CHAPTER 3
MATHEMATICAL INDUCTION

Principle of Mathematical Induction

It states that if $P(n)$ be a mathematical proposition such that

i) $P(1)$ is true, and

ii) $P(K+1)$ is true whenever $P(K)$ is true where K is an arbitrary value of n.

i.e. $P(K)$ is true $\Rightarrow P(K+1)$ is true

then $P(n)$ is true $\forall n \in N$

SOLUTIONS

EXERCISE 3.1

1. **If $P(n)$ is the statement n^2+2 is a multiple of 3, then show that $P(2)$ is true and $P(3)$ is false?**

Solution: When $n=2$, $n^2+2=2^2+2=6$, which is a multiple of 3.

When $n=3$, $n^2+2=3^2+2=11$, which is not a multiple of 3.

$\therefore P(2)$ is true and $P(3)$ is false.

2. If $P(n)$ is the statement $5^{2n}+3n-1$ is divisible by 9state $P(2)$.

Is (i) $P(1)$ is true?

(ii) $P(3)$ is false?

Solution: $P(2)$ is $5^{2\times2}+3\times2-1=630$is divisible by 9.

i) When $n=1$, $5^{2n}+3n-1=5^2+3-1=27$, whichis divisible by 9.

$\therefore P(2)$ is true – Yes.

ii) When $n=3$, $5^{2n}+3n-1=5^{2\times3}+3\times3-1=15625+9-1$

$= 15633$, whichis divisible by 9.

$\therefore P(3)$ is false – No.

3. If $P(n)$ is the statement $n^2+n>15$ and if $P(K)$ is true, prove that $P(K+1)$ is true.

Solution: $P(n): n^2+n>15$

Here, $P(K)$ is true

i.e. $P(K)=K^2+K>15$

$=K(K+1)>15$

Now, $P(K+1)=(K+1)^2+(K+1)$

$=K^2+2K+1+K+1$

$=K^2+K+2K+2$

$=K(K+1)+2(K+1)>15 \; [\because K(K+1)>15]$

$\therefore P(K+1)$ is true if $P(K)$ is true.

4. **Prove by mathematical induction that** $\forall n \in N$ **,**

i) $2+4+6+\ldots\ldots\ldots\ldots\ldots+2n = n(n+1)$

Solution: Let $P(n): 2+4+6+\ldots\ldots\ldots\ldots\ldots+2n = n(n+1)$

When $n=1$

L.H.S = 2

R.H.S = $1(1+1) = 2 =$ L.H.S.

$\therefore P(1)$ is true.

Let us assume that $P(K)$ is true

i.e. $2+4+6+\ldots\ldots\ldots\ldots\ldots+2K = K(K+1)$

Adding $2(K+1)$ on both sides, we get,

$$2+4+6+\ldots\ldots\ldots\ldots\ldots+2K+2(K+1) = K(K+1)+2(K+1)$$
$$= (K+1)(K+2)$$
$$= (K+1)[(K+1)+1]$$

$\therefore P(K+1)$ is true if $P(K)$ is true

Hence, by the principle of mathematical induction, $P(n)$ is true $\forall n \in N$

(ii) $1+4+7+\ldots\ldots\ldots\ldots\ldots+(3n-2) = \dfrac{n(3n-1)}{2}$

Solution: Let $P(n): 1+4+7+\ldots\ldots\ldots\ldots\ldots+(3n-2) = \dfrac{n(3n-1)}{2}$

If $n=1$

L.H.S. =1

R.H.S. = $\dfrac{1(3\times1-1)}{2} = \dfrac{3-1}{2} = 1$ = L.H.S.

$\therefore P(1)$ is true.

