2 \times 3 \times 5 \times 7 \times 17 = 7140$

(ii) 126, 882, 1617

Solution:-

$126 = 2 \times 3^2 \times 7$

$882 = 2 \times 3^2 \times 7^2$

$1617 = 3 \times 7^2 \times 11$

$\therefore$ HCF$= 3 \times 7 = 21$

and LCM$= 2 \times 3^2 \times 7^2 \times 11 = 9702$

(iii) 504, 2393, 4725

Solution:

$504 = 2^3 \times 3^2 \times 7$

$2394 = 2 \times 3^2 \times 7 \times 19$

$4725 = 3^3 \times 5^2 \times 7$

$\therefore$ HCF$= 3^2 \times 7 = 63$

and LCM$= 2^3 \times 3^3 \times 5^2 \times 7 \times 19 = 718200$

(iv) 1260, 1800, 3780, 7560

Solution:

$1260 = 2^2 \times 3^2 \times 5 \times 7$

$1800 = 2^3 \times 3^2 \times 5^2$

$3780 = 2^2 \times 3^3 \times 5 \times 7$

$7560 = 2^3 \times 3^3 \times 5 \times 7$

$\therefore$ HCF$= 2^2 \times 3^2 \times 5 = 180$

and LCM$= 2^3 \times 3^3 \times 5^2 \times 7 = 37800$

4. The HCF and LCM of two numbers are 27 and 29295 respectively. If one number is 837, find the other.

Solution:-

We have, HCF = 27

LCM = 29295

One number = 837

$\therefore$ other number $= \frac{\text{HCF}\times\text{LCM}}{837} = \frac{27\times29295}{837} = 27 \times 35 = 945$

5. Show that for any natural number *n*, the digit in unit's place of 3^n cannot be even.

Solution:- Since the only prime involved in the canonical decomposition of 3^n is 3, therefore the only prime that divides 3^n is 3. It means that 3^n is not divisible by any prime other than 3. In particular, 3^n is not divisible by 2.

We know that if a number is divisible by 2, the unit's place of the number is even. As the number is not divisible by 2, the unit's place of 3^n cannot be even.

6. Find any five consecutive composite numbers.

Solution:-

Let us consider the primes 2, 3 and 5 whose product is 30.

Clearly the five consecutive numbers

$$2 \times 3 \times 5 + 2 = 32$$
$$2 \times 3 \times 5 + 3 = 33$$
$$2 \times 3 \times 5 + 4 = 34$$
$$2 \times 3 \times 5 + 5 = 35$$
$$2 \times 3 \times 5 + 6 = 36$$

are consecutive composite numbers in which 32, 34 are divisible by 2; 33, 36 by 3 and 35 by 5.

7. Find any four consecutive odd composite numbers.

Solution:- Let us consider the primes 2, 3, 5 and 7 whose product is 210.

$2 \times 3 \times 5 \times 7 + 3 = 213$, which is divisible by 3.

$2 \times 3 \times 5 \times 7 + 5 = 215$, which is divisible by 5.

$2 \times 3 \times 5 \times 7 + 7 = 217$, which is divisible by 7.

$2 \times 3 \times 5 \times 7 + 9 = 219$, which is divisible by 3.

Therefore 213, 215, 217 and 219 are four consecutive odd composite numbers

8. Find the least number which when divided by 24, 36 and 60 will leave in each case the same remainder 7.

Solution:- We have, $24 = 2^3 \times 3$

$$36 = 2^2 \times 3^2$$
$$60 = 2^2 \times 3 \times 5$$
$$\text{LCM} = 2^3 \times 3^2 \times 5 = 360$$

So, the least number which when divided by 24, 36 and 60 will leave in each case the same remainder 7 is $360 + 7$ i.e. 367.

