CHAPTER 2
SEQUENCES, A.P., G.P. and H.P.

SEQUENCE : A sequence is an arrangement of numbers in a definite order according to some rules.

E.g. :- 2, 4, 6, 8, ……….. is a sequence.

A sequence is said to be finite if the number of its elements is finite, otherwise it is said to be infinite.

A finite sequence $a_1, a_2, a_3, ..., a_k$ is denoted by $\{a_n\}^k_{n=1}$ and an infinite sequence $a_1, a_2, a_3........, a_{n,.....}$ is denoted by $\{a_n\}^\alpha_{n=1}$ or simply by $\{a_n\}$, where a_n is the n^{th} term of the sequence.

Arithmetic Progression (A.P.): A sequence $\{a_n\}$ is called an arithmetic progression (AP) if there exists a no. d such that $a_{n+1} - a_n = d \ \forall n \in N$. The number d is called the common difference (c.d.) of the AP.

Notes:

1) The general term or n^{th} term (a_n) of an A.P. whose first term is a andcommon difference is d,is given by $a_n = a + (n-1)d$

2) Sum of the first n terms (s_n) of an A.P. is given by

$$S_n = \frac{n}{2}[a+l], \text{ or } S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Arithmetic Mean (AM) : The arithmetic mean (AM) between two numbers a and b is given by

$$A.M. = \frac{1}{2}(a+b)$$

Geometric Progression (GP): The sequence $\{a_n\}$ is called a geometric progression (GP) if there exists a non-zero number r such that $\frac{a_{n+1}}{a_n} = r, \ \forall n \in N$.

The number r is called the common ratio (c.r.) of the GP.

Notes: 1)The general term or n^{th} term a_n of a G.P. whose first term is a and common ratio is r, is given by

$$a_n = ar^{n-1}$$

2) The sum of the first n terms, S_n of a G.P. is given by

(i) $S_n = \frac{a(r^n-1)}{r-1}$ if $r > 1$

(ii) $S_n = \frac{a(1-r^n)}{1-r}$ if $r < 1$

and (iii) $S_n = na$, if $r = 1$

.Geometric Mean (G.M.): If a, x, b are in GP, then x is the geometric mean between a and b.

$\therefore$ GM between a and b is given by, $x = \sqrt{ab}$

Harmonic Progression (HP): A sequence $\{a_n\}$ is called a harmonic progression if the sequence $\left\{\frac{1}{a_n}\right\}$ is an AP. i.e. if there exists a number d such that $\frac{1}{a_{n+1}} - \frac{1}{a_n} = d, \forall n \in N$.

Harmonic Mean (HM) : If H be the harmonic mean between a and b, then a, H, b are in HP and consequently $\frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in AP.

HM between a and $b = \frac{2ab}{a+b}$

Relation between AM, G.M. and HM:

$i) A.M., G.M.$ and $H.M.$ are in $G.P.$

$ii) AM > GM > HM$ (for two unequal quantities)

SERIES

Sum of some important finite series are :

(i) $1+2+3+........+n = \frac{n(n+1)}{2}$

(ii) $1^2+2^2+3^2+..........+n^2 = \frac{n(n+1)(2n+1)}{6}$

(iii) $1^3+2^3+3^3+......+n^3 = \left[\frac{n(n+1)}{2}\right]^2$

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SOLUTIONS

EXERCISE – 2.1

Q1. Find the first five terms each of the following sequences.

(i) $\left\{1+(-1)^n\right\}$

Solution: We have, $a_n = \left\{1+(-1)^n\right\}$

Now, $a_1 = \left\{1+(-1)^1\right\} = 1-1=0$

$a_2 = \left\{1+(-1)^2\right\} = 1+1=2$

$a_3 = \left\{1+(-1)^3\right\} = 1-1=0$

$a_4 = \left\{1+(-1)^4\right\} = 1+1=2$

$a_5 = \left\{1+(-1)^5\right\} = 1-1=0$

$\therefore$ The first five terms of the given sequence are 0, 2, 0, 2, 0

(ii) $\left\{(-1)^{n-1}\right\}$

Solution: We have, $a_n = \left\{(-1)^{n-1}\right\}$

Now, $a_1 = \left\{(-1)\right\}^{1-1} = (-1)^0 = 1$

$a_2 = \left\{(-1)\right\}^{2-1} = (-1)^1 = -1$

$a_3 = \left\{(-1)\right\}^{3-1} = (-1)^2 = 1$

$a_4 = \left\{(-1)\right\}^{4-1} = (-1)^3 = -1$

$a_5 = \left\{(-1)\right\}^{5-1} = (-1)^4 = 1$

$\therefore$ The first five terms of the given sequence are 1, -1, 1, -1, 1.

(iii) $\left\{\dfrac{3n-1}{n+2}\right\}$

Solution: We have, $a_n = \left\{\dfrac{3n-1}{n+2}\right\}$

Now, $a_1 = \left\{\dfrac{3\times1-1}{1+2}\right\} = \dfrac{2}{3}$

$a_2 = \left\{\dfrac{3\times2-1}{2+2}\right\} = \dfrac{5}{4}$

$a_3 = \left\{\dfrac{3\times3-1}{3+2}\right\} = \dfrac{8}{5}$

$a_4 = \left\{\dfrac{3\times4-1}{4+2}\right\} = \dfrac{11}{6}$

$a_5 = \left\{\dfrac{3\times5-1}{5+2}\right\} = \dfrac{14}{7} = 2$

$\therefore$ The first five terms of the given sequence are $\dfrac{2}{3}, \dfrac{5}{4}, \dfrac{8}{5}, \dfrac{11}{6}, 2$.

(iv) $\left\{\dfrac{2n-1}{n}\right\}$

Solution: We have, $a_n = \left\{\dfrac{2n-1}{n}\right\}$

Now, $a_1 = \left\{\dfrac{2\times1-1}{1}\right\} = \dfrac{1}{1} = 1$

$a_2 = \left\{\dfrac{2\times2-1}{2}\right\} = \dfrac{3}{2}$

$a_3 = \left\{\dfrac{2\times3-1}{3}\right\} = \dfrac{5}{3}$

$a_4 = \left\{\dfrac{2\times4-1}{4}\right\} = \dfrac{7}{4}$

$a_5 = \left\{\dfrac{2\times5-1}{5}\right\} = \dfrac{9}{5}$

$\therefore$ The first five terms of the given sequence are $1, \dfrac{3}{2}, \dfrac{5}{3}, \dfrac{7}{4}, \dfrac{9}{5}$.

(v) $\{2^n + 3\}$

Solution: We have, $a_n = \{2^n + 3\}$

$$\therefore\ a_1 = \{2^1 + 3\} = 5$$

$$a_2 = \{2^2 + 3\} = 7$$

$$a_3 = \{2^3 + 3\} = 11$$

$$a_4 = \{2^4 + 3\} = 19$$

$$a_5 = \{2^5 + 3\} = 35$$

$\therefore$ The first five terms of the given sequence are 5, 7, 11, 19, 35.

(vi) $\left\{\dfrac{n^2 + 1}{3n - 1}\right\}$

Solution: We have, $a_n = \left\{\dfrac{n^2 + 1}{3n - 1}\right\}$

$$\therefore\ a_1 = \left\{\frac{1^2 + 1}{3\times 1 - 1}\right\} = \frac{2}{2} = 1$$

$$a_2 = \left\{\frac{2^2 + 1}{3\times 2 - 1}\right\} = \frac{5}{5} = 1$$

$$a_3 = \left\{\frac{3^2 + 1}{3\times 3 - 1}\right\} = \frac{10}{8} = \frac{5}{4}$$

$$a_4 = \left\{\frac{4^2 + 1}{3\times 4 - 1}\right\} = \frac{17}{11}$$

$$a_5 = \left\{\frac{5^2 + 1}{3\times 5 - 1}\right\} = \frac{26}{14} = \frac{13}{7}$$

$\therefore$ The first five terms of the given sequence are 1, 1, $\frac{5}{4}, \frac{17}{11}, \frac{13}{7}$.

(vii) $\left\{\frac{1}{(2n-1)^2}\right\}$

Solution: We have, $a_n = \frac{1}{(2n-1)^2}$

$$\therefore\ a_1 = \frac{1}{(2\times1-1)^2} = \frac{1}{1^2} = 1$$

$$a_2 = \frac{1}{(2\times2-1)^2} = \frac{1}{3^2} = \frac{1}{9}$$

$$a_3 = \frac{1}{(2\times3-1)^2} = \frac{1}{5^2} = \frac{1}{25}$$

$$a_4 = \frac{1}{(2\times4-1)^2} = \frac{1}{7^2} = \frac{1}{49}$$

$$a_5 = \frac{1}{(2\times5-1)^2} = \frac{1}{9^2} = \frac{1}{81}$$

$\therefore$ The first five terms of the given sequence are 1, $\frac{1}{9}, \frac{1}{25}, \frac{1}{49}, \frac{1}{81}$.

(viii) $\left\{\frac{(-1)^n}{n!}\right\}$

Solution: We have, $a_n = \frac{(-1)^n}{n!}$

$$\therefore\ a_1 = \frac{(-1)^1}{1!} = -1$$

$$a_2 = \frac{(-1)^2}{2!} = \frac{1}{2!}$$

$$a_3 = \frac{(-1)^3}{3!} = \frac{-1}{3!}$$

$$a_4 = \frac{(-1)^4}{4!} = \frac{1}{4!}$$

$$a_5 = \frac{(-1)^5}{5!} = \frac{-1}{5!}$$

Hence, the first five terms of the given sequence are -1, $\frac{1}{2!}, \frac{-1}{3!}, \frac{1}{4!}, \frac{-1}{5!}$.

Q2. Find the first four terms of each of the following sequence whose general term is:

(i) $\frac{n-1}{n}$

Solution: We have, $a_n = \frac{n-1}{n}$

$$a_1 = \frac{1-1}{1} = 0$$

$$a_2 = \frac{2-1}{2} = \frac{1}{2}$$

$$a_3 = \frac{3-1}{3} = \frac{2}{3}$$

$$a_4 = \frac{4-1}{4} = \frac{3}{4}$$

$\therefore$ The first four terms are $0, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}$.

(ii) $\frac{n+1}{n+2}$

Solution: We have, $a_n = \frac{n+1}{n+2}$

Now, $a_1 = \frac{1+1}{1+2} = \frac{2}{3}$

$$a_2 = \frac{2+1}{2+2} = \frac{3}{4}$$

$$a_3 = \frac{3+1}{3+2} = \frac{4}{5}$$

$$a_4 = \frac{4+1}{4+2} = \frac{5}{6}$$

$\therefore$ The first four terms are $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$.

(iii) $\frac{1}{3^{n-1}}$

Solution: We have, $a_n = \frac{1}{3^{n-1}}$

Then, $a_1 = \frac{1}{3^{1-1}} = \frac{1}{3^0} = 1$

$a_2 = \frac{1}{3^{2-1}} = \frac{1}{3}$

$a_3 = \frac{1}{3^{3-1}} = \frac{1}{3^2} = \frac{1}{9}$

$a_4 = \frac{1}{3^{4-1}} = \frac{1}{3^3} = \frac{1}{27}$

$\therefore$ The first four terms are $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$.

(iv) $\frac{n!}{n+1}$

Solution: Here, $a_n = \frac{n!}{n+1}$

Now, $a_1 = \frac{1!}{1+1} = \frac{1!}{2}$

$a_2 = \frac{2!}{2+1} = \frac{2!}{3}$

$a_3 = \frac{3!}{3+1} = \frac{3!}{4}$

$a_4 = \frac{4!}{4+1} = \frac{4!}{5}$

$\therefore$ The first four terms are $\frac{1!}{2}, \frac{2!}{3}, \frac{3!}{4}, \frac{4!}{5}$.

(v) $n(n+1)$

Solution: We have, $a_n = n(n+1)$

Now, $a_1 = 1(1+1) = 1\times 2 = 2$

$a_2 = 2(2+1) = 2\times 3 = 6$

$a_3 = 3(3+1) = 3\times 4 = 12$

$a_4 = 4(4+1) = 4\times 5 = 20$

$\therefore$ The first four terms are 2, 6, 12, 20.

(vi) $1+\frac{1}{2}+\frac{1}{3}+.......+\frac{1}{n}$

Solution: We have, $a_n = 1+\frac{1}{2}+\frac{1}{3}+.......+\frac{1}{n}$

Now, $a_1 = 1$

$$a_2 = 1+\frac{1}{2} = \frac{3}{2}$$

$$a_3 = 1+\frac{1}{2}+\frac{1}{3} = \frac{6+3+2}{6} = \frac{11}{6}$$

$$a_4 = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4} = \frac{12+6+4+3}{12} = \frac{25}{12}$$

$\therefore$ The first four terms are $1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}$.

(vii) $\sqrt{n-1}-\sqrt{n}$

Solution: Here, $a_n = \sqrt{n-1}-\sqrt{n}$

$\therefore$ $a_1 = \sqrt{1-1}-\sqrt{1} = -1$

$$a_2 = \sqrt{2-1}-\sqrt{2} = 1-\sqrt{2}$$

$$a_3 = \sqrt{3-1}-\sqrt{3} = \sqrt{2}-\sqrt{3}$$

$$a_4 = \sqrt{4-1}-\sqrt{4} = \sqrt{3}-2$$

$\therefore$ The first four terms are $-1, 1-\sqrt{2}, \sqrt{2}-\sqrt{3}, \sqrt{3}-2$.

(viii) $1-\frac{(-1)^n}{2}$

Solution: Here, $a_n = 1-\frac{(-1)^n}{2}$

Now, $a_1 = 1-\frac{(-1)^1}{2} = 1+\frac{1}{2} = \frac{3}{2}$

$$a_2 = 1-\frac{(-1)^2}{2} = 1-\frac{1}{2} = \frac{1}{2}$$

$$a_3 = 1-\frac{(-1)^3}{2} = 1+\frac{1}{2} = \frac{3}{2}$$

$$a_4 = 1-\frac{(-1)^4}{2} = 1-\frac{1}{2} = \frac{1}{2}$$

$\therefore$ The first four terms are $\frac{3}{2}, \frac{1}{2}, \frac{3}{2}, \frac{1}{2}$.

Q3. Find the general term (i.e. the n^th term) of each of the following sequences:-

(i) 0, 3, 8, 15, 24, …..

Solution: We have, $a_1 = 0 = 1^2 - 1$

$a_2 = 3 = 2^2 - 1$

$a_3 = 8 = 3^2 - 1$

$a_4 = 15 = 4^2 - 1$

$a_5 = 24 = 5^2 - 1$ and so on.

$\therefore$ The required general term is $a_n = n^2 - 1$

(ii) $1, \frac{3}{2}, \frac{5}{3}, \frac{7}{4}, \frac{9}{5}, \ldots\ldots\ldots\ldots$

Solution: We have, $a_1 = 1 = \frac{2\times 1 - 1}{1}$

$a_2 = \frac{3}{2} = \frac{2\times 2 - 1}{2}$

$a_3 = \frac{5}{3} = \frac{2\times 3 - 1}{3}$

$a_4 = \frac{7}{4} = \frac{2\times 4 - 1}{4}$

$a_5 = \frac{9}{5} = \frac{2\times 5 - 1}{5}$ and so on.

$\therefore$ The required general term is $a_n = \frac{2n-1}{n}$

(iii) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\ldots\ldots$

Solution: We have, $a_1 = 1 = \frac{1}{2^{1-1}}$

$a_2 = \frac{1}{2} = \frac{1}{2^{2-1}}$

$a_3 = \frac{1}{4} = \frac{1}{2^{3-1}}$

$a_4 = \frac{1}{8} = \frac{1}{2^{4-1}}$ and so on

$\therefore$ The required general term is $a_n = \frac{1}{2^{n-1}}$

(iv) 1, 0, 1, 0, 1, 0, …..

Solution: We have, $a_1 = 1 = \dfrac{1+(-1)^{1-1}}{2}$

$$a_2 = 0 = \frac{1+(-1)^{2-1}}{2}$$

$$a_3 = 1 = \frac{1+(-1)^{3-1}}{2}$$

$$a_4 = 0 = \frac{1+(-1)^{4-1}}{2}$$

$a_5 = 1 = \dfrac{1+(-1)^{5-1}}{2}$ and so on

Hence, the required general term is $a_n = \dfrac{1+(-1)^{n-1}}{2}$

(v) $\dfrac{1}{2}, \dfrac{3}{4}, \dfrac{5}{6}, \dfrac{7}{8}$

Solution: We have, $a_1 = \dfrac{1}{2} = \dfrac{2\times 1-1}{2\times 1}$

$$a_2 = \frac{3}{4} = \frac{2\times 2-1}{2\times 2}$$

$$a_3 = \frac{5}{6} = \frac{2\times 3-1}{2\times 3}$$

$a_4 = \dfrac{7}{8} = \dfrac{2\times 4-1}{2\times 4}$ and so on

Hence, the required general term is $a_n = \dfrac{2n-1}{2n}$

(vi) $\dfrac{3}{4}, \dfrac{5}{16}, \dfrac{7}{36}, \dfrac{9}{64}$

Solution: Here, $a_1 = \dfrac{3}{4} = \dfrac{2\times 1+1}{4\times 1^2}$

$$a_2 = \frac{5}{16} = \frac{2\times 2+1}{4\times 2^2}$$

$$a_3 = \frac{7}{36} = \frac{2\times 3+1}{4\times 3^2}$$

$a_4 = \dfrac{9}{64} = \dfrac{2\times 4+1}{4\times 4^2}$ and so on

Hence, the required general term is $a_n = \dfrac{2n+1}{4n^2}$

(vii) $0, \frac{3}{2}, \frac{-2}{3}, \frac{5}{4}, \frac{-4}{5}, \ldots\ldots\ldots\ldots\ldots$

Solution: Here,

$$a_1 = 0 = (-1)^1 + \frac{1}{1}$$

$$a_2 = \frac{3}{2} = (-1)^2 + \frac{1}{2}$$

$$a_3 = \frac{-2}{3} = (-1)^3 + \frac{1}{3}$$

$$a_4 = \frac{5}{4} = (-1)^4 + \frac{1}{4} \text{ and so on}$$

Hence, the required general term is $a_n = (-1)^n + \frac{1}{n}$

(viii) $\frac{1!}{2}, \frac{2!}{5}, \frac{3!}{8}, \frac{4!}{11}, \ldots\ldots$

Solution: Here,

$$a_1 = \frac{1!}{2} = \frac{1!}{3 \times 1 - 1}$$

$$a_2 = \frac{2!}{5} = \frac{2!}{3 \times 2 - 1}$$

$$a_3 = \frac{3!}{8} = \frac{3!}{3 \times 3 - 1}$$

$$a_4 = \frac{4!}{11} = \frac{4!}{3 \times 4 - 1} \text{ and so on}$$

Hence, the required general term is $a_n = \frac{n!}{3n - 1}$

(ix) $\frac{1}{1.2}, \frac{1}{2.3}, \frac{1}{3.4}, \frac{1}{4.5}, \ldots\ldots$

Solution: Here,

$$a_1 = \frac{1}{1.2} = \frac{1}{1(1+1)}$$

$$a_2 = \frac{1}{2.3} = \frac{1}{2(2+1)}$$

$$a_3 = \frac{1}{3.4} = \frac{1}{3(3+1)}$$

$$a_4 = \frac{1}{4.5} = \frac{1}{4(4+1)} \text{ and so on}$$

Hence, the required general term is $a_n = \frac{1}{n(n+1)}$

(x) $1, 1+\sqrt{2}, \sqrt{2}+\sqrt{3}, \sqrt{3}+\sqrt{4}$

Solution: Here, $a_1 = 1 = \sqrt{1-1}+\sqrt{1}$

$a_2 = 1+\sqrt{2} = \sqrt{2-1}+\sqrt{2}$

$a_3 = \sqrt{2}+\sqrt{3} = \sqrt{3-1}+\sqrt{3}$

$a_4 = \sqrt{3}+\sqrt{4} = \sqrt{4-1}+\sqrt{4}$ and so on.

Hence, the required general term is $a_n = \sqrt{n-1}+\sqrt{n}$

Q4. Find the n^th term of the sequence $\frac{1}{2}, \frac{-1}{3}, \frac{1}{4}, \frac{-1}{5}, \ldots\ldots$ hence obtain the 9^th term.

Solution: Here, $a_1 = \frac{1}{2} = \frac{(-1)^{1+1}}{1+1}$

$a_2 = -\frac{1}{3} = \frac{(-1)^{2+1}}{2+1}$

$a_3 = \frac{1}{4} = \frac{(-1)^{3+1}}{3+1}$

$a_4 = -\frac{1}{5} = \frac{(-1)^{4+1}}{4+1}$

$\therefore$ n^{th} term $= \frac{(-1)^{n+1}}{n+1}$

And 9^{th} term $= \frac{(-1)^{9+1}}{9+1} = \frac{(-1)^{10}}{10} = \frac{1}{10}$

Q5. Determine the following sequences:

(i) $\left\{\frac{1}{n^2+2}\right\}_{n=1}^{10}$

Solution: Here, $a_n = \frac{1}{n^2+2}$

$\therefore \quad a_1 = \frac{1}{1^2+2} = \frac{1}{3}$

$a_2 = \frac{1}{2^2+2} = \frac{1}{6}$

$a_3 = \frac{1}{3^2+2} = \frac{1}{11}$

… … … … … …

$a_{10} = \frac{1}{10^2+2} = \frac{1}{102}$

$\therefore$ The sequence is $\frac{1}{3}, \frac{1}{6}, \frac{1}{11}, \ldots\ldots\ldots, \frac{1}{102}$.