Let us assume that $P(K)$ is true.

i.e. $1+4+7+\ldots\ldots\ldots\ldots\ldots+(3K-2) = \dfrac{K(3K-1)}{2}$

Adding $[3(K+1)-2 = 3K+1]$ on both sides, we get,

$$1+4+7+\ldots\ldots\ldots\ldots+(3K-2)+(3K+1) = \frac{K(3K-1)}{2}+(3K+1)$$
$$= \frac{3K^2-K+6K+2}{2}$$
$$= \frac{3K^2+3K+2K+2}{2}$$
$$= \frac{(3K+2)(k+1)}{2}$$
$$= \frac{(K+1)[3(K+1)-1]}{2}$$

$\therefore P(K+1)$ is true if $P(K)$ is true

Hence, by the principle of mathematical induction, $P(n)$ is true $\forall n \in N$

(iii) $1+6+11+.................+(5n-4)=\frac{1}{2}n(5n-3)$

Solution: Let $P(n):1+6+11+...................(5n-4)=\frac{1}{2}n(5n-3)$

When $n=1$

L.H.S. = 1

R.H. S = $\frac{1}{2}\times 1(5\times 1-3)=1$ = L.H.S.

$\therefore P(1)$ is true.

Let us assume that $P(K)$ is true

i.e. $1+6+11+...............+(5K-4)=\frac{1}{2}K(5K-3)$

Adding $5(K+1)-4=5K+1$ on both sides, we get,

$$1+6+11+...............+(5K-4)+(5K+1)=\frac{1}{2}K(5K-3)+(5K+1)$$
$$=\frac{1}{2}[5K^2-3K+10K+2]$$
$$=\frac{1}{2}[(K+1)(5K+2)]$$
$$=\frac{1}{2}(K+1)[5(K+1)-3]$$

$\therefore P(K+1)$ is true if $P(K)$ is true.

Hence, by the principle of mathematical induction, P(n) is true $\forall n\in N$

(v) $1^3+2^3+3^3+..............+n^3=\left[\frac{n(n+1)}{2}\right]^2$

Solution: Let $P(n):1^3+2^3+3^3+...............+n^3=\left[\frac{n(n+1)}{2}\right]^2$

When $n=1$

L.H.S. = $1^3=1$

R.H.S $=\left[\frac{1(1+1)}{2}\right]^2=\left[\frac{2}{2}\right]^2=1=L.H.S.$

$\therefore P(1)$ is true.

Let us assume that $P(K)$ is true.

$$1^3+2^3+3^3+................+K^3=\left[\frac{K(K+1)}{2}\right]^2$$

Adding $(K+1)^3$ on both sides, we get,

$$1^3+2^3+3^3+.................+K^3+(K+1)^3=\frac{K^2(K+1)^2}{4}+(K+1)^3$$

$$= (K+1)^2\left[\frac{K^2}{4}+K+1\right]$$

$$= (K+1)^2\left[\frac{K^2+4K+4}{4}\right]$$

$$= (K+1)^2\,\frac{(K+2)^2}{4}$$

$$= (K+1)^2\left(\frac{K+2}{2}\right)^2$$

$$=\left[\frac{(k+1)\{(K+1)+1\}}{2}\right]^2$$

$\therefore P(K+1)$ is true if $P(K)$ is true

Hence, by the principle of mathematical induction , P(n) is true $\forall n \in N$

(vi) $1.3+3.5+5.7+\cdots\ldots\ldots\ldots\ldots\ldots.+(2n-1)(2n+1)=\frac{K(4K^2+6K-1)}{3}$

Solution: $P(n): 1.3+3.5+5.7+\ldots\ldots\ldots\ldots\ldots\ldots+(2n-1)(2n-1)=\dfrac{n(4n^2+6n-1)}{3}$

When $n=1$

L.H.S. = 1.3 = 3

R.H.S. = $\dfrac{1(4.1^2+6.1-1)}{3}=\dfrac{9}{3}=3$ = L.H.S.

$\therefore P(1)$ is true

Let us assume that $P(K)$ is true

i.e. $1.3+3.5+5.7+\cdots\ldots\ldots\ldots\ldots\ldots.+(2K-1)(2K+1)=\frac{K(4K^2+6K-1)}{3}$.