9. Find the least number which when divided by 7, 8 and 12 leaves the same remainder 5 in each case.

Solution:- We have, $7 = 1 \times 7$

$8 = 2^3$

$12 = 2^2 \times 3$

$\text{LCM} = 2^3 \times 3 \times 7 = 168$

So, least number which when divided by 7, 8 and 12 leaves the same remainder 5 in each case is 168 + 5 i.e. 173.

10. Find the least multiple of 13 which when divided by 5, 8 and 12 leaves the same remainder 2 in each case.

Solution:- LCM of 5, 8 and 12 = 120

By Euclid's division lemma, 120 = 13 × 9 + 3

The required number will be of the form

$120k + 2 = (13 \times 9 + 3)\text{k} + 2$

$= 13 \times 9\text{k} + (3\text{k} + 2)$

By inspection, the least positive integral value of k so that $120k + 2$ is divisible by 13 is 8.

Hence, the required number $= 120 \times 8 + 2$

$= 960 + 2$

$= 962$

11. By what prime numbers may 319 be divided so that the remainder is 4?

Solution:-

A required number will be a prime factor of 319 – 4 i.e. 315 but greater than 4.

$315 = 3^2 \times 5 \times 7$

The prime factors of 315 are 3, 5 and 7.

∴ The required numbers are 5 and 7.

12. By what numbers may 27 be divided so that the remainder is 3?

Solution:-

A required number will be greater than 3 and will be a factor of 27 – 3 i.e. 24.

We have $24 = 2^3 \times 3$

Hence the factors of 24 are *1, 2, 3,* $2 \times 2, 2 \times 3, 2 \times 2 \times 2, 2 \times 2 \times 3, 2 \times 2 \times 2 \times 3$

i.e. 1, 2, 3, 4, 6, 8, 12 and 24.

∴ The required numbers are 4, 6, 8, 12 and 24.

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- ***Field Properties of Real Numbers:***

1. ***Closure under addition:-** The sum of two real numbers is a real number i.e. $x + y \in R$ whenever $x, y \in R$.*
2. ***Associativity of addition:-** For every $x, y, z \in R, (x + y) + z = x + (y + z)$*
3. ***Commutativity of addition:-** $x + y = y + x$ for every $x, y \in R$.*
4. ***Existence of additive identity:-** There exists a real number 0 (zero) called the additive identity such that $x + 0 = x$ for every $x \in R$.*
5. ***Existence of additive inverse:-** For each $x \in R$, there exists $-x \in R$ called the additive inverse or negative of x such that $x + (-x) = 0$(additive identity).*
6. ***Closure under multiplication:-**The product of two real numbers is a real number i.e. $xy \in R$ whenever $x, y \in R$.*
7. ***Associativity of multiplication:-** For every $x, y, z \in R, (xy)z = x(yz)$*
8. ***Commutativity of multiplication:-** $xy = yx$ for every $x, y \in R$.*
9. ***Existence of multiplicative identity:-** There exists a real number 1, called the multiplicative identity such that $x \times 1 = x$ for any $x \in R$.*
10. ***Existence of multiplicative inverse:-** For each non-zero real number x, there exists $\frac{1}{x} \in R$ called the multiplicative inverse or reciprocal of x such that $x \times \frac{1}{x} = 1$(multiplicative identity).*
11. ***Multiplication distributes over addition:-** For any real number x, y, z, $x(y + z) = xy + xz$*

- ***Corollaries:***

1. ***Cancellation law for addition:** If $x, y, z \in R$ and $x + y = x + z$, then $y = z$.*
2. ***Cancellation law for multiplication:** If $x, y, z \in R, x \neq 0$ and $xy = xz$, then $y = z$.*
3. ***For any $x \in R$, $x.0 = 0$.***
4. ***For $x, y \in R, x(-y) = -xy$***
5. ***For $x, y \in R, (-x)(-y) = xy$***
6. ***If $x, y \in R$, and $xy = 0$, then $x = 0$ or $y = 0$.***