(ii) $\{3n-1\}_{n=1}^{15}$

Solution: Here, $a_n = \{3n-1\}$

$$a_1 = 3\times1-1 = 2$$

$$a_2 = 3\times2-1 = 5$$

$$a_3 = 3\times3-1 = 8$$

… … … … … …

$$a_{15} = 3\times15-1 = 44$$

$\therefore$ The sequence is 2, 5, 8, ………., 44.

(iii) $\{n(n+2)\}_{n=1}^{50}$

Solution: Here, $a_n = \{n(n+2)\}$

$$\therefore \quad a_1 = \{1(1+2)\} = 3$$

$$a_2 = \{2\times(2+2)\} = 8$$

$$a_3 = \{3\times(3+2)\} = 15$$

… … … … … …

$$a_{50} = \{50\times(50+2)\} = 50\times52 = 2600$$

$\therefore$ The sequence is 3, 8, 15, ………, 2600.

(iv) $\left\{\dfrac{1}{3^{n-1}}\right\}_{n=1}^{100}$

Solution: Here, $a_n = \dfrac{1}{3^{n-1}}$

$$a_1 = \frac{1}{3^{1-1}} = \frac{1}{1} = 1$$

$$a_2 = \frac{1}{3^{2-1}} = \frac{1}{3}$$

$$a_3 = \frac{1}{3^{3-1}} = \frac{1}{3^2}$$

… … … … … …

$$a_{100} = \frac{1}{3^{100-1}} = \frac{1}{3^{93}}$$

$\therefore$ The sequence is 1, $\dfrac{1}{3}, \dfrac{1}{3^2}, \ldots\ldots\ldots\ldots\ldots, \dfrac{1}{3^{99}}$.

SOLUTIONS

EXERCISE – 2.2

Q1. Find the 15th and 50th terms of the AP 1, 3, 5 ,7.

Solution: Here, $a = 1$, $d = 3 - 1 = 2$

$\therefore$ 15^{th} term $= 1+(15-1)2$

$= 1+28=29$

and 50^{th} term $= 1+(50 – 1).2$

$= 1+49\times2$

$= 1+98 = 99$

Q2. Find the 21st term of the AP. 7, 4, 1, -2, -5, 8, ……..

Solution: Here, $a = 7$ and $d = 4 - 7 = -3$

$\therefore$ 21^{st} term $= 7 + (21 - 1)(-3)$

$= 7 + 20 \times (-3)$

$= 7 - 60 = -53$

Q3. (i) Which term of the AP 1, 4, 7, 10, …….is 55?

Solution: Here, $a = 1$, $d = 4 - 1 = 3$

Let 55 be the n^{th} term of the AP.

$\therefore$ n^{th} term $= a + (n - 1)d$

$\Rightarrow 55 = 1 + (n - 1)3$

$\Rightarrow 55 = 3n - 2$

$\Rightarrow 3n = 57$

$\Rightarrow n = 19$

Hence, 55 is the 19^{th} term of the AP.

(ii) Which term of the AP 3, $\frac{11}{3}, \frac{13}{3}$, 5............. is 9?

Solution: Here, $a = 3,\ d = \frac{11}{3} - 3 = \frac{11-9}{3} = \frac{2}{3}$

Let 9 be the n^{th} term of the AP.

$$\therefore 9 = 3 + (n-1)\frac{2}{3}$$

$$\Rightarrow 27 = 9 + 2n - 2$$

$$\Rightarrow 2n == 20$$

$$\Rightarrow n = 10$$

$\therefore$ 9 is the 10^{th} term of the AP.

Q4. Is 216 a term of the AP., 3, 8, 13, 18, …….? If not, find the term nearest to it?

Solution: Here, $a = 3,\ d = 8 - 3 = 5$

$$\therefore a_n = 216$$

$$\Rightarrow 3 + (n-1)5 = 216$$

$$\Rightarrow 5n = 216 + 2$$

$$\Rightarrow n = \frac{218}{5} = 43.6$$

Since, n cannot be fractional, 216 is not a term of the given AP.

Now, the nearest term of the AP is 44^{th} term.

$$a_{44} = a + (44-1)d$$

$$= 3 + 43 \times 5$$

$$= 218$$

$\therefore$ Its nearest term is 218.

Q5. The first term and the common difference of an AP are respectively 39 and -7. Find the 10^{th} term.

Solution: Here, $a = 39$ and $d = -7$

$$\therefore \quad a_{10} = a + (n-1)d$$

$$= 39 + (10-1)(-7)$$

$$= 39 + 9 \times (-7)$$

$$= 39 - 63$$

$$= -24$$

Q6. The first term and 12th term of an AP are respectively 5 and 49. Find the common difference.

Solution: Here, $a = 5$, $a_{12} = 49$, $n = 12$

Let d be the common difference

Now, $a_{12} = 5 + (12 - 1)d$

$\Rightarrow 49 = 5 + 11d$

$\Rightarrow 11d = 44$

$\Rightarrow d = 4$

Q7. How many numbers divisible by 15 are there between 20 and 400 ?

Solution: The last term divisible by 15 is 390.

Numbers divisible by 15 between 20 and 400 are 30, 45, 60, 75, ……, 390.

This is an AP.

Here, $a = 30$, $d = 15$

Let $a_n = 390$

$\Rightarrow 390 = 30 + (n - 1)15$

$\Rightarrow 390 = 30 + 15n - 15$

$\Rightarrow 15n = 375$

$\Rightarrow n = 25$

$\therefore$ There are 25 numbers between 20 and 400 which are divisible by 15.

Q8. If the nth term of sequence is 3*n*+4, show that the sequence is an AP. Hence find the first term and common difference.

Solution: Here, nth term = $3n+4$

Now, $a_1 = 3.1 + 4 = 7$

$a_2 = 3.2 + 4 = 10$

$a_3 = 3.3 + 4 = 13$

$a_4 = 3.4 + 4 = 16$

Now, $10 - 7 = 13 - 10 = 16 - 13 = ... = 3$

Since the difference of any two consecutive terms takes in the some order is constant, the sequence 7, 10, 13, 16, …… is an A.P.

$\therefore$ first term = 7 and common difference = 3.

Q9. Find the 25th term and the common difference of the AP whose n^{th} term is $4n+1$.

Solution: Here, $a_n = 4n+1$

Now, $a_{25} = 4\times 25 + 1 = 101$

Also, $a_1 = 4.1+1=5$

$a_2 = 4.2+1=9$

$\therefore$ Common difference$= a_2 - a_1 = 9-5=4$

Q10. The 8th and 15th terms of an AP. are 4 and -24 respectively. Find its 12th term.

Solution: Here, $a_8 = 4$

$\Rightarrow a+(8-1)d = 4$

$\Rightarrow a+7d = 4$ ________________ (1)

& $a_{15} = -24$

$\Rightarrow a+(15-1)d = -24$

$\Rightarrow a+14d = -24$ ________________ (2)

Subtracting (1) from (2), we get

$7d = -28$

$\Rightarrow d = -4$

Putting the value of d in (1), we get

$a + 7\times(-4) = 4$

$\Rightarrow a = 4+28 = 32$

$\therefore$ 12^{th} term $= 32+(12-1)(-4)$

$= 32+11\times(-4)$

$= 32-44$

$= -12$

Q11. The 13th and 22nd terms of an AP are respectively 6 and 9, which term is 8?

Solution: Let a be the first term and d be the common difference.

Then, $a_{13} = a + (13-1)d$

$\Rightarrow 6 = a + 12d$ ---------------- (1)

& $a_{22} = a + (22-1)d$

$\Rightarrow 9 = a + 21d$ ---------------- (2)

Subtracting (1) from (2), we get

$9d = 3$

$\Rightarrow d = \frac{1}{3}$

Again, from (1), we get

$6 = a + 12 \times \frac{1}{3}$

$\Rightarrow 6 = a + 4$

$\Rightarrow a = 2$

Let $a_n = 8$

$\Rightarrow 8 = 2 + (n-1) \times \frac{1}{3}$

$\Rightarrow 8 = \frac{6+n-1}{3}$

$\Rightarrow 24 = n + 5$

$\Rightarrow n = 19$

$\therefore$ the 19th term is 8.

Q12. The pth and qth term of an AP are respectively *q* and *p*. Find the nth term.

Solution: Let a be the first term and d be the common difference.

Then, p^{th} term $= a + (p-1)d$

$\Rightarrow q = a + (p-1)d$ ---------------- (1)

& q^{th} term $= a + (q-1)d$

$\Rightarrow p = a + (q-1)d$ ---------------- (2)

Subtracting (1) from (2), we get

$p - q = (q - p)d$

$\Rightarrow d = \frac{p-q}{q-p} = -1$ ---------------- (3)

$\therefore n^{th}$ term $= a + (n-1)(-1)$

$= (p + q - 1) - n + 1$ [from (2)]

$= p + q - n$

Q13. A sequence $\{a_n\}$ is given by $a_n = n^2 - 1$, $n \in N$. Show that it is not an AP.

Solution: Here, $a_n = n^2 - 1$

Now, $a_{n+1} - a_n = \{(n+1)^2 - 1\} - \{n^2 - 1\}$

$= n^2 + 2n + 1 - 1 - n^2 + 1$

$= 2n + 1$, this is not a constant.

$\therefore$ the given equence is not an A.P.

Q14. If a, b, c are in AP, show that

(i) $b+c, c+a, a+b$ are also in AP.

Solution: Since a, b, c are in AP

$\therefore \quad b - a = c - b$

$\Rightarrow 2b = c + a$

Now, $(b+c), (c+a), (a+b)$ are in AP.

if $(c+a) - (b+c) = (a+b) - (c+a)$

if $a - b = b - c$

if $b - a = c - b$

if $2b = c + a$ which is true.

Hence $(b+c), (c+a), (a+b)$ are in AP.

(ii) $a^2(b+c), b^2(c+a), c^2(a+b)$ are also in AP.

Solution: Since *a, b, c* are in AP.

$2b = a + c$

$\therefore \quad a^2(b+c), b^2(c+a), c^2(a+b)$ are in AP

if $b^2(c+a) - a^2(b+c) = c^2(a+b) - b^2(c+a)$

if $b^2c + b^2a - a^2b - a^2c = c^2a + c^2b - b^2c - b^2a$

if $c(b^2 - a^2) + ab(b-a) = a(c^2 - b^2) + bc(c-b)$

if $(b-a)(bc + ca + ab) = (c-b)(ac + ab + bc)$

if $b - a = c - b$

$\Rightarrow 2b = a + c$ which is true.

Hence, $a^2(b+c), b^2(c+a), c^2(a+b)$ are in AP.

(iii) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are also in AP.

Solution: Since a, b, c are in AP.

$$b - a = c - b$$

$$\Rightarrow 2b = a + c \qquad \text{---------------- (1)}$$

Now, $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in AP

$$\text{if} \frac{1}{ca} - \frac{1}{bc} = \frac{1}{ab} - \frac{1}{ca}$$

$$\text{if} \frac{2}{ca} = \frac{c+a}{abc}$$

$$\text{if} \frac{2abc}{ca} = c + a$$

$\Rightarrow 2b = c + a$ which is true by (1)

Hence, $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$ are in AP

Q15. If x, y, z respectively p^{th}, q^{th}, r^{th} terms of an AP. Show that $p(y-z) + q(z-x) + r(x-y) = 0$

Solution: Let *a* be the first term and *d* be the common difference.

Now, $x = a + (p-1)d$

$$y = a + (q-1)d$$

$$z = a + (r-1)d$$

$$x - y = (p-q)d$$

$$y - z = (q-r)d$$

$$z - x = (r-p)d$$

Now, $p(y-z) + q(z-x) + r(x-y)$

$$= p(q-r)d + q(r-p)d + r(p-q)d$$

$$= (pq - pr + qr - qp + rp - rq)d$$

$$= 0 \times d$$

$$= 0$$

Q16. If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP, show that a^2, b^2, c^2 are in AP.

Solution: Since $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP.

$$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a} \text{ ---------------- (1)}$$

Now, a^2, b^2, c^2 are in AP.

$$\text{if } b^2 - a^2 = c^2 - b^2$$

$$\text{if } (b-a)(b+a) = (c-b)(c+b)$$

$$\text{if } \frac{b-a}{b+c} = \frac{c-b}{a+b}$$

$$\text{if } \frac{(b+c)-(c+a)}{(c+a)(b+c)} = \frac{(c+a)-(a+b)}{(a+b)(c+a)}$$

$$\text{if } \frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a} \text{ which is true by (1)}$$

Hence, a^2, b^2, c^2 are in AP.

Q17. If the n^{th} term of 3, 5, 7, 9, ……….is the same as that of 9, $10\frac{1}{2}, 12, 13\frac{1}{2}$, ….., find *n*.

Solution: By question, we have

$$n^{th} \text{ term of } 3, 5, 7, 9, \ldots\ldots = n^{th} \text{ term of } 9, 10\frac{1}{2}, 12, 13\frac{1}{2}, \ldots.$$

$$\Rightarrow 3 + (n-1)2 = 9 + (n-1)\frac{3}{2} \left[\because 10\frac{1}{2} - 9 = \frac{3}{2}\right]$$

$$\Rightarrow 3 + 2n - 2 = \frac{18 + 3n - 3}{2}$$

$$\Rightarrow 2(2n+1) = 3n + 15$$

$$\Rightarrow 4n + 2 = 3n + 15$$

$$\Rightarrow n = 13$$

Q18. The sum of three numbers in AP is 21 and the sum of their squares is 179. Find the numbers.

Solution: Let $a-d, a, a+d$ be the three numbers.

Then, $a-d+a+a+d=21$

$\Rightarrow 3a=21$

$\Rightarrow a=7$

and $(a-d)^2+a^2+(a+d)^2=179$

$\Rightarrow a^2-2ad+d^2+a^2+a^2+2ad+d^2=179$

$\Rightarrow 3a^2+2d^2=179$

$\Rightarrow 3\times 7^2+2d^2=179$

$\Rightarrow 2d^2=179-147$

$\Rightarrow d^2=\dfrac{32}{2}=16$

$\Rightarrow d=\pm 4$

When $d=4$, the three numbers are 3, 7 and 11.

When $d=-4$, the three numbers are 11, 7 and 3.

Q19. The sum of three numbers in AP is 24 and the product of the two extremes is 55. Find the numbers.

Solution: Let $a-d, a, a+d$ be the three numbers.

Then, $a-d+a+a+d=24$

$\Rightarrow 3a=24$

$\Rightarrow a=8$

Now, product of the extremes $(a-d)$ and $(a+d)=55$

$\Rightarrow (a-d)(a+d)=55$

$\Rightarrow a^2-d^2=55$

$\Rightarrow 64-d^2=55$

$\Rightarrow d^2=9$

$\Rightarrow d=\pm 3$

When d=3, the three numbers are 5, 8, 11

and when $d=-3$, the three numbers are 11, 8, 5.

Q20. The sum of four numbers in AP is 48 and the product of two extremes is 108. Find the numbers.

Solution: Let $a-3d, a-d, a+d$ and $a+3d$ be the numbers.

By question,

$$a-3d+a-d+a+d+a+3d=48$$
$$\Rightarrow 4a=48$$
$$\Rightarrow a=12$$

and, $(a-3d)(a+3d)=108$

$$\Rightarrow a^2-9d^2=108$$
$$\Rightarrow 12^2-9d^2=108$$
$$\Rightarrow 144-108=9d^2$$
$$\Rightarrow d^2=\frac{36}{9}=4$$
$$\Rightarrow d=\pm 2$$

When d=2, the four numbers are 6, 10, 14 and 18.

When d=-2, the four numbers are 18, 14, 10 and 6.

Q21. Find the arithmetic mean between

(i) 10 and 20

Solution: AM of 10 and 20 = $\frac{10+20}{2}=15$

(ii) -5 and 5

Solution: A.M. of -5 and 5 = $\frac{-5+5}{2}=0$

(iii) -5 and 9

Solution: A.M. of -5 and 9= $\frac{-5+9}{2}=\frac{4}{2}=2$

Q22. Insert

(i) 2 arithmetic means between 2 and 11.

Solution: Let a_1, a_2 be the AM between 2 and 11.

Then, $2, a_1, a_2, 11$ are in AP

$\therefore$ First term = 2, $n=4$

Let d be the common difference

Then, 4^{th} term = 11

$$\Rightarrow 2+(4-1)d=11$$
$$\Rightarrow 3d=9$$
$$\Rightarrow d=3$$

$\therefore$ The means are $a_1=2+3=5$ & $a_2=5+3=8$.

(ii) 3 arithmetic means between 6 and 22.

Solution: a_1, a_2, a_3 be the AMs between 6 and 22.

Then, $6, a_1, a_2, a_3, 22$ are in AP

Let $a =$ first term=6 and d be the common difference.

Now, 5^{th} term = 22

$$\Rightarrow 6+(5-1)d = 22$$

$$\Rightarrow 6+4d = 22$$

$$\Rightarrow d = \frac{16}{4} = 4$$

$\therefore$ The three AMs are $a_1 = 6+4=10,\ a_2 = 10+4=14$ and $a_3 = 14+4=18$.

(iii) 4 arithmetic means between 5 and 20.

Solution: Let a_1, a_2, a_3, a_4 be the four AMs between 5 and 20

Then, $5, a_1, a_2, a_3, a_4, 20$ are in AP.

Let $a =$ first term = 5 and d be the C.D.

Now, 6^{th} term = 20

$$\Rightarrow 5+(6-1)d = 20$$

$$\Rightarrow 5d = 15$$

$$\Rightarrow d = 3$$

$\therefore a_1 = 5+3=8,\ a_2 = 8+3=11,\ a_3 = 11+3=14,\ a_4 = 14+3=17$.

Hence the four AMs are 8, 11, 14, 17.

(iv) n arithmetic means between 1 and n^2.

Solution: Let $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ be the n AMs between 1 and n^2.

Then, $1, a_1, a_2, \ldots\ldots\ldots, a_n, n^2$ are in AP.

Let $a =$ first term=1, and d be the c.d.

Now, $(n+2)^{th}$ term = n^2

$$\Rightarrow 1+(n+2-1)d = n^2$$

$$\Rightarrow (n+1)d = n^2-1 = (n-1)(n+1)$$

$$\Rightarrow d = n-1$$

$\therefore\ a_1 = 1+n-1 = n$, $a_2 = n+n-1 = 2n-1$, $a_3 = 2n-1+n-1 = 3n-2$,

$a_4 = 3n-2+n-1 = 4n-3, \ldots\ldots\ldots\ a_n = n^2-(n-1) = n^2-n+1$.

$\therefore$ The AMs between 1 and n^2 are $n, 2n-1, 3n-2, 4n-3, \ldots\ldots, n^2-n+1$.

(v) 3 arithmetic means between $2n+1$ and $2n-1$

Solution: Let a_1, a_2, a_3 be the three AMs.

Then, $2n+1, a_1, a_2, a_3, 2n-1$ are in AP.

Let $a =$ first term $= 2n+1$ and d be the common difference.

Then, 5^{th} term $= 2n-1$

$$\Rightarrow 2n+1+(5-1)d = 2n-1$$
$$\Rightarrow 4d = -2$$
$$\Rightarrow d = -\frac{1}{2}$$

$$\therefore a_1 = (2n+1)+(-\frac{1}{2}) = 2n+\frac{1}{2}$$
$$a_2 = 2n+\frac{1}{2}+\left(-\frac{1}{2}\right) = 2n$$
$$a_3 = 2n+\left(-\frac{1}{2}\right) = 2n-\frac{1}{2}$$

$\therefore$ The three AMs are $2n+\frac{1}{2}, 2n$ and $2n-\frac{1}{2}$.

Q23. There are *n* arithmetic means between 4 and 64. If the ratio of the fourth mean to the eight is 7:13, find *n*.

Solution: Let a be the first term and d be the common difference.

Then, $a+(n+2-1)d = 64$

$$\Rightarrow 4+(n+1)d = 64$$
$$\Rightarrow (n+1)d = 60$$
$$\Rightarrow d = \frac{60}{n+1}$$

4^{th} mean is the 5^{th} term

$$\therefore 5^{th} \text{ term, } x_4 = 4+(5-1)\frac{60}{n+1} = 4+\frac{4\times 60}{n+1}$$

And 8^{th} mean, $x_8 = 4+(9-1)\frac{60}{n+1} = 4+\frac{8\times 60}{n+1}$

Then, $\left\{4+\frac{4\times 60}{n+1}\right\} : \left\{4+\frac{8\times 60}{n+1}\right\} = 7:13$

$$\Rightarrow \frac{4n+4+4\times 60}{n+1} : \frac{4n+4+8\times 60}{n+1} = 7:13$$
$$\Rightarrow \frac{4(n+1)+4\times 60}{n+1} \times \frac{n+1}{4(n+1)+8\times 60} = 7:13$$
$$\Rightarrow \frac{4(n+1)+4\times 60}{4(n+1)+8\times 60} = \frac{7}{13}$$
$$\Rightarrow \frac{n+1+60}{n+1+120} = \frac{7}{13}$$

$$\Rightarrow \frac{n+61}{n+121}=\frac{7}{13}$$

$$\Rightarrow 13n+793=7n+847$$

$$\Rightarrow 6n=54$$

$$\Rightarrow n=9$$

Q24. If $a+b+c\neq 0$ and $\frac{b+c}{a},\frac{c+a}{b},\frac{a+b}{c}$ are in AP, prove that $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are also in AP.