Adding $(2K+1)(2K+3)$ on both sides ,we get

$$1.3+3.5+5.7+\ldots\ldots\ldots\ldots\ldots(2K-1)(2K+1)+(2K+1)(2K+3)$$

$$=\frac{K(4K^2+6K-1)}{3}+(2K+1)(2K+3)$$

$$=\frac{K(4K^2+6K-1)+3(4K^2+8K+3)}{3}$$

$$=\frac{4K^3+6K^2-K+12K^2+24K+9}{3}$$

$$=\frac{4K^3+18K^2+23K+9}{3}$$

$$=\frac{4K^3+4K^2+14K^2+14K+9K+9}{3}$$

$$=\frac{(K+1)[4(K+1)^2+6(K+1)-1}{3}$$

$\therefore P(K+1)$ is true if $P(K)$ is true

Hence, by the principle of mathematical induction, P(n) is true $\forall n \in N$

(vii) $1+\frac{1}{2}+\frac{1}{2^2}+\ldots\ldots\ldots\ldots\ldots+\frac{1}{2^{n-1}}=\frac{2^n-1}{2^{n-1}}$

Solution: Let P(n): $1+\frac{1}{2}+\frac{1}{2^2}+\ldots\ldots\ldots\ldots\ldots+\frac{1}{2^{n-1}}=\frac{2^n-1}{2^{n-1}}$

When $n=1$

L.H.S. = 1

R.H.S. $=\frac{2^1-1}{2^{1-1}}=\frac{1}{2^0}=\frac{1}{1}=1$ = L.H.S.

$\therefore P(1)$ is true

Let us assume that $P(K)$ is true. Then,

$$1+\frac{1}{2}+\frac{1}{2^2}+\ldots\ldots\ldots+\frac{1}{2^{K-1}}=\frac{2^K-1}{2^{K-1}}$$

Adding $\frac{1}{2^K}$ on both sides, we get,

$$1+\frac{1}{2}+\frac{1}{2^2}+\ldots\ldots\ldots\ldots\ldots+\frac{1}{2^{K-1}}+\frac{1}{2^K}=\frac{2^K-1}{2^{K-1}}+\frac{1}{2^K}$$

$$=\frac{2(2^K-1)+1}{2^K}$$

$$=\frac{2^{K+1}-2+1}{2^K}$$

$$=\frac{2^{K+1}-1}{2^{(K+1)-1}}$$

$\therefore P(K+1)$ is true if $P(K)$ is true.

Hence, by the principle of mathematical induction, P(n) is true $\forall n \in N$

(viii) $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\ldots\ldots\ldots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$

Solution: Let P(n): $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\ldots\ldots\ldots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$

If $n=1$

L.H.S. = $\frac{1}{1.3}=\frac{1}{3}$

R.H.S. = $\frac{1}{2\times1+1}=\frac{1}{3}$ = L.H.S.

$\therefore P(1)$ is true.

Let us assume that $P(K)$ is true

i.e. $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\ldots\ldots\ldots+\frac{1}{(2K-1)(2K+1)}=\frac{K}{2K+1}$

Adding $\frac{1}{(2K+1)(2K+3)}$ on both sides, we get,

$$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\dots\dots\dots\dots+\frac{1}{(2K-1)(2K+1)}+\frac{1}{(2K+1)(2K+3)}$$

$$=\frac{K}{2K+1}+\frac{1}{(2K+1)(2K+3)}$$

$$=\frac{K(2K+3)+1}{(2K+1)(2K+3)}$$

$$=\frac{2K^2+3K+1}{(2K+1)(2K+3)}$$

$$=\frac{(2K+1)(K+1)}{(2K+1)(2K+3)}=\frac{K+1}{2(K+1)+1}$$

$\therefore P(K+1)$ is true if $P(K)$ is true.

Hence, by the principle of mathematical induction, P(n) is true $\forall n \in N$

(ix) $1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\dots\dots\dots\dots+\frac{1}{3^{n-1}}=\frac{3^n-1}{2\times 3^{n-1}}$

Solution: Let $P(n): 1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\dots\dots\dots\dots\dots+\frac{1}{3^{n-1}}=\frac{3^n-1}{2\times 3^{n-1}}$

When $n=1$

L.H.S. = 1

R.H.S. = $\frac{3^1-1}{2\times 3^{1-1}}=\frac{2}{2}=1$ = L.H.S.