❖ ***Absolute Value or Modulus of a Real Number: The absolute value or modulus of a real number x, denoted by $|x|$is defined by***

$$|x| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

or

$$|x| = \begin{cases} 0 \text{ if } x = 0 \\ \text{the greater of } x \text{ or } -x \text{ if } x \neq 0 \end{cases}$$

❖ ***Some fundamental properties of absolute values of real numbers:***

1. $|x| \geq 0$
2. $|-x| = |x|$
3. $|xy| = |x||y|$
4. $|x+y| \leq |x| + |y|$
5. $|x-y| \geq |x| - |y|$ *and* $|x-y| \geq |y| - |x|$
6. $|x-y| < \delta$ *if and only if* $y - \delta < x < y + \delta$

SOLUTIONS

EXERCISE 1.3

1. Examine whether the following statements are true or false:

(i) The reciprocal of an irrational number is irrational.

(ii) The set of natural numbers contains additive identity.

(iii) The set of integers has multiplicative identity.

(iv) The reciprocal of a non-zero rational number is rational.

(v) The operation of subtraction in R is commutative.

(vi) The operation of division in R is associative.

Ans:- (i) True (ii) False (iii) True (iv) True (v) False (vi) False

2. (a) Is there any real number x such that $\frac{1}{x} \notin$ R?

Ans:- Yes, the real number is 0.

(b) Is there any $x \in$ R such that $-x \notin$ R?

Ans:- No, there is no any $x \in$ R such that $-x \notin$ R.

(c)Is there any $x \in$ R such that x^2 is not positive?

Ans:- Yes, that is 0.

(d) Is there any $x \in$ R such that x^2 is negative?

Ans:- No, there is no any $x \in$ R such that x^2 is negative.

(e) Can two different real numbers have the same absolute value?
Ans:- Yes, two different real numbers can have the same absolute value.

3. If $|a| = |b|$, find all possible relations between a and b.

Solution:-

We have, $|a| = |b|$

$\Rightarrow |a|^2 = |b|^2$

$\Rightarrow a^2 = b^2$

$\Rightarrow a^2 - b^2 = 0$

$\Rightarrow (a - b)(a + b) = 0$

$\Rightarrow a - b = 0$ or $a + b = 0$

$\Rightarrow a = b$ or $a = -b$, which are the possible relations between a and b.

4. Give any three values of x satisfying $|x - 3| < 1$.

Solution:-

We have $|x - 3| < 1$

$\Rightarrow 3 - 1 < x < 3 + 1$ $\quad$ [$\because |x - y| < \delta$ if and only if $y - \delta < x < y + \delta$]

$\Rightarrow 2 < x < 4$

So, any three values of satisfying $|x - 3| < 1$are 2.5, 3 and 3.5.

5. Find x if

(i) $|x - 2| = 0$

Solution:-

We have, $|x - 2| = 0$

$\Rightarrow x - 2 = 0$

$\therefore x = 2$

(ii) $|x - 2| = 3$

Solution:-

We have, $|x - 2| = 3$

$\Rightarrow x - 2 = 3$ or $x - 2 = -3$

$\Rightarrow x = 3 + 2$ or $x = -3 + 2$

$\Rightarrow x = 5$ or $x = -1$

(iii) $|x - 3| = \sqrt{2}$

Solution:-

We have, $|x - 3| = \sqrt{2}$

$\Rightarrow x - 3 = \sqrt{2}$ or $x - 3 = -\sqrt{2}$

$\therefore x = 3 + \sqrt{2}$ or $x = 3 - \sqrt{2}$

(iv) $|x - 2| = x$

Solution:-

We have, $|x - 2| = x$

$\Rightarrow x - 2 = -x$ or $x - 2 = x$, which is impossible.

$\Rightarrow x + x = 2$

$\Rightarrow 2x = 2$

$\therefore x = 1$

6. If $a^2 + b^2 = 0$, prove that a=0 and b=0.

Solution:-

a^2 and b^2 are square numbers.

$\therefore a^2$ and b^2 are non − negative integers.