Solution: Since $\frac{b+c}{a},\frac{c+a}{b},\frac{a+b}{c}$ are in AP.

$$\therefore \frac{c+a}{b}-\frac{b+c}{a}=\frac{a+b}{c}-\frac{c+a}{b}$$

$$\Rightarrow \frac{a(c+a)-b(b+c)}{ab}=\frac{b(a+b)-c(c+a)}{bc}$$

$$\Rightarrow \frac{ac+a^2-b^2-bc}{ab}=\frac{ab+b^2-c^2-ac}{bc}$$

$$\Rightarrow \frac{(a^2-b^2)+c(a-b)}{ab}=\frac{(b^2-c^2)+a(b-c)}{bc}$$

$$\Rightarrow \frac{(a-b)(a+b+c)}{ab}=\frac{(b-c)(b+c+a)}{bc}$$

$$\Rightarrow \frac{a-b}{ab}=\frac{b-c}{bc} \qquad \text{-----------------(1)}$$

Now, $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in AP.

$$\text{if} \frac{1}{b}-\frac{1}{a}=\frac{1}{c}-\frac{1}{b}$$

$$\text{if} \frac{a-b}{ab}=\frac{b-c}{bc} \text{ which is true by (1)}$$

Hence $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in AP.

Q25. Find the sum of the first

(i) 20 terms of the AP 1, 5, 9, 13, …..

Solution: Here, $a=1, d=4, n=20$

$$\therefore \text{ The required sum } = \frac{20}{2}\{2\times1+(20-1)4\}$$

$$=10\{2+19\times4\}$$

$$=10\times\{2+76\}$$

$$=780$$

(ii) 25 terms of the AP. 9, 12, 15, 18,………..

Solution: Here $a=9,\ d=3,\ n=25$

$$\therefore \text{ The required sum } = \frac{25}{2}\{2\times9+(25-1)\times3\}$$

$$=\frac{25}{2}\{18+24\times3\}$$

$$=\frac{25}{2}\{18+72\}$$

$$=\frac{25}{2}\times90$$

$$=1125$$

(iii) 30 terms of the AP, $1\frac{2}{3}, 2, 2\frac{1}{3}, 2, \frac{2}{3}, \ldots\ldots\ldots$

Solution: Here, $a=1\frac{2}{3}=\frac{5}{3},\ d=2-1\frac{2}{3}=2-\frac{5}{3}=\frac{1}{3}$ and $n=30$

$$\therefore\quad S_{30}=\frac{30}{2}\left\{2\times\frac{5}{3}+(30-1)\times\frac{1}{3}\right\}$$

$$=15\left\{\frac{10}{3}+\frac{29}{3}\right\}$$

$$=15\times\frac{39}{3}$$

$$=195$$

(iv) 40 terms of the AP, 10, 8, 6, 4, ……….

Solution: Here, $a = 10$, $d = -2$, $n = 40$

$$\therefore\ S_{40} = \frac{40}{2}\{2\times 10 + (40-1)(-2)\}$$
$$= 20\{20-78\}$$
$$= 20\times(-58)$$
$$= -1160$$

(v) *n* term of the AP, $3n, 3n-1, 3n-2,$ **…….**

Solution: Here, $a = 3n$, $d = 3n-1-3n = -1$ and $n = n$

$$\therefore\ S_n = \frac{n}{2}[2\times 3n + (n-1)(-1)]$$
$$= \frac{n}{2}[6n - n + 1]$$
$$= \frac{n}{2}[5n+1]$$

(vi) *n* terms of the AP $\frac{1}{1+\sqrt{a}}, \frac{1}{1-a}, \frac{1}{1-\sqrt{a}}, ...$

Solution: Here, $a = \frac{1}{1+\sqrt{a}}$,

$$d = \frac{1}{1-a} - \frac{1}{1+\sqrt{a}}$$
$$= \frac{1}{(1-\sqrt{a})(1+\sqrt{a})} - \frac{1}{1+\sqrt{a}}$$
$$= \frac{1-(1-\sqrt{a})}{(1-\sqrt{a})(1+\sqrt{a})}$$
$$= \frac{\sqrt{a}}{1-a}$$

$$\therefore\ S_n = \frac{n}{2}\left[2\times\frac{1}{1+\sqrt{a}} + (n-1)\left(\frac{\sqrt{a}}{1-a}\right)\right]$$
$$= \frac{n}{2}\times\frac{1}{1-a}\left[2\left(1-\sqrt{a}\right) + (n-1)\sqrt{a}\right]$$
$$= \frac{n}{2(1-a)}\left[2 - 3\sqrt{a} + n\sqrt{a}\right] = \frac{n}{2(1-a)}\left[(n-3)\sqrt{a} + 2\right]$$

Q26. Find the sum of the following series:

(i) 5+8+11+…..+47

Solution: Here, $a = 5$, $d = 8 - 5 = 3$

Let n be the no. of terms.

Now, $a_n = 47$

$$\Rightarrow a + (n-1)d = 47$$

$$\Rightarrow 5 + (n-1) \times 3 = 47$$

$$\Rightarrow 3n - 3 = 42$$

$$\Rightarrow n = \frac{45}{3} = 15$$

$\therefore$ Required sum $= \frac{n}{2}[a + l]$

$$= \frac{15}{2}[5 + 47]$$

$$= \frac{15}{2} \times 52$$

$$= 390$$

(ii) 4+7+10+…..+49

Solution: Here, $a = 4$, $d = 7 - 4 = 3$

Let n be the no. of terms.

Now, $a_n = a + (n-1)d$

$$\Rightarrow 49 = 4 + (n-1) \times 3$$

$$\Rightarrow 45 = 3n - 3$$

$$\Rightarrow 3n = 48$$

$$\Rightarrow n = 16$$

$\therefore$ Required sum $= \frac{n}{2}[a + l]$

$$= \frac{16}{2}[4 + 49]$$

$$= 8 \times 53$$

$$= 424$$

(iii) 4+8+12+……+80

Solution: Here, $a = 4$, $d = 8 - 4 = 4$

Let n be the no. of terms.

Now, $a_n = 80$

$\Rightarrow 4 + (n-1)4 = 80$

$\Rightarrow 4n = 80$

$\Rightarrow n = 20$

$\therefore$ Required sum $= \frac{n}{2}[a + l]$

$= \frac{20}{2}[4 + 80]$

$= 10 \times 84$

$= 840$

(iv) $(\sqrt{2}+1)+\sqrt{2}+(\sqrt{2}-1)+\ldots\ldots\ldots+(\sqrt{2}-14)$

Solution: Here, $a = \sqrt{2}+1$, $d = \sqrt{2}-(\sqrt{2}+1) = -1$

Let n be the no. of terms.

Now, $a_n = \sqrt{2} - 14$

$\Rightarrow \sqrt{2}+1+(n-1)(-1) = \sqrt{2}-14$

$\Rightarrow -n + 2 = -14$

$\Rightarrow -n = -16$

$\Rightarrow n = 16$

$\therefore$ Required sum $= \frac{n}{2}[a + l]$

$= \frac{16}{2}\left[\sqrt{2}+1+\sqrt{2}-14\right]$

$= 8\left[2\sqrt{2}-13\right]$

(v) $(x-y)^2+(x^2+y^2)+(x+y)^2+\dots\dots\dots+(x^2+y^2+18xy)$

Solution: Here, $a=(x-y)^2$, $d=x^2+y^2-(x-y)^2$

$$= x^2+y^2-(x^2-2xy+y^2)$$
$$= 2xy$$

Let n be the no. of terms.

Now, $a_n = x^2+y^2+18xy$

$$\Rightarrow (x-y)^2+(n-1)2xy = x^2+y^2+18xy$$
$$\Rightarrow x^2+y^2-2xy+2nxy-2xy = x^2+y^2+18xy$$
$$\Rightarrow 2nxy = 22xy$$
$$\Rightarrow n=11$$

$\therefore$ Required sum $= \frac{11}{2}\left[(x-y)^2+x^2+y^2+18xy\right]$

$$= \frac{11}{2}\left(x^2+y^2-2xy+x^2+y^2+18xy\right)$$
$$= \frac{11}{2}\left(2x^2+2y^2+16xy\right)$$
$$= 11(x^2+y^2+8xy)$$

Q27. (i) How many terms of the AP, 5, 9, 13, 17, ……. Must be taken so that the sum be 1224?

Solution: Here, $a=5$, $d=9-5=4$

Let n be the no. of terms.

Now, Sum $= \frac{n}{2}\{2a+(n-1)d\}$

$$\Rightarrow 1224 = \frac{n}{2}\{2\times 5+(n-1)\times 4\}$$
$$\Rightarrow 2448 = n(10+4n-4)$$
$$\Rightarrow 2448 = n(4n+6)$$
$$\Rightarrow 1224 = 2n^2+3n$$
$$\Rightarrow 2n^2+3n-1224=0$$
$$\Rightarrow 2n^2+51n-48n-1224=0$$
$$\Rightarrow 2n(n-24)+51(n-24)=0$$
$$\Rightarrow (n-24)(2n+51)=0$$
$$\Rightarrow n=24 \text{ or } n=\frac{-51}{2}$$

Since number of terms cannot be negative therefore, $n=24$.

(ii) How many terms of the AP, 3, 8, 13, 18, ……..must be taken so that the sum be 1010.

Solution: Let n be the no. of terms so that the sum $S = 1010$.]

Here, $a = 3,\ d = 8 - 3 = 5$

Now, $S = \frac{n}{2}\{2a + (n-1)d\}$

$$\Rightarrow 1010 = \frac{n}{2}\{2 \times 3 + (n-1)5\}$$
$$\Rightarrow 2020 = n(6 + 5n - 5)$$
$$\Rightarrow 2020 = n(5n + 1)$$
$$\Rightarrow 5n^2 + n - 2020 = 0$$
$$\Rightarrow 5n^2 + 101n - 100n - 2020 = 0$$
$$\Rightarrow n(5n + 101) - 20(5n + 101) = 0$$
$$\Rightarrow (n - 20)(5n + 101) = 0$$
$$\Rightarrow n = 20 \text{ or } n = -\frac{101}{5}$$

Since n cannot be –ve, therefore $n = 20$.

Q28. How many terms of the AP 22, 18, 14, 10, ……………must be taken so that the sum may be 64. Explain the double answer.

Solution: Here, for the AP 22, 18, 14, 10, ……….,first term,(a) =22 ,c.d.$(d) = 18 - 22 = -4$

Let n be the number of terms so that the sum is 64.

Then, $S = 64$

$$\Rightarrow \frac{n}{2}\{2a + (n-1)d\} = 64$$
$$\Rightarrow \frac{n}{2}\{2 \times 22 + (n-1)(-4)\} = 64$$
$$\Rightarrow \frac{n}{2} \times 2\{22 - 2n + 2\} = 64$$
$$\Rightarrow n(24 - 2n) = 64$$
$$\Rightarrow 12n - n^2 = 32$$
$$\Rightarrow n^2 - 12n + 32 = 0$$
$$\Rightarrow n^2 - 8n - 4n + 32 = 0$$
$$\Rightarrow n(n-8) - 4(n-8) = 0$$
$$\Rightarrow (n-8)(n-4) = 0$$
$$\Rightarrow n = 8 \ or \ n = 4$$

Thus, the number of terms to be taken so that the sum is 64 is either 4 or 8.

Now, the sum of 4 additional terms $= a_5 + a_6 + a_7 + a_8$
$= 6 + 2 + (-2) + (-6)$
$= 0.$

∴ We can take both values of $n\ i.e.\ 4\ and\ 8$ as the sum of 4 additional terms is 0.

Q29. The 5th and 11th terms of an AP are 41 and 20 respectively. Find the sum of the first 12 terms.

Solution: Let a be the first term and d be the common difference.

Now, 5^{th} term $= 41$

$$\Rightarrow a+(5-1)d = 41$$

$$\Rightarrow a+4d = 41 \text{ ---------------- (1)}$$

& 11^{th} term $= 20$

$$\Rightarrow a+(11-1)d = 20$$

$$\Rightarrow a+10d = 20 \text{ ---------------- (2)}$$

Subtracting (2) from (1), we get

$$-6d = 21$$

$$\Rightarrow d = -\frac{21}{6} = -\frac{7}{2}$$

Substituting the value of d in (1), we get

$$a+4(-\frac{7}{2}) = 41$$

$$\Rightarrow a = 41+14 = 55$$

$\therefore$ Sum to first 12 terms $= \frac{12}{2}\left[2\times 55+(12-1)(-\frac{7}{2})\right]$

$$= 6\left[110-\frac{77}{2}\right]$$

$$= 6\left(\frac{220-77}{2}\right)$$

$$= 3\times 143$$

$$= 429$$

Q30. The 12th term of an AP=-13 and the sum of the first 4 terms is 24. Find the sum of the first 10 terms.

Solution: Let a be the first term and d be the common difference.

Now, 12^{th} term$= -13$

$\Rightarrow a+(12-1)d=-13$

$\Rightarrow a+11d=-13$ ---------------- (1)

and $24=\frac{4}{2}\{2a+(4-1)d\}$

$\Rightarrow 24=2(2a+3d)$

$\Rightarrow 24=4a+6d$

$\Rightarrow 24=4(-13-11d)+6d$ [From (1)]

$\Rightarrow 24=-52-44d+6d$

$\Rightarrow 76=-38d$

$\Rightarrow d=-2$ ---------------- (2)

and $a+11\times(-2)=-13$ [From (1) & (2)]

$\Rightarrow a-22=-13$

$\Rightarrow a=9$

$\therefore$ Sum to first 10 terms $=\frac{10}{2}\{2\times9+(10-1)(-2)\}$

$=5\{10+9\times(-2)\}$

$=5(18-18)$

$=0$

Q31. The first term of an AP is 7 and the sum of the first 15 terms is 420. Find the common difference of the AP.

Solution: Here $a=7$ and let d be the common difference.

Now, sum of the first 15 terms = 420

$\Rightarrow \frac{15}{2}\{2\times7+(15-1)d\}=420$

$\Rightarrow \frac{15}{2}(14+14d)=420$

$\Rightarrow 15(7+7d)=420$

$\Rightarrow 105+105d=420$

$\Rightarrow 105d=315$

$\Rightarrow d=\frac{315}{105}=3$

Q32. The sum of the first 15 terms and that of the first 22 terms of an AP are 495 and 1034 respectively. Find the sum of the first 18 terms.

Solution: Let a be the first term and d be the common difference.

Now, Sum of the first 15 terms = 495

$$\Rightarrow \frac{15}{2}\left[2a+(15-1)d\right]=495$$

$$\Rightarrow \frac{15}{2}\left[2a+14d\right]=495$$

$$\Rightarrow 15\left[a+7d\right]=495$$

$$\Rightarrow a+7d=33 \quad \text{---------------- (1)}$$

& Sum of the first 22 terms = 1034

$$\Rightarrow \frac{22}{2}\{2a+(22-1)d\}=1034$$

$$\Rightarrow 11(2a+21d)=1034$$

$$\Rightarrow 2a+21d=94$$

$$\Rightarrow 2(33-7d)+21d=94 \quad \text{[From (1)]}$$

$$\Rightarrow 66-14d+21d=94$$

$$\Rightarrow 7d=28$$

$$\Rightarrow d=4 \text{ ---------------- (2)}$$

From (1) and (2), we get

$$a+7\times 4=33$$

$$\Rightarrow a=33-28=5$$

$\therefore$ Sum of the first 18 terms $=\frac{18}{2}\{2\times 5+(18-1)4\}$

$= 9(10+17\times 4)$

$= 9(10+68)$

$= 9\times 78$

$= 702$

Q33. The sum of the first 21 terms of an AP is 28 and that of the first 28 terms is 21. Show that one term of the AP is zero and find the sum of the preceding terms.

Solution: Let *a* be the first term and *d* be the common difference.

Now, $28 = \frac{21}{2}[2a + (21-1)d]$

$\Rightarrow 28 = \frac{21}{2}[2a + 20d]$

$\Rightarrow 2a + 20d = \frac{28 \times 2}{21} = \frac{8}{3}$ ---------------- (1)

and $21 = \frac{28}{2}[2a + (28-1)d]$

$\Rightarrow 21 = 14\{2a + 27d\}$

$\Rightarrow 2a + 27d = \frac{3}{2}$ ---------------- (2)

Subtracting (1) from (2), we get

$7d = \frac{3}{2} - \frac{8}{3} = \frac{9-16}{6} = \frac{-7}{6}$

$\therefore d = -\frac{1}{6}$

$\therefore 2a + 20d = \frac{8}{3}$

$\Rightarrow 2a + 20 \times \left(-\frac{1}{6}\right) = \frac{8}{3}$

$\Rightarrow 2a - \frac{10}{3} = \frac{8}{3}$

$\Rightarrow 2a = \frac{18}{3} = 6$

$\Rightarrow a = 3$

Now, let $a_n = 0$

$\Rightarrow a + (n-1)d = 0$

$\Rightarrow 3 + (n-1)\left(-\frac{1}{6}\right) = 0$

$\Rightarrow 18 - n + 1 = 0$

$\Rightarrow n = 19 \in N$

Thus, 0 is one of the term of the AP.

Again, the sum of the preceding terms = the sum of the first 18 terms

$= \frac{18}{2}\{2a + (18-1)d\}$

$= 9\left\{2 \times 3 + 17 \times \left(-\frac{1}{6}\right)\right\}$

$= 9\left\{\frac{36-17}{6}\right\} = \frac{3 \times 19}{2}$

$= \frac{57}{2} = 28\frac{1}{2}$

Q34. The sum of the first 10 terms of an AP is 30 and the sum of the next 10 terms is -170. Find the sum of the next 10 terms following these.

Solution: Let a be the first term and d be the common difference.

Now, $30 = \frac{10}{2}\{2a + (10-1)d\}$

$\Rightarrow 30 = 5(2a + 9d)$

$\Rightarrow 2a + 9d = 6$ ---------------- (1)

& $-140 = \frac{20}{2}\{2a + (20-1)d)\}$

$\Rightarrow -140 = 10(2a + 19d)$

$\Rightarrow 2a + 19d = -14$ ---------------- (2)

Now, subtracting (1) from (2) , we get

$10d = -20$

$\Rightarrow d = -2$

Putting the value of d in equation (1), we get

$2a = 6 - \{9 \times (-2)\} = 24$

$\Rightarrow a = 12$

Then, 21^{st} term $= 12 + (21-1)(-2)$

$= 12 - 40 = -28$

$\therefore$ Sum of the next 10 terms $= \frac{10}{2}\{2 \times (-28) + (10-1)(-2)\}$

$= 5\{-56 - 18\}$

$= 5 \times (-74)$

$= -370$

Q35. Find the sum of the integers between 21 and 99 divisible by 6.

Solution: The integer divisible by 6 next to 21 is 24 and the integer divisible by 6 just below 94 is 96.

$\therefore a = 24$, $d = 6$ and the last term = 96

Then, $96 = 24 + (n-1)6$

$\Rightarrow 72 = 6n - 6$

$\Rightarrow 6n = 78$

$\Rightarrow n = 13$

$$\therefore \text{Sum} = \frac{13}{2}\{2\times 24 + (13-1)6\}$$

$$= \frac{13}{2}\{48 + 12\times 6\}$$

$$= \frac{13}{2}\{48 + 72\}$$

$$= \frac{13}{2}\times 120 = 780$$

Q36. Find the AP when the sum to n terms is

(i) n^2

Solution: Here, $S_n = n^2$

$S_1 = 1^2 = 1$

$S_2 = 2^2 = 4$ and so on.

Let a be the first term and d be the common difference.

Then, $S_1 = a = 1$

$S_2 = a + (a+d)$

$\Rightarrow 4 = 1 + (1+d)$

$\Rightarrow d = 2$

Hence, the required A.P. is 1, 3, 5, 7, 9, ...

(ii) $2n^2+5n$

Solution: Here, $S_n = 2n^2+5n$

$\therefore \quad S_1 = 2.1^2+5.1 = 2+5 = 7$

$S_2 = 2.2^2+5.2 = 8+10 = 18$ and so on.

Let a be the first term and d be the common difference,

Then, $S_1 = a = 7$

$S_2 = a+(a+d)$

$\Rightarrow 18 = 7+7+d$

$\Rightarrow d = 18-14 = 4$

$\therefore$ The required AP. Is 7, 11, 15, 19, 23, ……….

Q37. If the first term of an AP be *a*, its common difference be *2a* and the sum of the first *n* terms be S, prove that $n=\sqrt{\frac{S}{a}}$.