$\therefore P(1)$ is true

Let us assume that $P(K)$ is true.

i.e. $1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\dots\dots\dots\dots+\frac{1}{3^{K-1}}=\frac{3^K-1}{2\times 3^{K-1}}$

Adding $\frac{1}{3^{K+1-1}}=\frac{1}{3^K}$ on both sides we get,

$$1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\dots\dots\dots\dots+\frac{1}{3^{K-1}}+\frac{1}{3^K}=\frac{3^K-1}{2\times 3^{K-1}}+\frac{1}{3^K}$$

$$=\frac{3^k-1}{2\times 3^{k-1}}+\frac{1}{3^k}$$

$$=\frac{3^{K+1}-3+2}{2\times 3^K}$$

$$=\frac{3^{K+1}-1}{2\times 3^{(K+1)-1}}$$

$\therefore P(K+1)$ is true if $P(K)$ is true

Hence, by the principle of mathematical induction , P(n) is true $\forall n \in N$

(x) $2+2^2+2^3+...............+2^n=2(2^n-1)$

Solution: Let $P(n): 2+2^2+2^3+................+2^n=2(2^n-1)$

If $n=1$

L.H.S. = 2

R.H.S. = $2(2^1-1)=2$ = L.H.S.

$\therefore P(1)$ is true.

Let us assume that $P(K)$ is true. Then,

$$2+2^2+2^3+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+2^K=2(2^K-1).$$

Adding 2^{K+1} on both sides, we get

$$2+2^2+2^3+...................+2^K+2^{K+1}=2(2^K-1)+2^{K+1}$$
$$=2\times2^{K+1}-2$$
$$=2(2^{K+1}-1)$$

$\therefore P(K+1)$ is true if $P(K)$ is true

Hence, by the principle of mathematical induction , P(n) is true $\forall n\in N$

(xi) $1.2+2.2^2+3.2^2+..................+n.2^n=(n-1)2^{n+1}+2$

Solution: Let $P(n)=1.2+2.2^2+3.2^2+............+n.2^n=(n-1)2^{n+1}+2$

If $n=1$

L.H.S. = 1.2 =2

R.H.S. = $(1-1)2^{1+1}+2=2$ = L.H.S.

$\therefore P(1)$ is true

Let us assume that $P(K)$ is true

i.e. $1.2+2.2^2+3.2^3+..............+K.2^K=(K-1)2^{K+1}+2$

Adding $(K+1)2^{K+1}$ on the both sides, we get,

$$1.2+2.2^2+3.3^2+\cdots\ldots\ldots\ldots\ldots+K.2^K+(K+1)2^{K+1}$$
$$=(K-1)2^{K+1}+2+(K+1)2^{K+1}$$
$$=2^{K+1}(K-1+K+1)+2$$
$$=K.2.2^{K+1}+2$$
$$=(K+1-1)2^{(K+1)+1}+2$$

$\therefore P(K+1)$ is true if $P(K)$ is true.

Hence, by the principle of mathematical induction, P(n) is true $\forall n\in N$

(xii) $1.2.3+2.3.4+3.4.5+.........+n(n+1)(n+2)=\frac{1}{4}n(n+1)(n+2)(n+3)$

Solution: Let $P(n):1.2.3+2.3.4+3.4.5+......+n(n+1)(n+2)=\frac{1}{4}n(n+1)(n+2)(n+3)$