Then $a^2 + b^2 = 0$ is possible only when $a^2 = 0$ and $b^2 = 0$

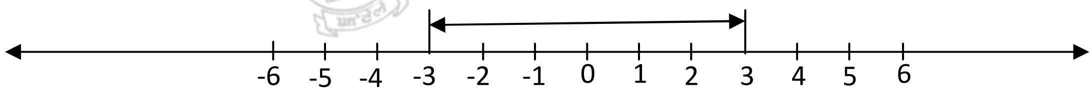
$\therefore a = 0$ and $b = 0$

7. Identify on the number line, the points x satisfying:

(i) $|x| \leq 3$

Solution:- $|x| \leq 3$

$\Rightarrow -3 \leq x \leq 3$

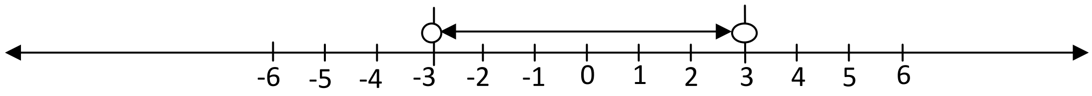


On the number line, the values of x satisfying $|x| \leq 3$ are represented by the points belong to the line segment joining -3 and 3.

(ii) $|\boldsymbol{x}| < 3$

Solution:- $|\boldsymbol{x}| < 3$

$\Rightarrow -3 < x < 3$

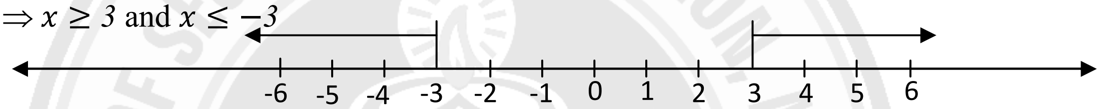


On the number line, the values of x satisfying $|\boldsymbol{x}| < 3$ are represented by the points belong to the line segment joining -3 and 3 excluding -3 and 3.

(iii) $|\boldsymbol{x}| \geq \mathbf{3}$

Solution:- $|x| \geq 3$

$\Rightarrow x \geq 3$ and $x \leq -3$

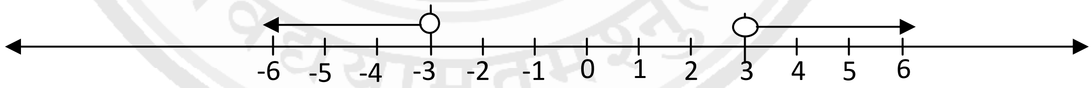


On the number line, the values of x satisfying $|x| \geq 3$ are represented by the points to the left of -3 and to the right of 3 including -3 and 3.

(iv) $|\boldsymbol{x}| > 3$

Solution:- $|\boldsymbol{x}| > 3$

$\Rightarrow x > 3$ and $x < -3$



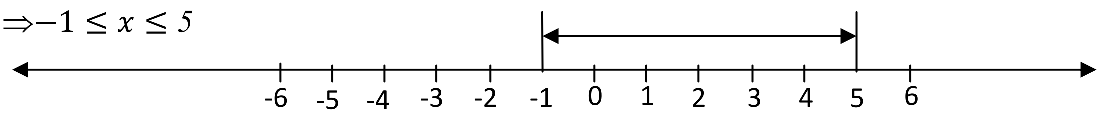
On the number line, the values of x satisfying $|\boldsymbol{x}| > 3$ are represented by the points to the left of -3 and to the right of 3 excluding -3 and 3.

(v) $|\boldsymbol{x} - \mathbf{2}| \leq \mathbf{3}$

Solution:- $|x - 2| \leq 3$

$\Rightarrow 2 - 3 \leq x \leq 2 + 3$

$\Rightarrow -1 \leq x \leq 5$



On the number line, the values of x satisfying $|x - 2| \leq 3$ are represented by the points belong to the line segment joining -1 and 5.

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