Solution: Here, a= first term, c.d.=2a

Now, Sum of the first n terms= $\frac{n}{2}\{2a+(n-1)2a\}$

$= \frac{n}{2}\{2a+2an-2a\}$

$= \frac{n}{2}\times 2an$

$= an^2 = \text{S}$

Then, $\sqrt{\frac{S}{a}} = \sqrt{\frac{an^2}{a}} = \sqrt{n^2} = n$

$\Rightarrow n = \sqrt{\frac{S}{a}}$

Q38. The sum of the first n terms and that of the first m terms of an AP are m and n respectively. Show that the sum of the first $(m+n)$ terms is $-(m+n)$.

Solution: Let a be the first term and d be the common difference.

Then, $S_n = m$

$$\Rightarrow \frac{n}{2}\{2a+(n-1)d\} = m$$

$$\Rightarrow 2an + n(n-1)d = 2m \quad \text{---------------- (1)}$$

and $S_m = n$

$$\Rightarrow \frac{m}{2}\{2a+(m-1)d\} = n$$

$$\Rightarrow 2am + m(m-1)d = 2n \quad \text{---------------- (2)}$$

Subtracting (1) from (2), we get

$$2a(m-n) + \{m(m-1) - n(n-1)d\}d = 2n - 2m$$

$$\Rightarrow 2a(m-n) + \{(m^2-n^2)-(m-n)\}d = -2(m-n)$$

$$\Rightarrow 2a + (m+n-1)d = -2 \quad \text{---------------- (3)}$$

Now, $S_{m+n} = \frac{m+n}{2}\{2a+(m+n-1)d\}$

$$= \frac{m+n}{2}(-2)) \qquad \text{[by (3)]}$$

$$\therefore S_{m+n} = -(m+n).$$

Q39. If the sum of m terms of an AP be equal to the sum of n terms. Prove that the sum of $(m+n)$ terms is zero.

Solution: Let a be the first term and d be the common difference.

Then, $S_m = S_n$

$$\Rightarrow \frac{m}{2}[2a+(m-1)d] = \frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow 2a(m-n) + \{m(m-1) - n(n-1)\}d = 0$$

$$\Rightarrow 2a(m-n) + \{m^2 - m - n^2 + n\}d = 0$$

$$\Rightarrow 2a(m-n) + \{(m^2-n^2)-(m-n)\}d = 0$$

$$\Rightarrow (m-n)[2a+(m+n-1)d] = 0$$

$$\Rightarrow 2a + (m+n-1)d = 0 \qquad [\because m-n \neq 0] \text{---------------- (1)}$$

$$\therefore S_{m+n} = \frac{m+n}{2}[2a+(m+n-1)d]$$

$$= \frac{m+n}{2} \times 0 \quad \text{[by (1)]}$$

$$= 0$$

Q40. If the p^{th} terms of an AP is $\frac{1}{q}$ and q^{th} terms is $\frac{1}{p}$, prove that the sum of first pq terms = $\frac{1}{2}(pq+1)$.

Solution: Let a be the first term and d be the c.d.

Then, $a_p = \frac{1}{q}$

$$\Rightarrow a+(p-1)d = \frac{1}{q}$$

$$\Rightarrow a+pd-d = \frac{1}{q} \quad \text{----------------- (1)}$$

and $a_q = \frac{1}{p}$

$$\Rightarrow a+(q-1)d = \frac{1}{p}$$

$$\Rightarrow a+qd-d = \frac{1}{p} \quad \text{----------------- (2)}$$

Subtracting (2) from (1), we get

$$a+pd-d-(a+qd-d) = \frac{1}{q}-\frac{1}{p}$$

$$\Rightarrow (p-q)d = \frac{p-q}{pq} \Rightarrow d = \frac{1}{pq}$$

From (1), we get

$$a+(p-1)\times\frac{1}{pq} = \frac{1}{q}$$

$$\Rightarrow a = \frac{1}{q}-\frac{p-1}{pq} = \frac{p-p+1}{pq} = \frac{1}{pq}$$

$$\therefore S_{pq} = \frac{pq}{2}\left[2\times\frac{1}{pq}+(pq-1)\times\frac{1}{pq}\right]$$

$$= \frac{pq}{2}\left[\frac{2+pq-1}{pq}\right]$$

$$= \frac{pq+1}{2}$$

$$= \frac{1}{2}(pq+1)$$

SOLUTIONS

EXERCISE – 2.3

Q1. Find the specified term of the following GP.

(i) 14^{th} term of $\frac{1}{8},\frac{1}{4},\frac{1}{2},1,..................$

Solution: Here, $a=\frac{1}{8},\ r=\frac{1}{4}\times\frac{8}{1}=2$

$\therefore\ 14^{th}$ term, $a_{14}=ar^{14-1}$

$=\frac{1}{8}\times 2^{13}$

$=\frac{1}{2^3}\times 2^{13}$

$=2^{10}=1024$

(ii) 7^{th} term of $81,-27,9,-3,................$

Solution: Here, $a=81,\ r=-\frac{27}{81}=-\frac{1}{3}$

$\therefore\ 7^{th}$ term, $a_7=ar^{7-1}$

$=81\times\left(-\frac{1}{3}\right)^6$

$=81\times\frac{1}{729}$

$=\frac{1}{9}$

(iii) 10^{th} term of $\frac{1}{\sqrt{2}},\sqrt{2},2\sqrt{2},...........$

Solution: Here, $a=\frac{1}{\sqrt{2}},\ r=\frac{\sqrt{2}}{1/\sqrt{2}}=2$

$\therefore 10^{th}$ term, $a_{10}=ar^{10-1}$

$=\frac{1}{\sqrt{2}}\times 2^{10-1}$

$=\frac{1}{\sqrt{2}}\times 2^9$

$=\frac{2^8\times\sqrt{2}\times\sqrt{2}}{\sqrt{2}}$

$=526\sqrt{2}$

(iv) 8th term of $p^2, pq, q^2, \ldots\ldots\ldots\ldots$

Solution: Here, $a = p^2$, $r = \dfrac{pq}{p^2} = \dfrac{q}{p}$

$\therefore 8^{\text{th}}$ term, $a_8 = ar^{8-1}$

$$= p^2 \times \left(\frac{q}{p}\right)^7$$

$$= \frac{q^7}{p^5}$$

Q2. Find the value of *k* so that the following may be in GP.

(i) $k+1, 2k+2, 5k-2$

Solution: Since $(k+1), (2k+2), (5k-2)$ are in GP,

$$(2k+2)^2 = (k+1)(5k-2)$$
$$\Rightarrow 4k^2 + 8k + 4 = 5k^2 - 2k + 5k - 2$$
$$\Rightarrow -k^2 + 5k + 6 = 0$$
$$\Rightarrow k^2 - 5k - 6 = 0$$
$$\Rightarrow k^2 - 6k + k - 6 = 0$$
$$\Rightarrow (k-6)(k+1) = 0$$
$$\Rightarrow k = 6 \text{ or } -1$$

When $k = -1$, the first term & 2^{nd} term are zero.

Hence $k = 6$.

(ii) $3k+1, 6k-4, 3k-2$

Solution: Since $3k+1, 6k-4, 3k-2$ are in GP,

$$\therefore (6k-4)^2 = (3k+1)(3k-2)$$
$$\Rightarrow 36k^2 - 48k + 16 = 9k^2 - 6k + 3k - 2$$
$$\Rightarrow 27k^2 - 45k + 18 = 0$$
$$\Rightarrow 3k^2 - 5k + 2 = 0$$
$$\Rightarrow (k-1)(3k-2) = 0$$
$$\Rightarrow k = 1 \text{ or } \frac{2}{3}$$

When $k = \dfrac{2}{3}$, the 2^{nd} and 3^{rd} terms are zero.

$\therefore k = 1$.

(iii) $k-1, 3k-3, 8k-2$

Solution: Since $k-1, 3k-3, 8k-2$ are in GP,

$$\therefore (3k-3)^2 = (k-1)(8k-2)$$

$$\Rightarrow 9k^2 - 18k + 9 = 8k^2 - 2k - 8k + 2$$

$$\Rightarrow k^2 - 8k + 7 = 0$$

$$\Rightarrow k^2 - 7k - k + 7 = 0$$

$$\Rightarrow (k-7)(k-1) = 0$$

$$\Rightarrow k = 7 \text{ or } 1$$

When $k = 1$, the first and 2nd terms are zero.

$$\therefore k = 7$$

Q3. The fourth and seventh terms of a GP.are 54 and 1458 respectively. Find the 10th term.

Solution: We have, $a_4 = ar^{4-1}$

$$\Rightarrow 54 = ar^3 \quad \text{-------------------- (1)}$$

$$\& \quad a_7 = ar^{7-1}$$

$$\Rightarrow 1458 = a..r^6 \quad \text{-------------------- (2)}$$

Dividing (2) by (1), we get

$$\frac{1458}{54} = \frac{ar^6}{ar^3}$$

$$\Rightarrow 27 = r^3$$

$$\Rightarrow r = 3$$

From (1), $54 = a \times 27$

$$\Rightarrow a = \frac{54}{27} = 2$$

$$\therefore 10^{th} \text{ term, } a_{10} = ar^{10-1}$$

$$= 2 \times 3^9$$

$$= 2 \times 19683$$

$$= 39366$$

Q4. (i) Which term of the GP. 9, 3, 1, ………. is $\frac{1}{243}$?

Solution: Let $\frac{1}{243}$ be the nth term.

Then, $ar^{n-1} = \frac{1}{243}$

$$\Rightarrow 9 \times \left(\frac{1}{3}\right)^{n-1} = \frac{1}{243}$$

$$\Rightarrow \left(\frac{1}{3}\right)^{n-1} = \frac{1}{9 \times 243} = \frac{1}{3^2 \times 3^5} = \left(\frac{1}{3}\right)^7$$

$$\Rightarrow n - 1 = 7$$

$$\Rightarrow n = 7 + 1 = 8$$

$\therefore \frac{1}{243}$ is the 8th term.

(ii) Which term of the GP 32, -16, 8, -4, …….., is $\frac{1}{32}$?

Solution: Let $\frac{1}{32}$ be the nth term.

Then, $ar^{n-1} = \frac{1}{32}$

$$\Rightarrow 32 \times \left(-\frac{16}{32}\right)^{n-1} = \frac{1}{32}$$

$$\Rightarrow \left(-\frac{1}{2}\right)^{n-1} = \frac{1}{32 \times 32} = \frac{1}{2^{10}}$$

$$\Rightarrow \left(-\frac{1}{2}\right)^{10} = \left(-\frac{1}{2}\right)^{n-1}$$

$$\Rightarrow n - 1 = 10$$

$$\Rightarrow n = 11$$

Hence, $\frac{1}{32}$ is the 11thterm.

Q5. If the sum of three nos. in GP.is 104 and their product is 13824, find the numbers.

Solution: Let $\frac{a}{r}$, a and ar be the three numbers.

Then, product = $\frac{a}{r} \times a \times ar$

$\Rightarrow 13824 = a^3$

$\Rightarrow (24)^3 = a^3$

$\Rightarrow a = 24$

& their sum = 104

$\Rightarrow \frac{a}{r} + a + ar = 104$

$\Rightarrow \frac{a + ar + ar^2}{r} = 104$

$\Rightarrow 24 + 24r + 24r^2 = 104r$

$\Rightarrow 1 + r + r^2 = \frac{104}{24} r = \frac{13r}{3}$

$\Rightarrow 3 + 3r + 3r^2 = 13r$

$\Rightarrow 3r^2 - 10r + 3 = 0$

$\Rightarrow 3r^2 - 9r - r + 3 = 0$

$\Rightarrow 3r(r-3) - 1(r-3) = 0$

$\Rightarrow (r-3)(3r-1) = 0$

$\Rightarrow r = 3 \text{ or } \frac{1}{3}$

Hence, the numbers are 8, 24, 72 or 72, 24, 8 ….

Q6. Divide 42 into three parts which are in GP such that their product is 512.

Solution: Let $\frac{a}{r}$, a and ar be the three numbers.

Then, $\frac{a}{r} \times a \times ar = 512$

$\Rightarrow a^3 = 512 = 8^3$

$\Rightarrow a = 8$

& $\frac{a}{r} + a + ar = 42$

$\Rightarrow a\left(\frac{1}{r} + 1 + r\right) = 42$

$\Rightarrow 8\left(\frac{1 + r + r^2}{r}\right) = 42$

$\Rightarrow \frac{1 + r + r^2}{r} = \frac{42}{8}$

$\Rightarrow 1 + r + r^2 = \frac{21r}{4}$

$\Rightarrow 4 + 4r + 4r^2 = 21r$

$\Rightarrow 4r^2 - 17r + 4 = 0$

$\Rightarrow 4r^2 - 16r - r + 4 = 0$

$\Rightarrow 4r(r - 4) - 1(r - 4) = 0$

$\Rightarrow (r - 4)(4r - 1) = 0$

$\Rightarrow r = 4 \text{ or } \frac{1}{4}$

Hence, the three numbers are 2, 8, 32.

Q7. Divide 31 into three parts which are in GP such that the sum of their squares is 651.

Solution: Let a, ar, ar^2 be the three parts which are in GP.

Then, $a + ar + ar^2 = 31$ ---------------- (i)

And $a^2 + a^2r^2 + a^2r^4 = 651$ ---------------- (ii)

Squaring both sides of (i), we get

$$a^2 + a^2r^2 + a^2r^4 + 2a^2r + 2a^2r^3 + 2a^2r^2 = 31^2$$

$$\Rightarrow (a^2 + a^2r^2 + a^2r^4) + 2ar(a + ar + ar^2) = 961$$

$$\Rightarrow 651 + 2ar \times 31 = 961$$

$$\Rightarrow 2 \times 31ar = 310$$

$$\Rightarrow ar = \frac{310}{31 \times 2}$$

$$\Rightarrow a = \frac{5}{r} \quad \text{---------------- (iii)}$$

Using (iii) in (i) we get

$$\frac{5}{r} + 5 + 5r = 31$$

$$\Rightarrow 5 + 5r + 5r^2 = 31r$$

$$\Rightarrow 5r^2 - 26r + 5 = 0$$

$$\Rightarrow 5r^2 - 25r - r + 5 = 0$$

$$\Rightarrow 5r(r - 5) - 1(r - 5) = 0$$

$$\Rightarrow (5r - 1)(r - 5) = 0$$

$$\Rightarrow r = \frac{1}{5}, 5$$

When $r = \frac{1}{5}$, $a = 25$ and the GP is 25, 5, 1

When $r = 5$, $a = 1$ and the GP is 1, 5, 25

Hence the three numbers are 1, 5 and 25.

Q8. Three numbers whose sum is 18 are in AP. when 2, 4, 11 are added to them respectively, the resulting numbers are in GP. Find the numbers.

Solution: Let $a-d, a, a+d$ be the three numbers.

Then, $a-d+a+a+d=18$

$\Rightarrow 3a=18$

$\Rightarrow a=6$

And $(a-d+2),(a+4),(a+d+11)$ are in GP

$$\Rightarrow \frac{a+4}{a-d+2}=\frac{a+d+11}{a+4}$$

$$\Rightarrow \frac{6+4}{6-d+2}=\frac{6+d+11}{6+4}$$

$$\Rightarrow \frac{10}{8-d}=\frac{17+d}{10}$$

$\Rightarrow 136+8d-17d-d^2=100$

$\Rightarrow d^2+9d-36=0$

$\Rightarrow d^2+12d-3d-36=0$

$\Rightarrow (d+12)(d-3)=0$

$\Rightarrow d=3 \text{ or} -12$

$\therefore$ The three nos. are 3, 6, 9 or 18, 6, -6.

Q9. The product of three numbers in GP is 729 and the sum of their product in pairs is 819. Find the numbers.

Solution: Let $\frac{a}{r}, a, ar$ be the three nos. in GP.

Then, $\frac{a}{r}\times a\times ar=729$

$\Rightarrow a^3=9^3$

$\Rightarrow a=9$

and $\frac{a}{r}\times a + a\times ar + ar\times\frac{a}{r} = 819$

$$\Rightarrow \frac{a^2}{r} + a^2 r + a^2 = 819$$

$$\Rightarrow \frac{(a^2 + a^2 r^2 + a^2 r)}{r} = 819$$

$$\Rightarrow a^2(1 + r^2 + r) = 819r$$

$$\Rightarrow 1 + r + r^2 = \frac{91r}{9}$$

$$\Rightarrow 9 + 9r + 9r^2 - 91r = 0$$

$$\Rightarrow 9r^2 - 82r + 9 = 0$$

$$\Rightarrow 9r^2 - 81r - r + 9 = 0$$

$$\Rightarrow (9r-1)(r-9) = 0$$

$$\Rightarrow r = \frac{1}{9}, 9$$

Hence, the numbers are 1, 9, 81 or 81, 9, 1.

10. If a, b, c are in GP, show that

(i) $a^2 + b^2, ab + bc, b^2 + c^2$ are in GP.

(ii) $\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ are in AP

Solution: (i) Since a, b, c are in GP

$b^2 = ac$ --------------- (1)

Now, $a^2 + b^2, ab + bc, b^2 + c^2$ are in GP

if $(ab + bc)^2 = (a^2 + b^2)(b^2 + c^2)$

if $a^2b^2 + b^2c^2 + 2b^2ac = a^2b^2 + a^2c^2 + b^4 + b^2c^2$

if $2b^2ac = a^2c^2 + b^2b^2$

if $2b^2ac = a^2c^2 + b^2.ac$ [by (1)]

if $b^2ac = a^2c^2$

if $b^2 = ac$ which is true by (1)

Hence $a^2 + b^2$, $ab + bc, b^2 + c^2$ are in GP.

(ii) Since a,b,c are in GP

$$b^2 = ac \text{ ---------------- (1)}$$

Now, $\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ are in AP

$$\text{if} \frac{1}{2b} - \frac{1}{a+b} = \frac{1}{b+c} - \frac{1}{2b}$$

$$\text{if} \frac{a+b-2b}{2b(a+b)} = \frac{2b-b-c}{(b+c)2b}$$

$$\text{if} \frac{a-b}{2b(a+b)} = \frac{b-c}{(b+c)2b}$$

$$\text{if} \frac{a-b}{a+b} = \frac{b-c}{b+c}$$

$$\text{if} (a-b)(b+c) = (a+b)(b-c)$$

$$\text{if} ab + ac - b^2 - bc = ab - ac + b^2 - bc$$

$$\text{if} 2ac = 2b^2$$

$$\Rightarrow b^2 = ac \text{ which is true by (1)}$$

Hence $\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ are in AP.

11. If a,b,c,d are in GP, prove that

(i) $a+b, b+c, c+d$ are in GP

Solution: Since a,b,c,d are in GP

$$ad = bc \text{ ---------------- (1)}$$

And $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ ----------------- (2)

Now, $a+b, b+c, c+d$ are in GP

$$\text{if} \frac{b+c}{a+b} = \frac{c+d}{b+c}$$

$$\text{if} (b+c)^2 = (a+b)(c+d)$$

$$\text{if} b^2 + 2bc + c^2 = ac + ad + bc + bd$$

$$\text{if} b^2 + c^2 + bc = ac + bc + bd \quad \text{[using (1)]}$$

$$\text{if} b^2 + c^2 = ac + bd$$

$$\text{if} ac + c^2 = ac + bd \left[\because \frac{b}{a} = \frac{c}{b}\right]$$

$$\text{if} c^2 = bd \text{ which is true by (2)}$$

Hence $a+b, b+c, c+d$ are in GP.

(ii) $a^2-b^2, b^2-c^2, c^2-d^2$ **are in GP**

Solution: Let $\frac{b}{a}=\frac{c}{b}=\frac{d}{c}=r$

$\therefore b=ar, c=br=ar^2, d=cr=ar^3$

Now, $a^2-b^2, b^2-c^2, c^2-d^2$ are in GP

$$\text{if}\frac{b^2-c^2}{a^2-b^2}=\frac{c^2-d^2}{b^2-c^2}$$

$$\text{if}(b^2-c^2)^2=(a^2-b^2)(c^2-d^2)$$

$$\text{if}(a^2r^2-a^2r^4)^2=(a^2-a^2r^2)(a^2r^4-a^2r^6)$$

$$\text{if}\left\{a^2r^2\left(1-r^2\right)\right\}^2=a^2(1-r^2)a^2r^4(1-r^2)$$

$$\text{if}\,a^4r^4(1-r^2)=a^4r^4(1-r^2) \text{ which is true.}$$

Hence, $a^2-b^2, b^2-c^2, c^2-d^2$ are in GP.

(iii) $(a-b)^2, (b-c)^2, (c-d)^2$ **are in GP**

Solution: Since a, b, c, d are in GP.

$$\frac{b}{a}=\frac{c}{b}=\frac{d}{c}=r$$

$\therefore b=ar,\ c=br=ar^2,\ d=cr=ar^3$

Now, $(a-b)^2, (b-c)^2, (c-d)^2$ are in GP.

$$\text{if}\frac{(b-c)^2}{(a-b)^2}=\frac{(c-d)^2}{(b-c)^2}$$

$$\text{if}(b-c)^4=(a-b)^2(c-d)^2$$

$$\text{if}(ar-ar^2)^4=(a-ar)^2(ar^2-ar^3)^2$$

$$\text{if}\{ar(1-r)\}^4=a^2(1-r)^2\{ar^2(1-r)\}^2$$

$$\text{if}\,a^4r^4(1-r)^4=a^2(1-r)^2a^2r^4(1-r)^2$$

$$\text{if}\,a^4r^4(1-r)^4=a^4r^4(1-r)^4 \text{ which is true.}$$

Hence, $(a-b)^2, (b-c)^2, (c-d)^2$ are in GP.