When $n=1$

L.H.S. = 1.2.3 = 6

R.H.s. = $\frac{1}{4}1(1+1)(1+2)(1+3)=\frac{1.2.3.\cancel{4}}{\cancel{4}}=6$

$\therefore P(1)$ is true

Let us assume that $P(K)$ is true

i.e. $1.2.3+2.3.4+3.4.5+..........+K(K+1)(K+2)=\frac{1}{4}K(K+1)(K+2)(K+3)$

Adding $(K+1)(K+2)(K+3)$ on both sides, we get

$$1.2.3+2.3.4+3.4.5+\cdots\ldots\ldots\ldots.+K(K+1)(K+2)+(K+1)(K+2)(K+3)$$

$$=\frac{1}{4}K(K+1)(K+2)(K+3)+(K+1)(K+2)(K+3)$$

$$=\frac{K(K+1)(K+2)(K+3)+4(K+1)(K+2)(K+3)}{4}$$

$$=\frac{(K+1)(K+2)(K+3)(K+4)}{4}$$

$\therefore P(K+1)$ is true if $P(K)$ is true

Hence, by the principle of mathematical induction, P(n) is true $\forall n\in N$

(xiii) $1^2+3^2+5^2+..............+(2n-1)^2=\frac{1}{3}n(2n-1)(2n+1)$

Solution: Let $P(n):1^2+3^2+5^2+.............+(2n-1)^2=\frac{1}{3}n(2n-1)(2n+1)$

When $n=1$

L.H.S. = $1^2=1$

R.H.S.= $\frac{1}{3}.1.(2-1)(2\times1+1)=1=$ L.H.S.

$\therefore P(1)$ is true

Let us assume that $P(K)$ is true

i.e. $1^2+3^2+5^2+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots+(2K-1)^2=\frac{1}{3}K(2K-1)(2K+1)$

Adding $(2K+1)^2$ on both sides, we get,

$$1^2+3^2+5^2+..........+(2K-1)^2+(2K+1)^2=\frac{K}{3}(2K-1)(2K+1)+(2K+1)^2$$

$$=\frac{K(2K-1)(2K+1)+3.(2K+1)^2}{3}$$

$$=\frac{(2K+1)[2K^2-K+3(2K+1)]}{3}$$

$$=\frac{(2K+1)(2K+3)(K+1)}{3}$$

$$=\frac{1}{3}(K+1)[2(K+1)-1][2(K+1)+1]$$

$\therefore P(K+1)$ is true if $P(K)$ is true

Hence, by the principle of mathematical induction , P(n) is true $\forall n \in N$

(xiv) $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+..........+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$

Solution: Let $P(n):\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+..........+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$

When $n=1$

L.H.S. = $\frac{1}{1.4}=\frac{1}{4}$

R.H.S. = $\frac{1}{3\times1+1}=\frac{1}{4}$ = L.H.S.

$\therefore P(1)$ is true

Let us assume that $P(K)$ is true

i.e. $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+..........+\frac{1}{(3K-2)(3K+1)}=\frac{K}{3K+1}$

Adding $\frac{1}{(3K+1)(3K+4)}$ on both sides,

$$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+...........+\frac{1}{(3K-2)(3K+1)}+\frac{1}{(3K+1)(3K+4)}$$

$$=\frac{K}{3K+1}+\frac{1}{(3K+1)(3K+4)}$$

$$=\frac{3K^2+4K+1}{(3K+1)(3K+4)}$$

$$=\frac{(K+1)(3K+1)}{(3K+4)(3K+1)}$$

$$=\frac{K+1}{3(K+1)+1}$$

$\therefore P(K+1)$ is true if $P(K)$ is true

Hence, by the principle of mathematical induction, P(n) is true $\forall n \in N$

(xv) $2^n > n$

Solution: Let $P(n): 2^n > n$

When $n = 1$

$2^1 > 1$ which is true

$\therefore P(1)$ true

Let us assume that $P(K)$ is true

i.e. $2^K > K$

$\Rightarrow 2 \times 2^K > 2K > K + 1$

$\Rightarrow 2^{K+1} > K + 1$

$\therefore P(K+1)$ is true if $P(K)$ is true

Hence, by the principle of mathematical induction , P(n) is true $\forall n \in N$

(xvi) $3^{2n} - 1$ is divisible by 4

Solution: Let $P(n): 3^{2n} - 1$ is divisible by 4

When $n = 1$

$3^{2\times1} - 1 = 3^2 - 1 = 8$,which is divisible by 4

$\therefore P(1)$ is true

Let us assume that P(K) is true for some K of n , then $3^{2K} - 1$ is divisible by 4

$\therefore 3^{2K} - 1 = 4q$, for some integer q.

Now, $P(K+1) = 3^{2(K+1)} - 1$

$= 3^{2K+2} - 1$

$= 3^{2K}.3^2 - 1$

$= 3^{2K}(8+1) - 1$

$= 3^{2K}.8 + 3^{2K} - 1$

$= 3^{2K}.8 + 4q$ $[\because 4q = 3^{2K} - 1]$

$= 4(3^{2K}.2 + q)$ which is divisible by 4.

$\therefore P(K+1)$ is true if $P(K)$ is true

Hence, by the principle of mathematical induction , $P(n)$ is true $\forall n \in N$

(xviii) $9^n - 1$ is divisible by 8

Solution: Let $P(n): 9^n - 1$ is divisible by 8

When $n = 1$

$9^1 - 1 = 9 - 1 = 8$,which is divisible by 8

$\therefore P(1)$ is true

Let us assume that $P(K)$ is true

i.e. $9^K - 1$ is divisible by 8,then

$9^K - 1 = 8q$ for some integer q.

Now, $P(K+1) = 9^{K+1} - 1$

$= 9^K.9 - 1$

$= 9^K(8+1) - 1$

$= 9^K.8 + 9^K - 1$

$= 9^K.8 + 8q$ $[\because 9^K - 1 = 8q]$

$= 8(9^K + q)$ which is divisible by 8

$\therefore P(K+1)$ is true if $P(K)$ is true

Hence, by the principle of mathematical induction, $P(n)$ is true $\forall n \in N$

(xviii) $a^n - b^n$ is divisible by $a - b$ $(a \neq b)$

Solution: Let $P(n): a^n - b^n$ is divisible by $a - b$

When $n = 1$,

$a^1 - b^1 = a - b$ which is divisible by a-b

$P(1)$ is true

Let us assume that $P(K)$ is true

i.e. $a^{K-}b^K$ is divisible by $a - b$

$\therefore a^{K-}b^K = (a-b)q$ for some polynomial q

Now, $P(K+1) = a^{K+1} - b^{K+1}$

$= a^K.a - b^K.b$

$= a^K.a - a^K.b + a^K.b - b^K.b$ $[-a^K.b + a^K.b = 0]$

$= a^K(a-b) + b(a^K - b^K)$

$= a^K(a-b) + b.(a-b)q$ $[\because a^K - b^K = (a-b)q]$

$= (a-b)[a^K + b.q]$ which is divisible by $a - b$

$\therefore P(K+1)$ is true if $P(K)$ is true

Hence, by the principle of mathematical induction , $P(n)$ is true $\forall n \in N$

(xix) $x^n - 1$ is divisible by $x-1 (x \neq 1)$

Solution: When $n=1$

$x^1 - 1 = x - 1$ which is divisible by $x-1$

$\therefore P(1)$ is true

Let us assume that $P(K)$ is true

i.e. $x^K - 1$ is divisible by $x-1$

$\therefore x^K - 1 = (x-1)q$ for some polynomial q

Now, $P(K+1) = x^{K+1} - 1$

$= x^K.x - 1$

$= x^K.x - x + x - 1$ $[\because -x + x = 0]$

$= x(x^K - 1) + x - 1$

$= x(x-1)q + (x-1)$ $[\because x^K - 1 = (x-1)q]$

$= (x-1)[xq+1]$ which is divisible by $x-1$.

$\therefore P(K+1)$ is true if $P(K)$ is true.