(iv) $\frac{1}{a^2+b^2}, \frac{1}{b^2+c^2}, \frac{1}{c^2+d^2}$ **are in GP**

Solution: Since a, b, c, d are in GP

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = r$$

$$\therefore b = ar,\ c = br = ar^2,\ d = cr = ar^3$$

Now, $\frac{1}{a^2+b^2}, \frac{1}{b^2+c^2}, \frac{1}{c^2+d^2}$ are in GP

$$\text{if} \frac{1}{b^2+c^2} \div \frac{1}{a^2+b^2} = \frac{1}{c^2+d^2} \div \frac{1}{b^2+c^2}$$

$$\text{if} \frac{a^2+b^2}{b^2+c^2} = \frac{b^2+c^2}{c^2+d^2}$$

$$\text{if} \frac{a^2+a^2r^2}{a^2r^2+a^2r^4} = \frac{a^2r^2+a^2r^4}{a^2r^4+a^2r^6}$$

$$\text{if} \frac{a^2(1+r^2)}{a^2r^2(1+r^2)} = \frac{a^2r^2(1+r^2)}{a^2r^4(1+r^2)}$$

$$\text{if} \frac{1}{r^2} = \frac{1}{r^2}$$ which is true.

Hence $\frac{1}{a^2+b^2}, \frac{1}{b^2+c^2}, \frac{1}{c^2+d^2}$ are in GP.

Q12. If p^{th}, q^{th}, r^{th} terms of a GP. are also in GP, show that p,q,r are in AP.

Solution: Let a be the first term and R be the common ratio.

Then, p^{th} term $= aR^{p-1}$

q^{th} term $= aR^{q-1}$

r^{th} term $= aR^{r-1}$

The terms are also in GP.

$$\therefore \frac{aR^{q-1}}{aR^{p-1}} = \frac{aR^{r-1}}{aR^{q-1}}$$

$$\Rightarrow (R^{q-1})^2 = R^{r-1}R^{p-1}$$

$$\Rightarrow R^{2q-2} = R^{r+p-2}$$

$$\Rightarrow 2q-2 = r+p-2$$

$$\Rightarrow 2q = r+p$$

$$\Rightarrow q-p = r-q$$

Hence p,q,r are in AP.

Q13. If a, b, c, d are in GP, show that

(i) $(b+c)(b+d) = (c+a)(c+d)$

Solution: Since a, b, c, d are in GP

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = r \text{ (common ratio)}$$

$$\Rightarrow b = ar,\ c = br = ar^2,\ d = cr = ar^3$$

Now, $(b+c)(b+d) = (ar+ar^2)(ar+ar^3)$

$= ar(1+r)ar(1+r^2)$

$= a^2r^2(1+r)(1+r^2)$

& $(c+a)(c+d) = (ar^2+a)(ar^2+ar^3)$

$= a(1+r^2).ar^2(1+r)$

$= a^2r^2(1+r)(1+r^2)$

Hence, $(b+c)(b+d) = (c+a)(c+d)$

(ii) $(a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2$

Solution: Since a, b, c, d are in GP

$$\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = r$$

$$\Rightarrow b = ar,\ c = br = ar^2,\ d = cr = ar^3$$

Now, $(a^2+b^2+c^2)(b^2+c^2+d^2)$

$= (a^2+a^2r^2+a^2r^4)(a^2r^2+a^2r^4+a^2r^6)$

$= a^2(1+r^2+r^4).a^2r^2(1+r^2+r^4)$

$= a^4r^2(1+r^2+r^4)^2$

$= \left\{a^2r\left(1+r^2+r^4\right)\right\}^2$

$= (a^2r+a^2r^3+a^2r^5)^2$

$= (a.ar+ar.ar^2+ar^2.ar^3)^2$

$= (ab+bc+cd)^2$

Hence proved.

(iii) $(b-c)^2+(c-a)^2+(d-b)^2=(a-d)^2$

Solution: Since a,b,c,d are in GP

$$\frac{b}{a}=\frac{c}{b}=\frac{d}{c}=r$$

$$\Rightarrow b=ar,\ c=br=ar^2,\ d=cr=ar^3$$

Now, $(b-c)^2+(c-a)^2+(d-b)^2$

$$=(ar-ar^2)^2+(ar^2-a)^2+(ar^3-ar)^2$$

$$=a^2r^2-2a^2r^3+a^2r^4+a^2r^4-2a^2r^2+a^2+a^2r^6-2a^2r^4+a^2r^2$$

$$=\cancel{2a^2r^2}-\cancel{2a^2r^2}+\cancel{2a^2r^4}-\cancel{2a^2r^4}-2a^2r^3+a^2r^6+a^2$$

$$=a^2-2a^2r^3+(ar^3)^2$$

$$=\left\{a-\left(ar^3\right)\right\}^2$$

$$=(a-d)^2$$

Q14. If 1, 1, 3, 9 be added respectively to the four terms of an AP, a GP results. Find the four terms of the AP.

Solution: Let $a-d,a,a+d,a+2d$ be the four terms of the AP.

Then, $a-d+1,a+1,a+d+3,a+2d+9$ are in GP.

Then, $$\frac{a+1}{a-d+1}=\frac{a+d+3}{a+1}=\frac{a+2d+9}{a+d+3}$$

Now, $$\frac{a+1}{a-d+1}=\frac{a+d+3}{a+1}$$

$$\Rightarrow a^2+2a+1=(a^2+ad+3a-ad-d^2-3d+a+d+3)$$

$$\Rightarrow 2a+1=4a-d^2-2d+3$$

$$\Rightarrow d^2=2a-2d+2 \text{-------------- (i)}$$

and, $\frac{a+1}{a-d+1}=\frac{a+2d+9}{a+d+3}$

$\Rightarrow a^2+ad+3a+a+d+3=a^2+2ad+9a-ad-2d^2-9d+a+2d+9$

$\Rightarrow 4a+d+3=10a-7d-2d^2+9$

$\Rightarrow 2d^2=6a-8d+6$

$\Rightarrow d^2=3a-4d+3$ -------------- (ii)

Again, $\frac{a+d+3}{a+1}=\frac{a+2d+9}{a+d+3}$

$\Rightarrow a^2+d^2+9+2ad+6d+6a=a^2+2ad+9a+a+2d+9$

$\Rightarrow d^2+6d+6a+9=10a+2d+9$

$\Rightarrow d^2+4d-4a=0$

$\Rightarrow d^2=4a-4d$ --------------- (iii)

From (i) and (ii), we get

$2a-2d+2=3a-4d+3$

$\Rightarrow 2d-a=1$ ------------------ (iv)

From (i) and (iii), we get

$2a-2d+2=4a-4d$

$\Rightarrow a-d=1$

$\Rightarrow (2d-1)-d=1$ [by using (iv)]

$\Rightarrow d=2$

Putting $d=2$ in(iv), we get

$2\times 2-a=1$

$\Rightarrow a=3$

$\therefore$ The required AP is $3-2,3,3+2,3+2\times 2$ i.e. 1, 3, 5, 7.

Q15. If a,b,c be the p^{th}, q^{th}, r^{th} terms both of an AP and of a GP. Show that $a^{b-c}.b^{c-a}.c^{a-b}=1$

Solution: Since a,b,c are in AP.

$$b-a=c-b$$

$$\Rightarrow 2b=a+c \quad \text{------------------ (i)}$$

Again, a,b,c are in GP

$$\therefore \frac{b}{a}=\frac{c}{b}$$

$$\Rightarrow b^2=ac$$

$$\therefore a^{b-c}b^{c-a}c^{a-b}=a^{b-c}.(\sqrt{ac})^{(c-a)}c^{a-b}$$

$$=a^{b-c}.a^{\frac{1}{2}(c-a)}.c^{\frac{1}{2}(c-a)}.c^{a-b}$$

$$=a^{b-c+\frac{1}{2}c-\frac{1}{2}a}.c^{\frac{1}{2}c-\frac{1}{2}a+a-b}$$

$$=a^{b-\frac{1}{2}(c+a)}.c^{\frac{1}{2}(c+a)-b}$$

$$=a^{b-b}c^{b-b} \quad \text{[from (i)]}$$

$$=a^0.c^0$$

$$=1\times 1=1$$

Q16. If a,b,c are in GP and $a^{\frac{1}{x}}=b^{1/y}=c^{1/z}$. Show that x,y,z are in AP.

Solution: Let $a^{\frac{1}{x}}=b^{1/y}=c^{1/z}=k$ (say)

$$\Rightarrow a=k^x$$

$$b=k^y$$

$$c=k^z$$

Since a,b,c are GP

$$\therefore \frac{b}{a}=\frac{c}{b}$$

$$\Rightarrow \frac{k^y}{k^x}=\frac{k^z}{k^y}$$

$$\Rightarrow k^{2y}=k^x.k^z$$

$$\Rightarrow k^{2y}=k^{x+z}$$

$$\Rightarrow y-x=z-y$$

Hence, x,y,z are in AP.

Q17. Find the Geometric mean between

(i) 3 and 27

Solution: GM between 3 and 27 $= \sqrt{3 \times 27}$

$= \sqrt{81} = 9$

(ii) $\sqrt{2}$ **and** $8\sqrt{2}$

Solution: GM between $\sqrt{2}$ and $8\sqrt{2}$ $= \sqrt{\sqrt{2} \times 8\sqrt{2}}$

$= \sqrt{16} = 4$

(iii) $\frac{1}{5}$ **and 125**

Solution: GM between $\frac{1}{5}$ and 125 $= \sqrt{\frac{1}{5} \times 125}$

$= \sqrt{25} = 5$

Q18. Insert (i) 2 geometric means between 2 and $\frac{1}{4}$

(ii) 3 geometric means between $-\frac{1}{3}$ **and** $\frac{9}{8}$

(iii) 3 geometric means between 3 and 48.

(iv) 3 geometric means between -2 **and** $-\frac{1}{8}$**.**

(v) 5 geometric means between 8 and $\frac{1}{8}$

(vi) 3 geometric means between a **and** $\frac{1}{a}$**.**

Solution: (i) Let x_1, x_2 be the two geometric means.

Then, $2, x, x_2, \frac{1}{4}$ are in GP.

Here, $a = 2$ & 4^{th} term $= \frac{1}{4}$

$\Rightarrow 2r^{4-1} = \frac{1}{4}$

$\Rightarrow r^3 = \frac{1}{8}$

$\Rightarrow r = \frac{1}{2}$

$\therefore x_1 = 2 \times \frac{1}{2} = 1$

$x_2 = 1 \times \frac{1}{2} = \frac{1}{2}$

$\therefore$ The two geometric means are 1 and $\frac{1}{2}$.

(ii) Let x_1, x_2 be the two geometric means.

Then, $-\frac{1}{3}, x_1, x_2, \frac{9}{8}$ are in GP.

$$\therefore a = -\frac{1}{3} \text{ and } \frac{9}{8} = \left(-\frac{1}{3}\right) r^{4-1}$$

$$\Rightarrow \frac{9}{8} = -\frac{1}{3} r^3$$

$$\Rightarrow r^3 = -\frac{9 \times 3}{8} = -\frac{3^3}{2^3}$$

$$\Rightarrow r = -\frac{3}{2}$$

Hence, the two GMs are $-\frac{1}{3} \times (-\frac{3}{2}) = \frac{1}{2}$ and $\frac{1}{2} \times \frac{-3}{2} = -\frac{3}{4}$.

(iii) Let x_1, x_2, x_3 be the three GMs

Then, $3, x_1, x_2, x_3, 48$ are in GP

$$\therefore a = 3 \text{ and } 48 = 3 \times r^{5-1}$$

$$\Rightarrow 48 = 3 \times r^4$$

$$\Rightarrow r^4 = 16 = 2^4$$

$$\Rightarrow r = 2$$

$$\therefore x_1 = 3 \times 2 = 6,\ x_2 = 6 \times 2 = 12,\ x_3 = 12 \times 2 = 24$$

$\therefore$ The three GMs are 6, 12, 24.

(iv) Let x_1, x_2, x_3 be the GMs

Then, $-2, x_1, x_2, x_3, \frac{-1}{8}$ are in GP

$$\therefore -\frac{1}{8} = (-2) r^{5-1}$$

$$\Rightarrow -\frac{1}{8} = (-2) r^4$$

$$\Rightarrow r^4 = \frac{1}{16} = \frac{1}{2^4}$$

$$\Rightarrow r = \frac{1}{2}$$

$$\therefore x_1 = -2 \times \frac{1}{2} = -1$$

$$x_2 = -1 \times \frac{1}{2} = -\frac{1}{2}$$

$$x_3 = -\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Hence, the three GMs. are $-1, -\frac{1}{2}, -\frac{1}{4}$.

(v) Let x_1, x_2, x_3, x_4, x_5 be the five GMs

Then, $8, x_1, x_2, x_3, x_4, x_5, \frac{1}{8}$ are in GP.

$$\therefore \quad 8r^{7-1} = \frac{1}{8}$$

$$\Rightarrow 8r^6 = \frac{1}{8}$$

$$\Rightarrow r^6 = \frac{1}{64} = \frac{1}{2^6}$$

$$\Rightarrow r = \frac{1}{2}$$

$$\therefore \quad x_1 = 8 \times \frac{1}{2} = 4,\ x_2 = 4 \times \frac{1}{2} = 2,\ x_3 = 2 \times \frac{1}{2} = 1$$

$$x_4 = 1 \times \frac{1}{2} = \frac{1}{2},\ x_5 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$\therefore$ The five GMs. are 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$.

(vi) Let x_1, x_2, x_3 be the three GMs.

Then, $a, x_1, x_2, x_3, \frac{1}{a}$ are in GP

$$\therefore \quad \frac{1}{a} = ar^{5-1}$$

$$\Rightarrow \frac{1}{a} = ar^4$$

$$\Rightarrow r^4 = \frac{1}{a^2}$$

$$\Rightarrow r^2 = \frac{1}{a}$$

$$\Rightarrow r = \frac{1}{\sqrt{a}}$$

$$\therefore x_1 = a \times \frac{1}{\sqrt{a}} = \sqrt{a},\ x_2 = \sqrt{a} \times \frac{1}{\sqrt{a}} = 1,\ x_3 = 1 \times \frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}}$$

Hence, the three GMs are $\sqrt{a}$, 1, $\frac{1}{\sqrt{a}}$.

Q19. The arithmetic mean between two number is 15 and their geometric mean is 9. Find the numbers.

Solution: Let a and b be the two numbers.

Then, $AM = \dfrac{a+b}{2}$

$\Rightarrow 15 = \dfrac{a+b}{2}$

$\Rightarrow a+b=30$ --------------- (i)

& $GM = \sqrt{ab}$

$\Rightarrow 9 = \sqrt{ab}$

$\Rightarrow ab = 81$

$\Rightarrow (30-b)b = 81$ [using(i)]

$\Rightarrow 30b - b^2 = 81$

$\Rightarrow b^2 - 30b + 81 = 0$

$\Rightarrow b^2 - 27b - 3b + 81 = 0$

$\Rightarrow (b-27)(b-3) = 0$

$\Rightarrow b = 3,\ 27$

$\therefore$ When $b=3,\ a=27$ and when $b=27$, $a=3$.

Hence the two nos. are 3 and 27.

Q20. If a be the arithmetic mean between b and c and p,q be the geometric mean between them, show that $p^3+q^3=2abc$.

Solution: AM. between b and c = $\dfrac{b+c}{2}$

$\Rightarrow a = \dfrac{b+c}{2}$ --------------- (i)

& since b, p, q, c are in GP

$$\frac{p}{b} = \frac{q}{p} = \frac{c}{q}$$

$\therefore p^2 = bq$, $q^2 = pc$ and $bc = pq$ ---------------- (ii)

Now, $p^3 + q^3 = p^2.p + q^2.q$

$= bq.p + pc.q$

$= pq(b+c)$

$= bc \times 2a$ [From (i) & (ii)]

$= 2abc$

Q21. If a,b,c be in GP and x,y be the arithmetic means between a,b and b,c respectively, show that (i) $\frac{1}{x}+\frac{1}{y}=\frac{2}{b}$ (ii) $\frac{a}{x}+\frac{c}{y}=2$

Solution: (i) Since a,b,c are in GP.

$$\frac{b}{a}=\frac{c}{b}$$

$$\Rightarrow b^2 = ac \text{ -------------- (i)}$$

& $x=\frac{a+b}{2},\ y=\frac{b+c}{2}$

$$\therefore \frac{1}{x}+\frac{1}{y} = \frac{1}{\frac{a+b}{2}}+\frac{1}{\frac{b+c}{2}}$$

$$= \frac{2}{a+b}+\frac{2}{b+c}$$

$$= \frac{2(b+c)+2(a+b)}{(a+b)(b+c)}$$

$$= \frac{2(a+2b+c)}{ab+ac+b^2+bc}$$

$$= \frac{2(a+2b+c)}{ab+b^2+b^2+bc} \quad [\because b^2 = ac]$$

$$= \frac{2(a+2b+c)}{b(a+2b+c)}$$

$$= \frac{2}{b}$$

(ii) $\frac{a}{x}+\frac{c}{y} = \frac{a}{\frac{a+b}{2}}+\frac{c}{\frac{b+c}{2}}$

$= \frac{2a}{a+b}+\frac{2c}{b+c}$

$= \frac{2a(b+c)+2c(a+b)}{(a+b)(b+c)}$

$= \frac{2(ab+ac+ca+bc)}{ab+ac+b^2+bc}$

$= \frac{2(ab+ac+ac+bc)}{(ab+ac+ac+bc)}$ $[\because b^2=ac]$

$=2$

Q22. Prove that the product of *n* geometric means between *a*&*b*is $(ab)^{n/2}$.

Solution: Let $ar, ar^2, ar^3, ..., ar^n$ be the geometric means between a and b.

Then, $b = ar^{(n+2)-1} = ar^{n+1}$

Now, Product of n geometric means $= ar.ar^2.ar^3 ...., ar^n$

$= a^n r^{1+2+3+....+n}$

$= a^n r^{\frac{n(n+1)}{2}}$

$= a^{2.\frac{n}{2}}.r^{\frac{n}{2}(n+1)}$

$= \left(a^2 r^{n+1}\right)^{\frac{n}{2}}$

$= \left(a.ar^{n+1}\right)^{\frac{n}{2}}$

$= (ab)^{n/2}$ $[\because b = ar^{n+1}]$

Hence proved.

Q23. Find the sum of the first

(i) 10 terms of the GP 1, 2, 4, 8, ………..

Solution: Here, $a=1$, $r=\frac{2}{1}=2$ and $n=10$

$$\therefore S_{10}=\frac{a(r^n-1)}{r-1} \qquad [\because r>1]$$

$$=\frac{1\times(2^{10}-1)}{2-1}$$

$$=2^{10}-1$$

$$=1024-1=1023$$

(ii) 8 terms of the GP. $1,\frac{1}{3},\frac{1}{9},\frac{1}{27},..........$

Solution: Here, $a=1$, $r=\frac{1}{3}$, $n=8$

$$\therefore S_8=\frac{a(1-r^8)}{1-r} \qquad [\because r<1]$$

$$=\frac{1\left(1-\frac{1}{3^8}\right)}{1-\frac{1}{3}}=\frac{\frac{3^8-1}{3^8}}{\frac{3-1}{3}}$$

$$=\frac{3^8-1}{2\times3^7}=\frac{6560}{2\times2187}=\frac{3280}{2187}$$

(iii) 12 terms of the GP. 8, 4, 2, 1, …………

Solution: Here, $a=8$, $r=\frac{4}{8}=\frac{1}{2}$ & $n=12$

$$\therefore S_{12}=\frac{a(1-r^{12})}{1-r} \qquad [\because r<1]$$

$$=\frac{8\left(1-\frac{1}{2^{12}}\right)}{1-\frac{1}{2}}$$

$$=\frac{8\times(2^{12}-1)}{\frac{1}{2}\times2^{12}}$$

$$=8\times\frac{4096-1}{2^{11}}$$

$$=\frac{8\times4095}{2048}=\frac{4095}{256}$$

(iv) 7 terms of the GP. 1, $-3, 9, -27, \ldots\ldots\ldots$

Solution: Here, $a = 1$, $r = -3$, $n = 7$

$$\therefore\ S_7 = \frac{a(1-r^7)}{1-r} \qquad [\because r < 1]$$

$$= \frac{1\{1-(-3)^7\}}{1+3} = \frac{\{1-(-3)^7\}}{4}$$

$$= \frac{1+2187}{4} = \frac{2188}{4} = 547$$

(v) 9 terms of the GP 1, $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots\ldots\ldots$

Solution: Here, $a = 1$, $r = -\frac{1}{2} < 1$, $n = 9$

$$S_9 = \frac{a(1-r^9)}{1-r} = \frac{1\left\{1-\left(-\frac{1}{2}\right)^9\right\}}{1-\left(-\frac{1}{2}\right)}$$

$$= \frac{1-\left(-\frac{1}{512}\right)}{1+\frac{1}{2}} = \frac{512+1}{512\times\frac{3}{2}}$$

$$= \frac{513}{256\times 3} = \frac{171}{256}$$

(vi) *n* terms of the GP. 1, $\frac{1}{5}, \frac{1}{25}, \frac{1}{125}, \ldots\ldots\ldots$

Solution: Here, $a = 1$, , $r = \frac{1}{5} < 1$ and $n = n$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{1\times\left\{1-\left(\frac{1}{5}\right)^n\right\}}{1-\frac{1}{5}}$$

$$= \frac{1-\frac{1}{5^n}}{\frac{4}{5}} = \frac{5}{4}\left(1-\frac{1}{5^n}\right)$$

(vii) n terms of the GP. 3, $-6, 12, -24, \ldots\ldots\ldots$

Solution: Here, $a = 3$, $r = -\frac{6}{3} = -2 < 1$

Now, $S_n = \frac{a(1-r^n)}{1-r}$

$= \frac{3\{1-(-2)^n\}}{1-(-2)}$

$= \frac{3\{1-(-2)^n\}}{3}$

$= 1-(-2)^n$

Q24. (i) How many terms of the GP. 1, 3, 9, 27, ………. Must be taken so that their sum is equal to 3280?