Hence, by the principle of mathematical induction , $P(n)$ is true $\forall n \in N$

(xx) $a^{2n} - b^{2n}$ is divisible by $(a+b)$, $(a \neq -b)$

Solution: Let $P(n): a^{2n} - b^{2n}$ is divisible by $a+b$

When $n=1$

$a^{2.1} - b^{2.1} = a^2 - b^2 = (a+b)(a-b)$ which is divisible by $a+b$

$\therefore P(1)$ is true

Let us assume that $P(K)$ is true

i.e. $a^{2K} - b^{2K}$ is divisible by $a+b$

$\therefore a^{2K} - b^{2K} = (a+b)q$ for some poly. q

Now, $P(K+1) = a^{2(K+1)} - b^{2(K+1)}$

$= a^{2K+2} - b^{2K+2}$

$= a^{2K}.a^2 - b^{2K}.b^2$

$= a^{2K}.a^2 - a^{2K}.b^2 + a^{2K}.b^2 - b^{2K}.b^2$ $[-a^{2K}b^2 + a^{2K}.b^2 = 0]$

$= a^{2K}(a^2 - b^2) + b^2(a^{2K} - b^{2K})$

$= a^{2K}.(a^2 - b^2) + b^2.(a+b)q$ $[\because (a+b)q = a^{2K} - b^{2K}]$

$= a^{2K}.(a+b)(a-b) + b^2(a+b)q$

$= (a+b)[a^{2K}(a-b) + b^2 q]$ which is divisible by $a+b$.

$\therefore P(K+1)$ is true if $P(K)$ is true

Hence, by the principle of mathematical induction , $P(n)$ is true $\forall n \in N$

(xxi) $a^{2n+1}+b^{2n+1}$ is divisible by $a+b,(a\neq -b)$

Solution: Let $P(n): a^{2n+1}+b^{2n+1}$ is divisible by $a+b$

When $n=1$

$a^{2.1+1}+b^{2.1+1}=a^3+b^3$

$=(a+b)(a^2-ab+b^2)$ which is divisible by $a+b$

$\therefore P(1)$ is true

Let us assume that $P(K)$ is true

i.e. $a^{2K+1}+b^{2K+1}$ is divisible by $a+b$

$\therefore a^{2K+1}+b^{2K+1}=(a+b)q$ for some poly. q

Now, $a^{2(K+1)+1}+b^{2(K+1)+1}$

$=a^{2K+3}+b^{2K+3}$

$=a^{2K+1}.a^2+b^{2K+1}.b^2$

$=a^{2K+1}.a^2-a^{2K+1}.b^2+a^{2K+1}.b^2+b^{2K+1}.b^2$ $[\because -a^{2K+1}.b^2+a^{2K+1}.b^2=0]$

$=a^{2K+1}(a^2-b^2)+b^2(a^{2K+1}+b^{2K+1})$

$=a^{2K+1}(a+b)(a-b)+b^2(a+b)q$ $[\because a^{2K+1}+b^{2K+1}=(a+b)q]$

$=(a+b)[a^{2K+1}(a-b)+b^2q]$ which is divisible by $a+b$

$\therefore P(K+1)$ is true if $P(K)$ is true

Hence, by the principle of mathematical induction, $P(n)$ is true $\forall n\in N$

(xxii) $\left(1+\frac{1}{n}\right)^n \leq n+1$

Solution: Let $P(n): \left(1+\frac{1}{n}\right)^1 \leq n+1$

When $n=1$

$\left(1+\frac{1}{1}\right)^1 \leq 1+1$

$\Rightarrow 2\leq 2$ which is true

$\therefore P(1)$ is trueLet us assume that $P(K)$ is true

i.e. $\left(1+\frac{1}{K}\right)^K \leq K+1$

Now,

$$\left(1+\frac{1}{K+1}\right)^{K+1} = \left(1+\frac{1}{K+1}\right)^{K}.\left(1+\frac{1}{K+1}\right)$$

$$< \left(1+\frac{1}{K}\right)^{K}.\frac{K+1+1}{K+1}\left[\because \frac{1}{K+1} < \frac{1}{K}\right]$$

$$\leq (K+1).\frac{K+2}{K+1}\left[\because \left(1+\frac{1}{K}\right)^{K} \leq K+1\right]$$

$$\leq K+2$$

$$\leq (K+1)+1$$

$$\therefore \left(1+\frac{1}{K+1}\right)^{K+1} \leq (K+1)+1$$

$\therefore P(K+1)$ is true if $P(K)$ is true.

Hence, by the principle of mathematical induction, $P(n)$ is true $\forall n \in N$