Solution: Let n be the no. of terms of the GP.

Now, $a = 1$, $r = 3 > 1$

Then, $S_n = \frac{a(r^n - 1)}{r-1}$

$\Rightarrow 3280 = \frac{1(3^n - 1)}{3-1}$

$\Rightarrow 3280 \times 2 = 3^n - 1$

$\Rightarrow 3^n = 6560 + 1$

$\Rightarrow 3^n = 6561 = 3^8$

$\Rightarrow n = 8$

(ii) How many terms of the GP. $1\frac{1}{3},2,3,......$ **must be taken so that their sum is equal to** $\frac{211}{12}$ **?**

Solution: Let n be the no. of terms of the GP.

We have, $a=1\frac{1}{3}=\frac{4}{3}$, $r=\frac{2}{\frac{4}{3}}=\frac{6}{4}=\frac{3}{2}>1$

$$\therefore \quad S_n=\frac{a(r^n-1)}{r-1}$$

$$\Rightarrow \frac{211}{12}=\frac{\frac{4}{3}\left\{\left(\frac{3}{2}\right)^n-1\right\}}{\frac{3}{2}-1}$$

$$\Rightarrow \frac{211}{12}\times\frac{3}{4}=\frac{\left(\frac{3}{2}\right)^n-1}{\frac{1}{2}}$$

$$\Rightarrow \frac{211}{12}\times\frac{3}{4}\times\frac{1}{2}=\left(\frac{3}{2}\right)^n-1$$

$$\Rightarrow \frac{211}{32}+1=\left(\frac{3}{2}\right)^n$$

$$\Rightarrow \frac{243}{32}=\left(\frac{3}{2}\right)^n$$

$$\Rightarrow \left(\frac{3}{2}\right)^5=\left(\frac{3}{2}\right)^n$$

$$\Rightarrow n=5$$

Q25. Find the least value of *n*, for which

$1+3+3^2+........+3^n>1000$

Solution: Here, $a=1$, $r=3>1$

Now, $\frac{a(r^{n+1}-1)}{r-1}>1000$

$$\Rightarrow \frac{1(3^{n+1}-1)}{3-1}>1000$$

$$\Rightarrow 3^{n+1}-1>2000$$

$$\Rightarrow 3^{n+1}>2001$$

By inspection, the least value of n is given by $n+1=7\Rightarrow n=7-1=6$.

Hence, the least value of n is 6.

Q26. The 5th term of a GP is 48 and the 12th term is 6144. Find the sum of the first 10 terms.

Solution: Let a be the first term and r be the common ratio.

Then, 5th term = 48

$\Rightarrow ar^4 = 48$ - (i)

& 12th term = 6144

$\Rightarrow ar^{11} = 6144$ - (ii)

Dividing (ii) by (i), we get

$$\frac{ar^{11}}{ar^4} = \frac{6144}{48}$$

$\Rightarrow r^7 = 128 = 2^7$

$\Rightarrow r = 2$

From (i),

$a \times 2^4 = 48$

$\Rightarrow a = \frac{48}{16} = 3$

Hence, $S_{10} = \frac{a(r^{10}-1)}{r-1}$ $[\because r > 1]$

$= \frac{3(2^{10}-1)}{2-1}$

$= 3(1024-1)$

$= 3 \times 1023$

$= 3069$

Q27. In a GP, the first term is 5, the last term is 320 and the sum is 635. Find the 4th term.

Solution: Here, $a = 5$, $a_n = 320$

$\Rightarrow ar^{n-1} = 320$

$\Rightarrow 5r^{n-1} = 320$

$\Rightarrow r^{n-1} = 64$

$\Rightarrow r^n = 64r$ …………………(i)

Also, $S_n = \dfrac{a(r^n - 1)}{r-1}$

$\Rightarrow 635 = \dfrac{5(r^n - 1)}{r-1}$

$\Rightarrow 127(r-1) = r^n - 1$

$\Rightarrow 127r - 127 = 64r - 1$ [using (i)]

$\Rightarrow 63r = 126$

$\Rightarrow r = 2$

$\therefore$ 4th term $= 5 \times 2^{4-1}$

$= 5 \times 2^3$

$= 5\times8=40$

Q28. The sum of the first 6 terms of a GP is 9 times the sum of the first 3terms. If the 7th term be 384, find the sum of the first 10 terms.

Solution: Let a be the first term and r be the common ratio.

Then, $S_6 = 9 \times S_3$

$\Rightarrow \dfrac{a(r^6 - 1)}{r-1} = 9 \times \dfrac{a(r^3 - 1)}{r-1}$

$\Rightarrow \dfrac{a(r^6 - 1)}{a(r^3 - 1)} = 9$

$\Rightarrow \dfrac{\left\{\left(r^3\right)^2 - 1^2\right\}}{r^3 - 1} = 9$

$\Rightarrow \dfrac{(r^3 - 1)(r^3 + 1)}{r^3 - 1} = 9$

$\Rightarrow r^3 + 1 = 9$

$\Rightarrow r^3 = 8 = 2^3$

$\Rightarrow r = 2$

& $ar^{7-1} = 384$

$\Rightarrow a \times 2^6 = 384$

$\Rightarrow a = \dfrac{384}{64} = 6$

$\therefore S_{10} = \dfrac{a(r^{10} - 1)}{r-1}$

$= \dfrac{6(2^{10} - 1)}{2-1}$

$= 6(1024 - 1)$

$= 6 \times 1023 = 6138$

Q29. The sum of the first 10 terms of a GP is 33 times the sum of the first 5 terms. Find the common ratio.

Solution: Let a be the first term and r be the common ratio.

Now, $S_{10} = 33 \times S_5$

$$\Rightarrow \frac{a(r^{10}-1)}{r-1} = 33 \times \frac{a(r^5-1)}{r-1}$$

$$\Rightarrow \frac{r^{10}-1}{r^5-1} = 33$$

$$\Rightarrow \frac{(r^5)^2-1}{r^5-1} = 33$$

$$\Rightarrow r^5 + 1 = 33$$

$$\Rightarrow r^5 = 32 = 2^5$$

$$\Rightarrow r = 2$$

Q30. The first and the last terms of a GP are respectively 3 and 768 and the sum is 1533. Find the number of terms and the common ratio.

Solution: Let n be the no. of terms and r be the common ratio.

We have, $a = 3$

& $ar^{n-1} = 768$

$$\Rightarrow 3.r^{n-1} = 768$$

$$\Rightarrow r^{n-1} = 256$$

$$\Rightarrow r^n = 256r \quad \text{....................(i)}$$

Also, $S_n = \frac{a(r^n-1)}{r-1}$

$$\Rightarrow 1533 = \frac{3(r^n-1)}{r-1}$$

$$\Rightarrow 511(r-1) = r^n - 1$$

$$\Rightarrow 511r - 511 = 256r - 1 \quad \text{[using (i)]}$$

$$\Rightarrow 255r = 510$$

$$\Rightarrow r = 2$$

Again, from (i), $\Rightarrow r^n = 256r$

$$\Rightarrow 2^n = 256 \times 2 = 512 = 2^9$$

$$\Rightarrow n = 9$$

Hence, the no. of terms = 9 and the common ratio is 2.

Q31. If S_1, S_2, S_3 be the sums of the first n terms, $2n$ terms, $3n$ terms respectively of a GP. Prove that

(i) $S_1^2 + s_2^2 = S_1(S_2 + S_3)$

(ii) $S_1(S_3 - S_2) = (S_1 + S_2)^2$

Solution: Let a be the first term, r be the common ratio and n be the no. of terms.

$$S_1 = \frac{a(r^n - 1)}{r - 1}$$

$$S_2 = \frac{a(r^{2n} - 1)}{r - 1}$$

$$S_3 = \frac{a(r^{3n} - 1)}{r - 1}$$

(i) Now, $$S_1^2 + S_2^2 = \left[\frac{a(r^n - 1)}{r - 1}\right]^2 + \left[\frac{a(r^{2n} - 1)}{r - 1}\right]^2$$

$$= \frac{a^2(r^n - 1)^2}{(r-1)^2} + \frac{a^2(r^{2n} - 1)^2}{(r-1)^2}$$

$$= \frac{a^2\left[r^{2n} - 2r^n + 1 + r^{4n} - 2r^{2n} + 1\right]}{(r-1)^2}$$

$$= \frac{a^2}{(r-1)^2}\left[r^{4n} - r^{2n} - 2r^n + 2\right]$$

& $$S_1(S_2 + S_3) = \frac{a(r^n - 1)}{r - 1}\left[\frac{a(r^{2n} - 1)}{r - 1} + \frac{a(r^{3n} - 1)}{r - 1}\right]$$

$$= \frac{a(r^n - 1)}{r - 1}\left[\frac{a(r^{2n} - 1 + r^{3n} - 1)}{r - 1}\right]$$

$$= \frac{a^2}{(r-1)^2}\left[r^{4n} + r^{3n} - 2r^n - r^{3n} - r^{2n} + 2\right]$$

$$= \frac{a^2}{(r-1)^2}\left[r^{4n} - r^{2n} - 2r^n + 2\right]$$

Hence, $S_1^2 + S_2^2 = S_1(S_2 + S_3)$.

(ii) $S_1(S_3 - S_2) = \frac{a(r^n-1)}{r-1}\left[\frac{a(r^{3n}-1)}{r-1} - \frac{a(r^{2n}-1)}{r-1}\right]$

$$= \frac{a^2(r^n-1)}{r-1}\left[\frac{r^{3n}-1-r^{2n}+1}{r-1}\right]$$

$$= \frac{a^2(r^n-1)}{(r-1)(r-1)}\left[r^{3n}-r^{2n}\right]$$

$$= \frac{a^2(r^n-1)(r^{3n}-r^{2n})}{(r-1)^2}$$

and, $(S_1 - S_2)^2 = \left[\frac{a(r^n-1)}{r-1} - \frac{a(r^{2n}-1)}{r-1}\right]^2$

$$= \left[\frac{a(r^n-1-r^{2n}+1)}{r-1}\right]^2$$

$$= \frac{a^2\left[r^n - r^{2n}\right]^2}{(r-1)^2}$$

$$= \frac{a^2}{(r-1)^2}\left[r^n(1-r^n)\right]^2$$

$$= \frac{a^2}{(r-1)^2}\left[(r^n-1)(-r^n)\right]^2$$

$$= \frac{a^2}{(r-1)^2}(r^n-1)^2 r^{2n}$$

$$= \frac{a^2(r^n-1)(r^{3n}-r^{2n})}{(r-1)^2}$$

Hence, $S_1(S_3 - S_2) = (S_1 - S_2)^2$

SOLUTIONS

EXERCISE – 2.4

Q1. Find the specified term of each of the following HP.

(i) 10^{th} term of $1,\frac{1}{4},\frac{1}{7},\frac{1}{10}$

Solution: Let a be the first term and d be the common difference of the AP corresponding to the given HP.

Then, $a=1$, $d=4-1=3$

$\therefore$ 10^{th} term of the corresponding AP $=1+(10-1)3$

$=1+9\times 3$

$=28$

$\therefore$ The 10^{th} term of the HP is $\frac{1}{28}$

(ii) 5^{th} Term of $\frac{3}{4},1,\frac{3}{2},$............

Solution: Let a be the first term and d be the common difference of the AP corresponding to the given HP.

Then, $a=\frac{4}{3}$, $d=1-\frac{4}{3}=-\frac{1}{3}$

$\therefore$ 5^{th} term of the AP $=a+(5-1)d$

$$=\frac{4}{3}+4\times\left(-\frac{1}{3}\right)$$

$$=0$$

$\therefore$ The 5^{th} term of the HP does not exists for the reciprocal of 0 is not defined.

(iii) 6^{th} term of 3, $1\frac{1}{2},1,$.................

Solution: The corresponding AP is $\frac{1}{3},\frac{2}{3},1,$.............

Then, $a=\frac{1}{3}$, $d=\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$

$\therefore$ 6^{th} term of the AP $=\frac{1}{3}+(6-1)\frac{1}{3}$

$$=\frac{1}{3}+\frac{5}{3}$$

$$=\frac{6}{3}=2$$

$\therefore$ 6^{th} term of the HP$=\frac{1}{2}$

(iv) 20th term of 1, $1\frac{3}{5}, 4, ...$

Solution: The corresponding AP is $1, \frac{5}{8}, \frac{1}{4}, ...$

Then, $a = 1$, $d = \frac{5}{8} - 1 = \frac{-3}{8}$

$\therefore$ 20^{th} term of the AP $= 1 + (20-1)\left(-\frac{3}{8}\right)$

$= 1 + 19 \times \left(-\frac{3}{8}\right)$

$= 1 - \frac{57}{8} = -\frac{49}{8}$

$\therefore$ 20^{th} term of the HP $= -\frac{8}{49}$

(v) The n^{th} term of $11\frac{2}{3}, 8\frac{3}{4}, 7, 5\frac{5}{6}$

Solution: The corresponding AP is

$\frac{3}{35}, \frac{4}{35}, \frac{1}{7}, \frac{6}{35}$

Here, $a = \frac{3}{35}$, $d = \frac{4}{35} - \frac{3}{35} = \frac{1}{35}$

$\therefore n^{th}$ term of the AP $= a + (n-1)d$

$= \frac{3}{35} + (n-1)\frac{1}{35}$

$= \frac{3}{35} + \frac{n}{35} - \frac{1}{35}$

$= \frac{3+n-1}{35}$

$= \frac{n+2}{35}$

$\therefore n^{th}$ term of the HP $= \frac{35}{n+2}$

Q2. Find the H.P. whose

(i) 1st term is $3\frac{1}{8}$ and 4th term is $1\frac{7}{13}$

Solution: Let a be the first term and d be the common difference of the AP corresponding to the given HP

Then, $a = \frac{1}{\frac{25}{8}} = \frac{8}{25}$, $a_4 = \frac{1}{\frac{20}{15}} = \frac{13}{20}$

$\therefore$ 4th term = $a + (n-1)d$

$\Rightarrow \frac{13}{20} = \frac{8}{25} + (4-1)d$

$\Rightarrow \frac{13}{20} - \frac{8}{25} = 3d$

$\Rightarrow \frac{65-32}{100} = 3d$

$\Rightarrow 3d = \frac{33}{100}$

$\Rightarrow d = \frac{11}{100}$

The corresponding AP is $\frac{8}{25}, \frac{43}{100}, \frac{54}{100}$.

Hence the required HP is $\frac{25}{8}, \frac{100}{43}, \frac{100}{54}, ...$ i.e. $3\frac{1}{8}, 2\frac{14}{43}, 1\frac{23}{27}, ...$

(ii) 4th term is $\frac{1}{12}$ and 14th term is $\frac{1}{42}$

Solution: Let a be the first term and d be the c.d. of the corresponding AP.

Then, $a_4 = 12$

$\Rightarrow a + 3d = 12$ - (i)

& $a_{14} = 42$

$\Rightarrow a + 13d = 42$ - (ii)

Subtracting (i) from (ii),we get

$10d = 30$

$\Rightarrow d = 3$

From(i), $a + 3 \times 3 = 12$

$\Rightarrow a = 12 - 9 = 3$

$\therefore$ The corresponding AP = 3, 6, 9, 12, 15.

Hence, the required HP = $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \frac{1}{15}, ...$

Q3. Find the 19th term of the HP whose 5th and 10th terms are $-\dfrac{36}{151}$ and $-\dfrac{4}{35}$ respectively.

Solution: Here, 5th term of the corresponding AP = $-\dfrac{151}{36}$

$$\Rightarrow a+4d=-\frac{151}{36} \qquad - \qquad \text{(i)}$$

& 10th term of the corresponding AP = $-\dfrac{35}{4}$

$$\Rightarrow a+9d=-\frac{35}{4} \qquad - \qquad \text{(ii)}$$

Subtracting (ii) from (1), we get

$$-5d=-\frac{151}{36}+\frac{35}{4}$$

$$=\frac{-151+315}{36}$$

$$\Rightarrow -5d=\frac{164}{36}=\frac{41}{9}$$

$$\Rightarrow d=-\frac{41}{45}$$

From (i), we get

$$a+4\times\left(-\frac{41}{45}\right)=-\frac{151}{36}$$

$$\Rightarrow a=-\frac{151}{36}+\frac{164}{45}$$

$$=\frac{-755+656}{180}$$

$$=-\frac{99}{180}=\frac{-11}{20}$$

$$\therefore \quad a_{19}=a+(19-1)d$$

$$=-\frac{11}{20}+18\times\left(-\frac{41}{45}\right)$$

$$=-\frac{11}{20}-\frac{82}{5}$$

$$\frac{-11-328}{20}=-\frac{339}{20}$$

$\therefore$ The 19th term of the HP = $-\dfrac{20}{339}$.

Q4. Insert:

(i) two harmonic means between $\frac{1}{3}$ and $\frac{1}{81}$

Solution: Let x_1, x_2 be the two harmonic means:

Then $\frac{1}{3}, x_1, x_2, \frac{1}{81}$ are in HP

$\therefore 3, \frac{1}{x_1}, \frac{1}{x_2}, 81$ are in AP.

Let $a = 3$ be the first term and d be the c.d. of the AP.

Now, $81 = 3 + (4-1)d$

$\Rightarrow 81 = 3 + 3d$

$\Rightarrow 3d = 78$

$\Rightarrow d = 26$

$\frac{1}{x_1} = 3 + 26 = 29 \quad \frac{1}{x_2} = 29 + 26 = 55$

$\therefore x_1 = \frac{1}{29}, \; x_2 = \frac{1}{55}$

(ii) Three harmonic means between $2\frac{2}{5}$ and 12.

Solution: Let x_1, x_2, x_3 be the three harmonic means

Then, $\frac{12}{5}, x_1, x_2, x_3, 12$ are in HP

$\therefore \frac{5}{12}, \frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \frac{1}{12}$ are in AP

Let $a = \frac{5}{12}$ and d be the c.d. of the AP.

Now, $\frac{1}{12} = \frac{5}{12} + (5-1)d$

$$\Rightarrow \frac{1}{12} - \frac{5}{12} = 4d$$

$$\Rightarrow -\frac{4}{12} = 4d$$

$$\Rightarrow d = \frac{-1}{12}$$

$$\frac{1}{x_1} = \frac{5}{12} - \frac{1}{12} = \frac{4}{12} = \frac{1}{3}$$

$$\frac{1}{x_2} = \frac{1}{3} - \frac{1}{12} = \frac{4-1}{12} = \frac{3}{12} = \frac{1}{4}$$

$$\frac{1}{x_3} = \frac{1}{4} - \frac{1}{12} = \frac{3-1}{12} = \frac{2}{12} = \frac{1}{6}$$

Hence, the three harmonic means are 3, 4 and 6.

(iii) four harmonic means between 1 and 6.

Solution: Let x_1, x_2, x_3, x_4 be the four harmonic means.

Then 1, $x_1, x_2, x_3, x_4, 6$ are in HP

$\therefore 1, \frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \frac{1}{x_4}, \frac{1}{6}$ are in AP

Here, $a = 1$

Now, $\frac{1}{6} = 1 + (6-1)d$

$$\Rightarrow \frac{1}{6} = 1 + 5d$$

$$\Rightarrow \frac{1}{6} - 1 = 5d$$

$$\Rightarrow -\frac{5}{6} = 5d$$

$$\Rightarrow d = -\frac{1}{6}$$

$$\therefore \quad \frac{1}{x_1}=1-\frac{1}{6}=\frac{5}{6}$$

$$\frac{1}{x_2}=\frac{5}{6}-\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$$

$$\frac{1}{x_3}=\frac{2}{3}-\frac{1}{6}=\frac{4-1}{6}=\frac{3}{6}=\frac{1}{2}$$

$$\frac{1}{x_4}=\frac{3}{6}-\frac{1}{6}=\frac{2}{6}=\frac{1}{3}$$

$\therefore$ The four harmonic means are $\frac{6}{5},\frac{3}{2}$, 2 and 3.

(iv) Three harmonic means between *a* and *b*.

Solution: Let x_1,x_2,x_3 the three harmonic means.

Then, a,x_1,x_2,x_3,b are in HP .

$\therefore \frac{1}{a},\frac{1}{x_1},\frac{1}{x_2},\frac{1}{x_3},\frac{1}{b}$ are in AP.

Now, $$\frac{1}{b}=\frac{1}{a}+(5-1)d$$

$$\Rightarrow \frac{1}{b}-\frac{1}{a}=4d$$

$$\Rightarrow \frac{a-b}{ab}=4d$$

$$\Rightarrow d=\frac{a-b}{4ab}$$

$$\therefore \quad \frac{1}{x_1}=\frac{1}{a}+\frac{a-b}{4ab}=\frac{4b+a-b}{4ab}=\frac{a+3b}{4ab}$$

$$\frac{1}{x_2}=\frac{a+3b}{4ab}+\frac{a-b}{4ab}=\frac{a+3b+a-b}{4ab}=\frac{2a+2b}{4ab}=\frac{a+b}{2ab}$$

$$\frac{1}{x_3}=\frac{a+b}{2ab}+\frac{a-b}{4ab}=\frac{2a+2b+a-b}{4ab}=\frac{3a+b}{4ab}$$

$\therefore$ The three harmonic means are $\frac{4ab}{a+3b},\frac{2ab}{a+b}$ and $\frac{4ab}{3a+b}$

Q5. If the p^{th} term of an HP be q and the q^{th} term be p, prove that

(i) $(p+q)^{th}$ **term is** $\frac{pq}{p+q}$

(ii) n^{th} **term is** $\frac{pq}{n}$

(iii) $(pq)^{th}$ **term is 1.**

Solution:

(i) Let a be the first term and d be the c.d. of the corresponding AP.

Then, p^{th} term $= \frac{1}{q}$

$$\Rightarrow a+(p-1)d = \frac{1}{q} \quad - \quad \text{(i)}$$

& q^{th} term$= \frac{1}{p}$

$$\Rightarrow a+(q-1)d = \frac{1}{p} \quad - \quad \text{(ii)}$$

Subtracting (i) from (ii), we get

$$(q-1-p+1)d = \frac{1}{p} - \frac{1}{q}$$

$$\Rightarrow (q-p)d = \frac{q-p}{pq}$$

$$\Rightarrow d = \frac{1}{pq}$$

From (i), we get

$$a+(p-1)\frac{1}{pq} = \frac{1}{q}$$

$$\Rightarrow a = \frac{1}{q} - \frac{p-1}{pq} = \frac{p-p+1}{pq} = \frac{1}{pq}$$

$(p+q)^{th}$ term $= a+(p+q-1)d$

$$= \frac{1}{pq}+(p+q-1)\frac{1}{pq}$$

$$= \frac{1+p+q-1}{pq}$$

$$= \frac{p+q}{pq}$$

Hence, $(p+q)^{th}$ term of HP = $\frac{pq}{p+q}$.

(ii) n^{th} term of the AP = $a+(n-1)d$

$$= \frac{1}{pq}+(n-1)\frac{1}{pq}$$

$$= \frac{1+n-1}{pq} = \frac{n}{pq}$$

$\therefore n^{\text{th}}$ term of the HP = $\frac{pq}{n}$

(iii) $(pq)^{th}$ term of the AP $= a+(pq-1)d$

$$= \frac{1}{pq}+(pq-1)\frac{1}{pq}$$

$$= \frac{1+pq-1}{pq} = 1$$

$\therefore (pq)^{th}$ term of the HP = 1

Q6. If the p^{th}, q^{th} and r^{th} terms of the HP be a, b, c respectively, show that $(q-r)bc+(r-p)ca\_(p-q)ab=0$.

Solution: Let A be the first term and D be the c.d. of the corresponding A.P.

Then, p^{th}term = $A+(p-1)D$

$$\Rightarrow \frac{1}{a} = A+(p-1)D \quad - \quad \text{(i)}$$

$$\frac{1}{b} = A+(q-1)D \quad - \quad \text{(ii)}$$

$$\& \quad \frac{1}{c} = A+(r-1)D \quad - \quad \text{(iii)}$$

Multiplying (i) by $abc(q-r)$, (ii) by $abc(r-p)$ and (iii) by $abc(p-q)$ and adding, we get,

$$(q-r)bc+(r-p)ac+(p-q)ab = A\{(q-r)abc+(r-p)abc+(p-q)abc\}$$

$$+D\{abc(q-r)(p-1)+abc(r-p)(q-1)+abc(p-q)(r-1)\}$$

$$= A.abc(q-r+r-p+p-q)+Dabc\{qp-q-pr+r+rq-r-pq+p+pr-p-qr+q\}$$

$$= abcA\times 0+abc.D\times 0$$

=0

Q7. If a^2, b^2, c^2 are in AP, prove that $b+c, c+a, a+b$ are in HP.

Solution: Since a^2, b^2, c^2 are in AP,

$$b^2-a^2=c^2-b^2$$

Now, $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in AP

$$\text{if} \frac{1}{c+a}-\frac{1}{b+c}=\frac{1}{a+b}-\frac{1}{c+a}$$

$$\text{if} \frac{b+c-c-a}{(c+a)(b+c)}=\frac{c+a-a-b}{(a+b)(c+a)}$$

$$\text{if} \frac{b-a}{b+c}=\frac{c-b}{a+b}$$

$$\text{if} (b-a)(a+b)=(c-b)(b+c)$$

$$\text{if} ab-a^2+b^2-ab=bc+c^2-b^2-bc$$

$$\text{if} b^2-a^2=c^2-b^2 \text{ which is true}$$

Hence, $b+c, c+a, a+b$ are in HP.

Q8. If a,b,c be in AP and p,q,r be in HP show that

$$\frac{a+c}{bq}=\frac{p+r}{pr}$$

Solution: Since a,b,c are in AP

$$b-a=c-b$$

$$\Rightarrow 2b=a+c \qquad - \qquad \text{(i)}$$

and p,q,r are in HP

$$\Rightarrow \frac{1}{p},\frac{1}{q},\frac{1}{r} \text{ are in AP}$$

Now, $\frac{1}{q}-\frac{1}{p}=\frac{1}{r}-\frac{1}{q}$

$$\Rightarrow \frac{p-q}{pq}=\frac{q-r}{qr}$$

$$\Rightarrow \frac{p-q}{p}=\frac{q-r}{r}$$

$$\Rightarrow 1-\frac{q}{p}=\frac{q}{r}-1$$

$$\Rightarrow 2=\frac{q}{r}+\frac{q}{p}$$

$$\Rightarrow \frac{a+c}{b}=\frac{pq+qr}{rp} \qquad \text{[by using (i)]}$$

$$\Rightarrow \frac{a+c}{bq}=\frac{p+r}{rp}$$

Q9. If $\frac{a+b}{2}, b, \frac{b+c}{2}$ be in HP, show that a, b, c are in GP.

Solution: Since $\frac{2}{a+b}, \frac{1}{b}, \frac{2}{b+c}$ are in AP

Then $\frac{1}{b} - \frac{2}{a+b} = \frac{2}{b+c} - \frac{1}{b}$

$$\Rightarrow \frac{2}{b} = \frac{2}{a+b} + \frac{2}{b+c}$$

$$\Rightarrow \frac{1}{b} = \frac{1}{a+b} + \frac{1}{b+c}$$

$$\Rightarrow \frac{1}{b} = \frac{b+c+a+b}{(a+b)(b+c)}$$

$$\Rightarrow \frac{1}{b} = \frac{2b+c+a}{(a+b)(b+c)}$$

$$\Rightarrow (a+b)(b+c) = 2b^2 + bc + ba$$

$$\Rightarrow ab + ac + b^2 + bc = 2b^2 + bc + ba$$

$$\Rightarrow b^2 = ac$$

$$\Rightarrow \frac{b}{a} = \frac{c}{b}$$

Hence, a, b, c are in GP.

Q10. If a, b, c are in HP, show that $\frac{1}{a} + \frac{1}{b+c}, \frac{1}{b} + \frac{1}{c+a}, \frac{1}{c} + \frac{1}{a+b}$ are also in HP.

Solution: Since a, b, c are in HP.

$\therefore \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP.

$$\Rightarrow \frac{1}{b} - \frac{1}{a} = \frac{1}{c} - \frac{1}{b}$$

$$\Rightarrow \frac{1}{a} + \frac{1}{c} = \frac{2}{b} \qquad - \qquad (i)$$

Now, $a(b+c), b(c+a), c(a+b)$ are in AP

if $b(c+a)-a(b+c)=c(a+b)-b(c+a)$

if $bc+ba-ab-ac=ca+bc-bc-ab$

if $bc-ac=ca-ab$

if $bc+ba=2ac$

if $\frac{bc+ba}{abc}=\frac{2ac}{abc}$

if $\frac{1}{a}+\frac{1}{c}=\frac{2}{b}$ which is true by (i)

Hence, $\frac{1}{a(b+c)}, \frac{1}{b(c+a)}, \frac{1}{c(a+b)}$ are in HP.

$\Rightarrow \frac{a+b+c}{a(b+c)}, \frac{a+b+c}{b(c+a)}, \frac{a+b+c}{c(a+b)}$ are in HP.

$\Rightarrow \frac{a}{a(b+c)}+\frac{b+c}{a(b+c)}, \frac{b}{b(c+a)}+\frac{a+c}{b(c+a)}, \frac{c}{c(a+b)}+\frac{a+b}{c(a+b)}$ are in HP.

$\Rightarrow \frac{1}{b+c}+\frac{1}{a}, \frac{1}{c+a}+\frac{1}{b}, \frac{1}{a+b}+\frac{1}{c}$ are in HP

$\Rightarrow \frac{1}{a}+\frac{1}{b+c}, \frac{1}{b}+\frac{1}{c+a}, \frac{1}{c}+\frac{1}{a+b}$ are in HP.

Q11. If a,b,c be in AP., b,c,d in GP and c,d,e in HP, prove that a,c,e are in GP.

Solution: Since a,b,c are in AP

$b-a=c-b$

$\Rightarrow 2b=a+c$

$\Rightarrow b=\frac{a+c}{2}$ - (i)

& b,c,d are in GP

$\Rightarrow \frac{c}{b}=\frac{d}{c}$

$\Rightarrow c^2=bd$ - (ii)

Also, c,d,e are in HP

$$\therefore \frac{1}{c},\frac{1}{d},\frac{1}{e} \text{ are in AP}$$

$$\Rightarrow \frac{1}{d}-\frac{1}{c}=\frac{1}{e}-\frac{1}{d}$$

$$\Rightarrow \frac{2}{d}=\frac{1}{e}+\frac{1}{c}=\frac{c+e}{ce}$$

$$\Rightarrow d=\frac{2ce}{c+e} \qquad - \quad \text{(iii)}$$

From (i), (ii) and (iii), we get

$$c^2=\frac{a+c}{2}\times\frac{2ce}{c+e}=\frac{ce(a+c)}{c+e}$$

$$\Rightarrow c=\frac{e(a+c)}{c+e}$$

$$\Rightarrow c^2+ce=e(a+c)$$

$$c^2=ae$$

$\therefore$ a,c,e are in GP.

Q12. If a,b,c are in GP, show that $\log_a x, \log_b x, \log_c x$ are in HP.

Solution: Since a,b,c are in GP

$$\therefore \frac{b}{a}=\frac{c}{b}$$

$$\Rightarrow b^2=ac$$

$$\Rightarrow \log_x(b^2)=\log_x(ac) \text{ [Taking log on both sides]}$$

$$\Rightarrow 2\log_x b=\log_x a+\log_x c$$

$$\Rightarrow \log_x b-\log_x a=\log_x c-\log_x b$$

$$\Rightarrow \log_x a, \log_x b, \log_x c \text{ are in A.P.}$$

$$\Rightarrow \frac{1}{\log_x a},\frac{1}{\log_x b},\frac{1}{\log_x c} \text{ are in HP.}$$

$$\Rightarrow \log_a x, \log_b x, \log_c x \text{ are in HP.}$$

Q13. If a, b, c are in AP and b, c, d are in HP, prove that $ad = bc$

Solution: Since a, b, c are in AP,

$$b - a = c - b$$

$$\Rightarrow 2b = a + c \qquad - \qquad \text{(i)}$$

And b, c, d are in HP

$$\therefore \frac{1}{b}, \frac{1}{c}, \frac{1}{d} \text{ are in AP}$$

$$\Rightarrow \frac{1}{c} - \frac{1}{b} = \frac{1}{d} - \frac{1}{c}$$

$$\Rightarrow \frac{b-c}{bc} = \frac{c-d}{cd}$$

$$\Rightarrow \frac{b-c}{b} = \frac{c-d}{d}$$

$$\Rightarrow bd - cd = bc - bd$$

$$\Rightarrow 2bd - cd = bc$$

$$\Rightarrow (a+c)d - cd = bc \qquad \text{[by (i)]}$$

$$\Rightarrow ad = bc$$

Q14. If $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ are in HP, show that

$$x_1x_2 + x_2x_3 + \ldots\ldots\ldots\ldots + x_{n-1}x_n = (n-1)x_1x_n.$$

Solution: Since $x_1, x_2, x_3, \ldots\ldots\ldots, x_n$ are in HP

$$\therefore \frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \ldots\ldots\ldots\ldots, \frac{1}{x_n} \text{ are in AP}$$

$$\Rightarrow \frac{1}{x_2} - \frac{1}{x_1} = \frac{1}{x_3} - \frac{1}{x_2} = \frac{1}{x_4} - \frac{1}{x_3} = \ldots\ldots\ldots = \frac{1}{x_n} - \frac{1}{x_{n-1}} = d$$

$$\Rightarrow \frac{x_1 - x_2}{x_1x_2} = \frac{x_2 - x_3}{x_2x_3} = \frac{x_3 - x_4}{x_3x_4} = \cdots \ldots\ldots\ldots\ldots\ldots. = \frac{x_{n-1} - x_n}{x_{n-1}x_n} = d$$

$$\Rightarrow \frac{x_1 - x_2 + x_2 - x_3 + x_3 - x_4 + \ldots\ldots\ldots + x_{n-1} - x_n}{x_1x_2 + x_2x_3 + x_3x_4 + \ldots\ldots\ldots + x_{n-1}x_n} = d \quad \text{[By addendo]}$$

$$\Rightarrow \frac{x_1 - x_n}{d} = x_1x_2 + x_2x_3 + x_3x_4 + \ldots\ldots\ldots + x_{n-1}x_n \qquad - \qquad \text{(i)}$$

Now, $\frac{1}{x_n}=\frac{1}{x_1}+(n-1)d$

$$\Rightarrow \frac{1}{x_n}-\frac{1}{x_1}=(n-1)d$$

$$\Rightarrow \frac{x_1-x_n}{x_1x_n}=(n-1)d$$

$$\Rightarrow x_1-x_n=(n-1)d\times x_1x_n \quad - \quad \text{(ii)}$$

From (i) & (ii), we get,

$$\frac{(n-1)d\times x_1x_n}{d}=x_1x_2+x_2x_3+x_3x_4+........+x_{n-1}x_n$$

$$\Rightarrow x_1x_2+x_2x_3+x_3x_4+.........+x_{n-1}x_n=(n-1)x_1x_n$$

Q15. If a,b,c,d are in HP, prove that $a+d>b+c$.

Solution: Since a,b,c,d are in HP,

HM. between a and $c=b$

& HM between b and $d=c$

Again, AM between a and $c=\frac{a+c}{2}$

& AM between b and $d=\frac{b+d}{2}$

Now, A.M > H.M.

$$\Rightarrow \frac{a+c}{2}>b \text{ and } \frac{b+d}{2}>c$$

Then, $\frac{a+c}{2}+\frac{b+d}{2}>b+c$

$$\Rightarrow \frac{a+c+b+d}{2}>b+c$$

$$\Rightarrow a+b+c+d>2(b+c)$$

$$\Rightarrow a+d>b+c$$

Q16. The GM and HM between two numbers are 9 and $\frac{27}{5}$ respectively. Find the numbers.

Solution: Let a and b be the two numbers

Then, GM=9

$\Rightarrow \sqrt{ab} = 9$

$\Rightarrow ab = 81$ - (i)

& HM = $\frac{27}{5}$

$\Rightarrow \frac{2ab}{a+b} = \frac{27}{5}$

$\Rightarrow \frac{2\times 81}{a+b} = \frac{27}{5}$

$\Rightarrow a+b = \frac{81\times 2\times 5}{27}$

$\Rightarrow a+b = 30$

$\Rightarrow a = 30-b$ - (ii)

From (i) & (ii), we get

$(30-b)b = 81$

$\Rightarrow 30b - b^2 = 81$

$\Rightarrow b^2 - 30b + 81 = 0$

$\Rightarrow b^2 - 27b - 3b + 81 = 0$

$\Rightarrow b(b-27) - 3(b-27) = 0$

$\Rightarrow (b-27)(b-3) = 0$

$\Rightarrow (b-3)(b-27) = 0$

$\Rightarrow b = 3, 27$

When $b = 3,\ a = 30-3 = 27$

When $b = 27,\ a = 30-27 = 3$

$\therefore$ The two numbers are 3 and 27.

Q17. If AM and GM of two positive numbers are 12 and 6 respectively. Find their HM.

Solution: Let a and b be the two positive numbers.

Then, $AM = 12$

$\Rightarrow \frac{a+b}{2} = 12$

$\Rightarrow a+b = 24$

& GM= $\sqrt{ab}$

$\Rightarrow 6 = \sqrt{ab}$

$\Rightarrow ab = 36$

$\therefore$ HM $= \frac{2ab}{a+b}$

$= \frac{2\times 36}{24} = 3$

Q18. If $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ in AP, show that $a,b,c,$ are in HP.

Solution: Since $\frac{b+c-a}{a}, \frac{c+a-b}{b}, \frac{a+b-c}{c}$ arein AP.

$\Rightarrow \frac{a+b+c-2a}{a}, \frac{c+a+b-2b}{b}, \frac{a+b+c-2c}{c}$ are in AP

$\Rightarrow \frac{a+b+c}{a} - 2, \frac{a+b+c}{b} - 2, \frac{a+b+c}{c} - 2$ *are in AP*.

$\Rightarrow \frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ *are in AP*

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP

$\Rightarrow a,b,c$ are in HP.

Q19. Which tern of the HP 1, $\frac{8}{5}$, 4,........... is $-\frac{1}{8}$?

Solution: We have, the corresponding AP is 1, $\frac{5}{8}, \frac{1}{4}$,........

$\therefore\ a = 1,\ d = \frac{5}{8} - 1 = -\frac{3}{8}$

n^{th} term of the HP= $-\frac{1}{8}$

$\therefore$ n^{th} term of the AP= -8

$\Rightarrow 1 + (n-1)\left(-\frac{3}{8}\right) = -8$

$\Rightarrow (n-1)\left(-\frac{3}{8}\right) = -9$

$\Rightarrow (n-1) = -9 \times \left(-\frac{8}{3}\right) = 24$

$\Rightarrow n = 25$

Hence $-\frac{1}{8}$ is the 25^{th} term of the HP.

Q20. If *A* be the AM and *H* be the HM between *a* and *b*.

Prove that $\frac{a-A}{a-H}\times\frac{b-A}{b-H}=\frac{A}{H}$

Solution: We have, $A=\frac{a+b}{2}$

$$H=\frac{2ab}{a+b}$$

Now, $$\frac{a-A}{a-H}\times\frac{b-A}{b-H}=\frac{a-\frac{a+b}{2}}{a-\frac{2ab}{a+b}}\times\frac{b-\frac{a+b}{2}}{b-\frac{2ab}{a+b}}$$

$$=\frac{\frac{2a-a-b}{2}}{\frac{a^2+ab-2ab}{a+b}}\times\frac{\frac{2b-a-b}{2}}{\frac{ab+b^2-2ab}{a+b}}$$

$$=\frac{a-b}{2}\times\frac{a+b}{a^2-ab}\times\frac{b-a}{2}\times\frac{a+b}{b^2-ab}$$

$$=\frac{a-b}{2}\times\frac{a+b}{a(a-b)}\times\frac{b-a}{2}\times\frac{a+b}{b(b-a)}$$

$$=\frac{(a+b)^2}{4ab}$$

$$=\frac{a+b}{2}\times\frac{a+b}{2ab}$$

$$=\frac{\frac{a+b}{2}}{\frac{2ab}{a+b}}$$

$$=\frac{A}{H}$$

SOLUTIONS

EXERCISE 2.5

Find the sum of the following series to n terms.

1. 1+4+7+10+……..

Solution: Here, n^{th} term of the series, $t_n = 1 + (n-1)3$

$$= 1 + 3n - 3$$

$$= 3n - 2$$

$\therefore$ Required sum $= \sum t_n$

$$= \sum (3n-2)$$

$$= 3\sum n - 2n$$

$$= \frac{3n(n+1)}{2} - 2$$

$$= \frac{n}{2}\left[3(n+1) - 4\right]$$

$$= \frac{n}{2}(3n-1)$$

Q2. $1^2 + 3^2 + 5^2 + ...$

Solution: Here, n^{th} term of the series, $t_n = \left[1 + (n-1)2\right]^2$

$$= (2n-1)^2 = 4n^2 - 4n + 1$$

$\therefore$ Required sum $= \sum (4n^2 - 4n + 1)$

$$= 4\sum n^2 - 4\sum n + n$$

$$= 4.\frac{n(n+1)(2n+1)}{6} - 4.\frac{n(n+1)}{2} + n$$

$$= n\left[\frac{2(n+1)(2n+1)}{3} - 2(n+1) + 1\right]$$

$$= n\left[\frac{2(2n^2 + n + 2n + 1)}{3} - 2n - 2 + 1\right]$$

$$= n\left[\frac{4n^2 + 6n + 2 - 6n - 6 + 3}{3}\right]$$

$$= \frac{n}{3}\left[4n^2 - 1\right]$$

Q3. 1.2+2.3+3.4+……..

Solution: We have, t_n = [n^{th} term of 1,2,3,……] [n^{th} term of 2, 3, 4, …….]

$$= [1+(n-1)\times 1][2+(n-1)\times 1]$$

$$= n(n+1) = n^2 + n$$

$\therefore$ Required sum $= \sum(n^2 + n)$

$$= \sum n^2 + \sum n$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{2n+1}{3} + 1\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{2n+1+3}{3}\right]$$

$$= \frac{n}{6}(n+1)(2n+4)$$

$$= \frac{n}{3}(n+1)(n+2)$$

Q4. 1.3+3.5+5.7+………..

Solution: Here, t_n = [n^{th} term of 1,3,5,……] [n^{th} term of 3,5,7, …….]

$$= [1 + (n-1)2][3 + (n-1)2]$$

$$= (2n-1)(2n+1) = 4n^2 - 1$$

$\therefore$ $S_n = \sum(4n^2 - 1)$

$$= 4\sum n^2 - n$$

$$= \frac{4n(n+1)(2n+1)}{6} - n$$

$$= \frac{n}{3}[2(n+1)(2n+1) - 3]$$

$$= \frac{n}{3}\left[4n^2 + 2n + 4n + 2 - 3\right]$$

$$= \frac{n}{3}\left[4n^2 + 6n - 1\right]$$

Q5. $1.2^2 + 2.3^2 + 3.4^2 + \ldots\ldots\ldots\ldots$

Solution: Here, $t_n = [n^{th} \text{ term of } 1,2,3,\ldots\ldots] \, [n^{th} \text{ term of } 2,3,4,\ldots\ldots]^2$

$$= n(n+1)^2 = n^3 + 2n^2 + n$$

$$\therefore \quad S_n = \sum(n^3 + 2n^2 + n)$$

$$= \sum n^3 + 2\sum n^2 + \sum n$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{2n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{2(2n+1)}{3} + 1\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+3n+8n+4+6}{6}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+11n+10}{6}\right]$$

$$= \frac{n}{12}(n+1)(3n^2+11n+10)$$

$$= \frac{1}{12}n(n+1)(n+2)(3n+5)$$

Q6. $1.3^2 + 2.4^2 + 3.5^2 + \ldots\ldots\ldots\ldots$

Solution: Here, $t_n = [n^{th} \text{ term of } 1,2,3,\ldots\ldots] \, [n^{th} \text{ term of } 3,4,5,\ldots\ldots\ldots]^2$

$$= [1+(n-1)1][3+(n-1)1]^2$$

$$= n(n+2)^2$$

$$= n(n^2+4n+4)$$

$$= n^3 + 4n^2 + 4n$$

$$S_n = \sum(n^3 + 4n^3 + 4n)$$

$$= \sum n^3 + 4\sum n^2 + 4\sum n$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{4.n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{4(2n+1)}{3} + 4\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+3n+16n+8+24}{6}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+19n+32}{6}\right]$$

$$= \frac{n}{12}(n+1)(3n^2+19n+32)$$

Q7. $1.2^2 + 3.5^2 + 5.8^2 + ...$

Solution: Here, $t_n = [n^{th} \text{ term of } 1,3,5,......]\ [n^{th} \text{ term of } 2,5,8,.........]^2$

$= [1+(n-1).2][2+(n-1).3]^2$

$= (2n-1)(3n-1)^2$

$= (2n-1)(9n^2-6n+1)$

$= 18n^3 - 12n^2 + 2n - 9n^2 + 6n - 1$

$= 18n^3 - 21n^2 + 8n - 1$

Now, $S_n = \Sigma t_n$

$$= \sum(18n^3 - 21n^2 + 8n - 1)$$

$$= 18\sum n^3 - 21\sum n^2 + 8\sum n - n$$

$$= 18\left[\frac{n(n+1)}{2}\right]^2 - 21.\frac{n(n+1)(2n+1)}{6} + 8.\frac{n(n+1)}{2} - n$$

$$= \frac{n(n+1)}{2}\left[\frac{18n(n+1)}{2} - \frac{21(2n+1)}{3} + 8\right] - n$$

$$= \frac{n(n+1)}{2}\left[9n^2 + 9n - 14n - 7 + 8\right] - n$$

$$= \frac{n}{2}\left[(n+1)(9n^2 - 5n + 1) - 2\right]$$

$$= \frac{n}{2}\left[9n^3 - 5n^2 + n + 9n^2 - 5n + 1 - 2\right]$$

$$= \frac{n}{2}[9n^3 + 4n^2 - 4n - 1]$$

Q8. **1+(1+3)+(1+3+5)+……….**

Solution: Here, $t_n = (1+3+5+..........+n)$

$= \frac{n}{2}[2.1+(n-1)2]$

$= \frac{n}{2}[2+2n-2]$

$= n^2$

Now, $S_n = \Sigma t_n$

$= \sum n^2$

$= \frac{n(n+1)(2n+1)}{6}$

Q9. $1^3+3^3+5^3+$……….

Solution: Here, $t_n = [1+(n-1)2]^3$

$= (2n-1)^3$

$= 8n^3 - 12n^2 + 6n - 1$

$\therefore S_n = = \sum(8n^3 - 12n^2 + 6n - 1)$

$= 8\sum n^3 - 12\sum n^2 + 6\sum n - n$

$= 8\left[\frac{n(n+1)}{2}\right]^2 - 12\times\frac{n(n+1)(2n+1)}{6} + 6.\frac{n(n+1)}{2} - n$

$= \frac{n(n+1)}{2}\left[\frac{8n(n+1)}{2} - \frac{12(2n+1)}{3} + 6\right] - n$

$= \frac{n(n+1)}{2}\left[4n^2 + 4n - 8n - 4 + 6\right] - n$

$= \frac{n(n+1)}{2}\left[4n^2 - 4n + 2\right] - n$

$= n(n+1)(2n^2 - 2n + 1) - n$

$= n(2n^3 - 2n^2 + n + 2n^2 - 2n + 1 - 1)$

$= n(2n^3 - n)$

$= n^2(2n^2 - 1)$

Q10. 1.1+2.3+3.5+…………

Solution: Here, t_n = [n^{th} term of 1,2,3,……] [n^{th} term of 1,3,5,………]

$= [1+(n-1)1][1+(n-1)2]$

$= n(2n-1)$

$= 2n^2 - n$

$\therefore S_n = \sum(2n^2 - n)$

$= 2\sum n^2 - \sum n$

$$= \frac{2n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{2(2n+1)}{3} - 1\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{4n+2-3}{3}\right]$$

$$= \frac{n}{6}(n+1)(4n-1)$$

Q11. 1.3+2.5+3.7+………..

Solution: Here, t_n = [n^{th} term of 1,2,3,……] [n^{th} term of 3,5,7,..…….]

$$= n(2n+1)$$

$$= 2n^2 + n$$

$$\therefore S_n = \sum(2n^2 + n)$$

$$= 2\sum n^2 + \sum n$$

$$= 2\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{2(2n+1)}{3} + 1\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{4n+2+3}{3}\right]$$

$$= \frac{n}{6}(n+1)(4n+5)$$

Q12. 1.2.4+2.3.7+3.4.10+……….

Solution: Here, $t_n = [1+(n-1)1][2+(n-1)1][4+(n-1)3]$

$$= n(n+1)(3n+1)$$

$$= (n^2+n)(3n+1)$$

$$= 3n^3 + n^2 + 3n^2 + n$$

$$= 3n^3 + 4n^2 + n$$

$$\therefore S_n = \sum(3n^3+4n^2+n)$$

$$= 3\sum n^3 + 4\sum n^2 + \sum n$$

$$= 3\left[\frac{n(n+1)}{2}\right]^2 + 4\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{3n(n+1)}{2} + \frac{4(2n+1)}{3} + 1\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{9n^2+9n+16n+8+6}{6}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{9n^2+25n+14}{6}\right]$$

$$= \frac{n}{12}(n+1)(9n^2+25n+14)$$

Q13. $1+\frac{1+2}{2}+\frac{1+2+3}{3}+\dots\dots$

Solution: Here, $t_n = \frac{1+2+3+\dots\dots+n}{n}$

$$= \frac{n(n+1)}{2n}$$

$$= \frac{n+1}{2}$$

$$= \frac{n}{2}+\frac{1}{2}$$

$$\therefore S_n = \sum\left(\frac{n}{2}+\frac{1}{2}\right)$$

$$= \frac{1}{2}\sum n + \frac{1}{2}n$$

$$= \frac{1}{2}\times\frac{n(n+1)}{2} + \frac{1}{2}n$$

$$= \frac{n}{4}[n+1+2]$$

$$= \frac{n(n+3)}{4}$$

Q14. 2.4+4.6+6.8+………..

Solution: Here, t_n $= [n^{th} \text{ term of } 2,4,6,\ldots\ldots] \, [n^{\text{th}} \text{ term of } 4,6,8,\ldots\ldots]$

$$= [2+(n-1)2][4+(n-1)2]$$

$$= [2+2n-2][4+2n-2]$$

$$= 2n(2n+2)$$

$$= 4n^2+4n$$

$$\therefore S_n = \sum\left(4n^2+4n\right)$$

$$= 4\sum n^2 + 4\sum n$$

$$= 4\times\frac{n(n+1)(2n-11)}{6}+\frac{4n(n+1)}{2}$$

$$= \frac{4n(n+1)}{2}\left[\frac{2n+1}{3}+1\right]$$

$$= 2n(n+1)\left[\frac{2n+1+3}{3}\right]$$

$$= \frac{4}{3}n(n+1)(n+2)$$

Q15. 1.3.5+3.5.7+5.7.9+………..

Solution: Here, t_n $= [1+(n-1)2][3+(n-1)2][5+(n-1)2]$

$$= (2n-1)(2n+1)(2n+3)$$

$$= (4n^2-1)(2n+3)$$

$$= 8n^3+12n^2-2n-3$$

$$\therefore S_n = \sum\left(8n^3+12n^2-2n-3\right)$$

$$= 8\sum n^3+12\sum n^2-2\sum n-3n$$

$$= 8\left[\frac{n(n+1)}{2}\right]^2+12\left[\frac{n(n+1)(2n+1)}{6}\right]-2\frac{n(n+1)}{2}-3n$$

$$= \frac{n(n+1)}{2}\left[8\frac{n(n+1)}{2} + 12\frac{(2n+1)}{3} - 2\right] - 3n$$

$$= \frac{n(n+1)}{2}\left[4n^2 + 4n + 8n + 4 - 2\right] - 3n$$

$$= \frac{n(n+1)}{2}\left[4n^2 + 12n + 2\right] - 3n$$

$$= \frac{n}{2}\left[4n^3 + 12n^2 + 2n + 4n^2 + 12n + 2\right] - 3n$$

$$= \frac{n}{2}\left[4n^3 + 16n^2 + 14n + 2\right] - 3n$$

$$= n\left[2n^3 + 8n^2 + 7n + 1 - 3\right]$$

$$= n(2n^3 + 8n^2 + 7n - 2)$$

Q16. 1.4.7+2.5.8+3.6.9+……….

Solution: Here, t_n

$$= \left[1+(n-1)1\right]\left[4+(n-1)1\right]\left[7+(n-1)1\right]$$

$$= n(n+3)(n+6)$$

$$= (n^2 + 3n)(n+6)$$

$$= n^3 + 6n^2 + 3n^2 + 18n$$

$$= n^3 + 9n^2 + 18n$$

$$\therefore S_n = \sum\left(n^3 + 9n^2 + 18n\right)$$

$$= \sum n^3 + 9\sum n^2 + 18\sum n$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{9n(n+1)(2n+1)}{6} + \frac{18n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{9(2n+1)}{3} + 18\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{n^2+n+12n+6+36}{2}\right]$$

$$= \frac{n}{4}(n+1)\left(n^2 + 13n + 42\right)$$

$$= \frac{n}{4}(n+1)(n+6)(n+7))$$

Q17. 1.5.9+2.6.10+3.7.11+…….

Solution: Here, $t_n = [1+(n-1)1][5+(n-1)1][9+(n-1)1]$

$$= n(n+4)(n+8)$$

$$= (n^2+4n)(n+8)$$

$$= n^3+8n^2+4n^2+32n$$

$$= n^3+12n^2+32n$$

$$\therefore S_n = \sum(n^3+12n^2+32n)$$

$$= \sum n^3 + 12\sum n^2 + 32\sum n$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{12n(n+1)(2n+1)}{6} + \frac{32n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{12(2n+1)}{3} + 32\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{n^2+n+16n+8+64}{2}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{n^2+17n+72}{2}\right]$$

$$= \frac{n}{4}(n+1)(n+8)(n+9)$$

Q18. 1.2.4+2.3.5+3.4.6+………..

Solution: Here, $t_n = [1+(n-1)1][2+(n-1)1][4+(n-1)1]$

$$= n(n+1)(n+3)$$

$$= (n^2+n)(n+3)$$

$$= n^3+3n^2+n^2+3n$$

$$= n^3+4n^2+3n$$

$$\therefore S_n = \sum(n^3+4n^2+3n)$$

$$= \sum n^3 + 4\sum n^2 + 3\sum n$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{4n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{4(2n+1)}{3} + 3\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+3n+16n+8+18}{6}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+19n+26}{6}\right]$$

$$= \frac{n}{12}(n+1))\left(3n^2+6n+13n+26\right)$$

$$= \frac{n}{12}(n+1)(n+2)(3n+13)$$

Q19. 1.3.4+2.4.5+3.4.6+………..

Solution: Here, $t_n = [1+(n-1)1][3+(n-1)1][4+(n-1)1]$

$$= n(n+2)(n+3)$$

$$= (n^2+2n)(n+3)$$

$$= n^3+3n^2+2n^2+6n$$

$$= n^3+5n^2+6n$$

$$\therefore S_n = \sum\left(n^3+5n^2+6n\right)$$

$$= \sum n^3 + 5\sum n^2 + 6\sum n$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{5n(n+1)(2n+1)}{6} + \frac{6n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{5(2n+1)}{3} + 6\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+3n+20n+10+36}{6}\right]$$

$$= \frac{n(n+1)}{2}\left[3n^2+23n+46\right]$$

Q20. 1.3.5+2.4.6+3.5.7+………..

Solution: Here, $t_n = [1+(n-1)1][3+(n-1)1][5+(n-1)1]$

$$= n(n+2)(n+4)$$

$$= (n^2+2n)(n+4)$$

$$= n^3+4n^2+2n^2+8n$$

$$= n^3+6n^2+8n$$

$$\therefore S_n = \sum(n^3+6n^2+8n)$$

$$= \sum n^3 + 6\sum n^2 + 8\sum n$$

$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{6n(n+1)(2n+1)}{6} + \frac{8n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + 2(2n+1) + 8\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{n^2+n+8n+4+16}{2}\right]$$

$$= \frac{n(n+1)}{4}\left[n^2+9n+20\right]$$

$$= \frac{n(n+1)}{4}(n+4)(n+5)$$

Q21. $1^2 + \frac{1^2+2^2}{2} + \frac{1^2+2^2+3^2}{3} + \ldots\ldots\ldots\ldots$

Solution: Here, $t_n = \frac{1^2+2^2+3^2+\ldots\ldots\ldots+n^2}{n}$

$$= \frac{n(n+1)(2n+1)}{6n}$$

$$= \frac{1}{6}\left[2n^2+n+2n+1\right]$$

$$= \frac{n^2}{3} + \frac{3n}{6} + \frac{1}{6} \quad = \frac{n^2}{3} + \frac{n}{2} + \frac{1}{6}$$

$$\therefore S_n = \sum\left(\frac{n^2}{3}+\frac{n}{2}+\frac{1}{6}\right)$$

$$= \frac{1}{3}\sum n^2 + \frac{1}{2}\sum n + \frac{1}{6}n$$

$$= \frac{n(n+1)(2n+1)}{3\times 6} + \frac{n(n+1)}{2\times 2} + \frac{1}{6}n$$

$$= \frac{n(n+1)}{2}\left[\frac{2n+1}{9}+\frac{1}{2}\right] + \frac{1}{6}n$$

$$= \frac{n(n+1)}{2}\left[\frac{4n+2+9}{18}\right] + \frac{1}{6}n$$

$$= \frac{n(n+1)}{2}\left(\frac{4n+11}{18}\right) + \frac{1}{6}n$$

$$= \frac{n}{36}[4n^2 + 11n + 4n + 11 + 6]$$

$$= \frac{n}{36}\left[4n^2 + 15n + 17\right]$$

Q22. $1^2 + \left(1^2+2^2\right) + \left(1^2+2^2+3^2+......\right) + ...$

Solution: Here, $t_n = 1^2 + 2^2 + 3^2 + ... + n^2$

$$= \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{1}{6}\left[\left(n^2+n\right)(2n+1)\right]$$

$$= \frac{1}{6}\left[\left(2n^3+n^2+2n^2+n\right)\right]$$

$$= \frac{1}{6}\left[\left(2n^3+3n^2+n\right)\right]$$

$$= \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

$$\therefore S_n = \sum\left(\frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n\right)$$

$$= \frac{1}{3}\sum n^3 + \frac{1}{2}\sum n^2 + \frac{1}{6}\sum n$$

$$= \frac{1}{3}\left[\frac{n(n+1)}{2}\right]^2 + \frac{1}{2}\times\frac{n(n+1)(2n+1)}{6} + \frac{1}{6}\times\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{12}\left[n(n+1)+(2n+1)+1\right]$$

$$= \frac{n(n+1)}{12}\left[n^2+n+2n+1+1\right]$$

$$= \frac{n(n+1)}{12}\left[n^2+3n+2\right]$$

$$= \frac{n(n+1)(n+1)(n+2)}{12}$$

$$= \frac{n(n+1)^2(n+2)}{12}$$

Q23. $\frac{1^3}{1} + \frac{1^3+2^3}{1+2} + \frac{1^3+2^3+3^3}{1+2+3} + ...$

Solution: Here, $t_n = \frac{1^3+2^3+3^3+.......+n^3}{1+2+3+......+n}$

$$= \frac{\left[\frac{n(n+1)}{2}\right]^2}{\left[\frac{n(n+1)}{2}\right]}$$

$$= \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{1}{2}n$$

$$\therefore S_n = \sum\left(\frac{1}{2}n^2 + \frac{1}{2}n\right)$$

$$= \frac{1}{2}\sum n^2 + \frac{1}{2}\sum n$$

$$= \frac{1}{2} \times \frac{n(n+1)(2n+1)}{6} + \frac{1}{2} \times \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{4}\left[\frac{2n+1}{3} + 1\right]$$

$$= \frac{n(n+1)}{4}\left[\frac{2n+1+3}{3}\right]$$

$$= \frac{n(n+1)}{4} \times \frac{(2n+4)}{3}$$

$$= \frac{n(n+1)(n+2)}{6}$$

Q24. $\frac{1^2}{2} + \frac{1^2+2^2}{3} + \frac{1^2+2^2+3^2}{4} + \ldots\ldots\ldots\ldots$

Solution: Here, $t_n = \frac{1^2+2^2+\ldots\ldots\ldots+n^2}{n+1}$

$$= \frac{n(n+1)(2n+1)}{6(n+1)}$$

$$= \frac{2n^2+n}{6}$$

$$= \frac{1}{3}n^2 + \frac{1}{6}n$$

$$\therefore S_n = \sum\left(\frac{1}{3}n^2 + \frac{1}{6}n\right)$$

$$= \frac{1}{3}\sum n^2 + \frac{1}{6}\sum n$$

$$= \frac{1}{3} \times \frac{n(n+1)(2n+1)}{6} + \frac{1}{6} \times \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{6}\left[\left(\frac{2n+1}{3}\right) + \frac{1}{2}\right]$$

$$= \frac{n(n+1)}{6} \frac{4n+2+3}{6}$$

$$= \frac{n}{36}(n+1)(4n+5)$$

Q25. 2+(2+5)+(2+5+8)+……….

Solution: Here, $t_n = 2 + 5 + 8 + \ldots\ldots$ to n terms

$$= \frac{n}{2}[2.2 + (n-1).3]$$

$$= \frac{n}{2}[4 + 3n - 3]$$

$$= \frac{n}{2}(3n+1)$$

$$= \frac{3}{2}n^2 + \frac{1}{2}n$$

$$\therefore S_n = \sum\left(\frac{3}{2}n^2 + \frac{1}{2}n\right)$$

$$= \frac{3}{2}\sum n^2 + \frac{1}{2}\sum n$$

$$= \frac{3}{2} \times \frac{n(n+1)(2n+1)}{6} + \frac{1}{2} \times \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{4}[2n+1+1]$$

$$= \frac{n(n+1)}{4}[2n+2]$$

$$= \frac{n}{2}(n+1)^2$$

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